Infinitely Many Twin Primes
Proof

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Abstract

After dissecting the mechanics of locating valid twin primes, I was able to establish a Proof through contradiction. I start by creating a table to easily display the potential list of twin primes. Using an elimination matrix scheme, I systematically remove twin prime candidates from the list if either half of the pair are multiples of an already known 'Prime Number'. Multiples of prime numbers, primes squared and primes multiplied by other primes are not prime numbers themselves (examples 5*5=25; 5*7=35; 5*11=55; 7*7=49; 7*11=77; and so on). It's an easy approach with repeatable patterns for each prime number. It quickly becomes obvious that these elimination patterns are repeating for all non-prime removals. All these elimination patterns are of the form remove-skip(n)-remove-skip(m)...repeated to infinity. Note that n+m+2 is the prime number. The first non-prime removal for any prime is in essense that prime^2 (prime squared). A prime number squared will always fall into the sixth column (the column starting with 7)! Further, two adjacent patterns will slightly overlap if those two primes form a twin prime pair. I then proceed to make the 'silly' assumption that there will be no potential twin prime candidates in the initial skip(n) plus skip(m) regions for the two overlapped twin primes (entire initial pattern for a given twin prime pair) in this elimination matrix. If we assume that 11 & 13 are the last twin primes possible, we would have to make the assumption that there are no twin primes candidates in the elimination overlapped pattern regions for either prime 11 or 13 combined at minimum. The contradiction arises because we can show that there is always at least one twin prime pair in this combined/overlapped region. As long as there are infinitely many primes there will be infinitely many twin primes. Euclid proved there are infinitely many primes with his proof. I have simply extended his proof into my own.

Introduction

What are twin prime pairs one might ask. Simply put, they are two prime numbers separated by two...5&7; 11&13; 17&19; and so forth. Note that twin prime candidates are separated by '6'...5+6=11; 7+6=13. 11+6=17; 13+6=19. That's the idea. 23&25 is a candidate pair in this list but 25 is not prime! It is a multiple of prime number 5. So the 23&25 candidate is eliminated. 47&49 are eliminated as well because 49 is a multiple of prime 7. 77&79 likewise because 77 is a multiple of prime 7. There is a very systematic removal/elimination process that can be used against itself to show that there will always be at least one twin prime pair left in contention and bypassed by the elimination of multiples of primes. This process is clearly discussed below. Combinatorics plays a role in this proof.

Infinitely Many 'Primes' – Euclid's Theorem

One can not look at solving the Twin Primes Conjecture until they have explored the proof of the simpler Infinitely Many Primes Conjecture. This of course is a well established theory – theorem. It is a straight forward inductive proof using contradiction. My potential proof for Infinitely Many Twin Primes relies on there being Infinitely Many Primes; which makes perfect sense. There must be an infinite supply of them.

So without further 'ado' here is Euclid's Proof for Infinitely Many Primes. He begins by assuming there are a finite number of primes. This simply means P1, P2, P3, P4,...Pn. So if there are a known finite number of primes one should not be able to create a Pn+1 that is greater than Pn. Euclid realized that if he multiplied all the known finite primes together and added one to that total he could have a potentially larger prime. And that larger potential prime would not be divisible by any of the 'finite' known primes – hence his adding 1 to ensure that is the case. If this potential new large prime is infact a prime it should only be divisible by itself and 1. In some cases this yields a new prime, but not always. He then realized that if that new potential prime was not actually a prime, then it had to have a prime factor that is larger than our known finite prime subset. So we end up having two cases where the new number is a 'prime' or it has a factor 'prime' larger than the finite set of primes. In both cases the new prime is larger than 'Pn', which is contradictory to his initial assumption...so there must be an infinite supply of primes. Well done Euclid!

Locating Primes

The following chart makes it much easier to visualize the potential primes and their relationship to one another...hence my preferred layout. You can quickly see where this twin prime conjecture originates.

					1
2	3	4	5	6	7
8	9	10	11	12	13
14	15	16	17	18	19
20	21	22	23	24	25
26	27	28	29	30	31
32	33	34	35	36	37
38	39	40	41	42	43
44	45	46	47	48	49
50	51	52	53	54	55
56	57	58	59	60	61
62	63	64	65	66	67
68	69	70	71	72	73
74	75	76	77	78	79
80	81	82	83	84	85
86	87	88	89	90	91
92	93	94	95	96	97
98	99	100	101	102	103
104	105	106	107	108	109
110	111	112	113	114	115
116	117	118	119	120	121
122	123	124	125	126	127
128	129	130	131	132	133
134	135	136	137	138	139
140	141	142	143	144	145
146	147	148	149	150	151
152	153	154	155	156	157
158	159	160	161	162	163
164	165	166	167	168	169
170	171	172	173	174	175
176	177	178	179	180	181
182	183	184	185	186	187
188	189	190	191	192	193
194	195	196	197	198	199
200	201	202	203	204	205
206	207	208	209	210	211
212	213	214	215	216	217
218	219	220	221	222	223
224	225	226	227	228	229
230	231	232	233	234	235
236	237	238	239	240	241

This table has '6' columns...so 6 is a very important number. Each column starts with the lowest whole number...note the first column starts with 2. Because of the terminolgy of prime numbers – it must be divisible by itself and 1 (only) – excludes 1 from being prime. The next element in each column is found by adding 6 to the previous element over and over. There's that pesky 6 again! Column 1, 3 & 5 are even numbers; 2, 4 & 6 are odd.

So, considering the numbers in row two for each column, technically the first (excluding 1 from being prime so we can ignore), we have 2, 3, 4, 5, 6, and 7. Of those 2, 3, 5 and 7 are clearly prime. They are divisible by themselves and 1 only. Those are the only two factors allowed.

The remainder of columns 1, 3 and 5 are even numbers that are divisible by '2' so these all have an additional factor of '2'. We can exclude them from further consideration. The remainder of column 2 – starting with prime 3 can also be excluded from further consideration since the remaining elements in that column are all multiples of '3'. Adding multiple of 6 to '3' gives multiples of 3 in all cases so all those numbers have an additional factor of '3'. Exclude the remainder of column 2.

Having excluded columns 3 and 5 outright (they are even numbers divisible by 2); with the remainder of columns 1 and 2, we have in effect eliminated all remaining even numbers greater than prime '2' and 1/3 of the odd numbers which happen to be divisible by prime '3'.

This leaves two columns for consideration for additional prime numbers. The definition of twin primes are those separated by two. For example 5 and 7 are twin primes. The placement of these two columns ensures that additonal twin primes occur in the same row of both those columns so long as they are both prime to start. Non-yellow highlights are not prime numbers. We have the following reduced chart:

5	7
11	13
17	19
23	25
29	31
35	37
41	43
47	49
53	55
59	61
65	67
71	73
77	79
83	85
89	91
95	97
101	103
107	109
113	115
119	121
125	127
131	133
137	139
143	145

As Euclid has so kindly provided a proof that these two columns go on to infinity is visually clear; and that there should be twin primes approaching infinity as well. Note that the first 'ones-place' digit in each of these two columns is a repeating pattern 5, 1, 7, 3, 9, 5, 1, 7, 3, 9, ... These patterns are offset by exactly 2; 7 in first column is two ahead of 7 in the next column. We'll use this to our advantage later in this proof.

Elinimation of Twin Prime Candidates (Identifying Non-Primes)

PT1	PT2		5		7		11		13		17		19		23		29		31		37		41		43		47		53		59		61	
5	7	X		X		X		X		X		X		X		X		X		X		X		X		X		X		X		X		X
11	13	X		X		X		X		X		X		X		X		X		X		X		X		X		X		X		X		X
17	19	X		X		X		X		X		X		X		X		X		X		X		X		X		X		X		X		X
23	25	X																																
29	31	X		X		X		X		X		X		X		X		X		X		X		X		X		X		X		X		X
35	37	X																																
41	43	X		X		X		X		X		X		X		X		X		X		X		X		X		X		X		X		X
47	49	X		X																														
53	55	X																																
59	61	X		X		X		X		X		X		X		X		X		X		X		X		X		X		X		X		X
65	67	X																																
71	73	X		X		X		X		X		X		X		X		X		X		X		X		X		X		X		X		X
77	79	X		X																														
83	85	X																																
89	91	X		X																														
95	97	X																																
101	103	X		X		X		X		X		X		X		X		X		X		X		X		X		X		X		X		X
107	109	X		X		X		X		X		X		X		X		X		X		X		X		X		X		X		X		X
113	115	X																																
119	121	X		X																														
125	127	X																																
131	133	X		X																														
137	139	X		X		X		X		X		X		X		X		X		X		X		X		X		X		X		X		X
143	145	X																																
149	151	X		X		X		X		X		X		X		X		X		X		X		X		X		X		X		X		X
155	157	X																																
161	163	X		X																														
167	169	X		X		X		X																										
173	175	X																																
179	181	X		X		X		X		X		X		X		X		X		X		X		X		X		X		X		X		X
185	187	X																																
191	193	X		X		X		X		X		X		X		X		X		X		X		X		X		X		X		X		X
197	199	X		X		X		X		X		X		X		X		X		X		X		X		X		X		X		X		X
203	205	X																																
209	211	X		X		X																												
215	217	X																																
221	223	X		X		X		X																										
227	229	X		X		X		X		X		X		X		X		X		X		X		X		X		X		X		X		X
233	235	X																																
239	241	X		X		X		X		X		X		X		X		X		X		X		X		X		X		X		X		X
245	247	X																																
251	253	X		X		X																												
257	259	X		X																														
263	265	X																																
269	271	X		X		X		X		X		X		X		X		X		X		X		X		X		X		X		X		X
275	277	X																																
281	283	X		X		X		X		X		X		X		X		X		X		X		X		X		X		X		X		X
287	289	X		X																														
293	295	X																																
299	301	X		X																														
305	307	X																																
311	313	X		X		X		X		X		X		X		X		X		X		X		X		X		X		X		X		X
317	319	X		X		X																												
323	325	X																																

I believe the above chart is self explanatory but here goes anyways. You can see the two prime towers PT1 and PT2 on the left. These are clearly the two remaining columns from earlier discussions. I start by marking every pair as a potential twin prime with 'X'. Prime numbers '2' and '3' have resulted in these two columns 'only' for consideration. The next prime number is '5'; so we pass through and mark every pair where either PT1 or PT2 are divisible by prime '5'. There is a repeating pattern. The next column of 'X's shows the remaining potential twin prime pairs that do not contain a multiple of 5. We can clearly see that two fifths (40%) of the potential primes have been eliminated.

The remainder of the chart shows multiples of each additional prime number being pulled out...each following it's own distinct repeatable pattern. Each additional prime as they grow in magnatude start to remove potential twin primes candidates a bit further down the chart than all prior smaller primes. The intervals in these patterns grow slightly larger as well. This allows for skipping over 'clumps' that seem to be developing. Twin primes appear to be grouping/clumping. We need not concentrate on this clumping since it is not required for this proof. Just an interesting fact to point out.

Eliminating '5' results in the elimination pattern 'exexx'. Eliminating '7' results in the elimination pattern 'exxxxex'. Eliminating '11' results in the elimination pattern 'exxxexxxxxx'. Note the pattern length is the prime.

Let's take a peek further down this chart to see what is happening:

767	769	X		X		X		X																										
773	775	X																																
779	781	X		X		X																												
785	787	X																																
791	793	X		X																														
797	799	X		X		X		X		X																								
803	805	X																																
809	811	X		X		X		X		X		X		X		X		X		X		X		X		X		X		X		X		X
815	817	X																																
821	823	X		X		X		X		X		X		X		X		X		X		X		X		X		X		X		X		X
827	829	X		X		X		X		X		X		X		X		X		X		X		X		X		X		X		X		X
833	835	X																																
839	841	X		X		X		X		X		X		X		X																		
845	847	X																																
851	853	X		X		X		X		X		X		X																				
857	859	X		X		X		X		X		X		X		X		X		X		X		X		X		X		X		X		X
863	865	X																																
869	871	X		X		X																												
875	877	X																																
881	883	X		X		X		X		X		X		X		X		X		X		X		X		X		X		X		X		X
887	889	X		X																														
893	895	X																																
899	901	X		X		X		X		X																								
905	907	X																																
911	913	X		X		X																												
917	919	X		X																														
923	925	X																																
929	931	X		X																														
935	937	X																																
941	943	X		X		X		X		X		X		X																				
947	949	X		X		X		X																										

This is pretty cool. Here's another snippet from further down still:

3119	3121	X		X		X		X		X		X		X		X		X		X		X		X		X		X		X		X		X
3125	3127	X																																
3131	3133	X		X		X		X																										
3137	3139	X		X		X		X		X		X		X		X		X		X		X		X										
3143	3145	X																																
3149	3151	X		X		X		X		X		X		X																				
3155	3157	X																																
3161	3163	X		X		X		X		X		X		X		X																		
3167	3169	X		X		X		X		X		X		X		X		X		X		X		X		X		X		X		X		X
3173	3175	X																																
3179	3181	X		X		X																												
3185	3187	X																																
3191	3193	X		X		X		X		X		X		X		X		X																
3197	3199	X		X																														
3203	3205	X																																
3209	3211	X		X		X		X																										
3215	3217	X																																
3221	3223	X		X		X																												
3227	3229	X		X																														
3233	3235	X																																
3239	3241	X		X																														
3245	3247	X																																
3251	3253	X		X		X		X		X		X		X		X		X		X		X		X		X		X		X		X		X
3257	3259	X		X		X		X		X		X		X		X		X		X		X		X		X		X		X		X		X
3263	3265	X																																
3269	3271	X		X																														
3275	3277	X																																
3281	3283	X		X																														
3287	3289	X		X		X																												
3293	3295	X																																
3299	3301	X		X		X		X		X		X		X		X		X		X		X		X		X		X		X		X		X
3305	3307	X																																
3311	3313	X		X																														
3317	3319	X		X		X		X		X		X		X		X		X																
3323	3325	X																																
3329	3331	X		X		X		X		X		X		X		X		X		X		X		X		X		X		X		X		X
3335	3337	X																																
3341	3343	X		X		X		X																										
3347	3349	X		X		X		X		X																								
3353	3355	X																																
3359	3361	X		X		X		X		X		X		X		X		X		X		X		X		X		X		X		X		X
3365	3367	X																																
3371	3373	X		X		X		X		X		X		X		X		X		X		X		X		X		X		X		X		X
3377	3379	X		X		X																												
3383	3385	X																																
3389	3391	X		X		X		X		X		X		X		X		X		X		X		X		X		X		X		X		X
3395	3397	X																																
3401	3403	X		X		X		X		X		X																						

Let's take a closer look at the elimination patterns. There is a pattern in those patterns that we will be able to use later to formulate a proof by contradiction much like Euclid did for proving infinately many primes.

In the following table, I display the primes 5 up to 97. You will notice that any prime that fell in column 4, I've highlighted in yellow. The remainder fall in column 6. These two columns form the potential twin primes. For those yellow primes the first skip is exactly half the second skip (exactly 1/3 of total skipped; the second skip is 2/3 of the total). For the non-yellow primes the second skip is the smaller of the two with the first skip being

(2*second skip + 2). For all intents and purpose the split is roughly 1/3 vs 2/3 of the total.

Prime		Eliminate 1	Skip n	Eliminate 1	Skip m
5		e	1	e	2
7		e	4	e	1
11		e	3	e	6
13		e	8	e	3
17		e	5	e	10
19		e	12	e	5
23		e	7	e	14
29		e	9	e	18
31		e	20	e	9
37		e	24	e	11
41		e	13	e	26
43		e	28	e	13
47		e	15	e	30
53		e	17	e	34
59		e	19	e	38
61		e	40	e	19
67		e	44	e	21
71		e	23	e	46
73		e	48	e	23
79		e	52	e	25
83		e	27	e	54
89		e	29	e	58
97		e	64	e	31

This is all the preliminary analysis that is required before jumping into the proof.

<u>Formulating the Proof</u>

In the above table we can easily see that the first skip (skip(n)) alternates between being the smallest of the two; to the largest. No nice convenient clean pattern emerges. Let's make an assumption to simplify the elimination process. Let's assume that the smaller of the two skips is always first. If we make this simple assumption we can now show that in ideal circumstances with the least amount of overlap, we can eliminate all the potential twin primes that occur in the first skip region of a given prime (and that is just exactly; with none to spare; if there is an overlap there will remain at least one twin prime that will not be eliminated). This is easily show by overlapping 5 onto 7.

The following tables display this reasoning. You can easily see in the follow up table where I overlap 5 & 7, and can no longer eliminate all the potential twin primes. This is an obvious overlap that gives the desired results. Not all overlaps will will yield similar results.

So one can now see that we may be onto something here. Continue onto the third table below that has some relaxed assumptions and based more in reality. This means that I do not consider all smaller skip(n) to occur first but instead mix them up. This manipulation makes use of combinatorics to show that not all candidates can be eliminated.

It also important to note that we've only been looking at the initial skip region of one half a twin prime and not the overall combined overlapping region of the halves. That is introduced after looking at the third chart.

97																						
	89																					
		83																				
			79																			
				73																		
					71																	
						67																
							61															
								59														
									53													
										47												
											43											
												41										
													37									
														31								
															29							
																23						
																	19					
																		17				
																			13			
																				11		
																					7	
																						5
																	19		13		7	
														31		23		17		11		5
											43		37		29							
										47		41										
							61		53													
				73		67		59														
			79		71																	
		83																				
	89																					
97																						

97																						
	89																					
		83																				
			79																			
				73																		
					71																	
						67																
							61															
								59														
									53													
										47												
											43											
												41										
													37									
														31								
															29							
																23						
																	19					
																		17				
																			13			
																				11		
																					7	5
																	19		13		7	5
														31		23		17		11		
											43		37		29							
										47		41										
							61		53													
				73		67		59														
			79		71																	
		83																				
	89																					
97																						

Let's ignore our original assumption that all the shorter skips occur first. But let's keep minimal overlap to show that there will be at least one twin prime in all the first skip areas though infinity...

97																						
	89																					
		83																				
			79																			
				73																		
					71																	
						67																
							61															
								59														
									53													
										47												
											43											
												41										
													37									
														31								
															29							
																23						
																	19					
																		17				
																			13			
																				11		
																					7	
																						5
																23		17		11		5
															29							
										47		41									7	
									53													
								59											13			
					71																	
		83															19					
	89																					
														31								
													37									
											43											
							61															
						67																
				73																		
			79																			
97																						

Displayed in the above fashion you can easily see that there will always be at least one available twin prime. If there is overlap which undoubtedly there is, the potential number of twin primes can only increase. So, this is proof that there are infinitely many twin primes. And those potential twin primes occur near the prime^2, The next three charts show the entire region for a prime using combinatorics to show that potentials exist in all cases of optimizing. The first of these charts show all short skips first optimized using combinatorics. Potentials are shown in green. These do not include overlapped of two primes and that interplay.

97																						
	89																					
		83																				
			79																			
				73																		
					71																	
						67																
							61															
								59														
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																		17				
																			13			
																				11		
																					7	
																						5
																	19		13		7	
														31		23		17		11		5
											43		37		29							
										47		41										
							61		53													5
				73		67		59													7	
			79		71																	5
		83																			7	
	89																			11		
97																			13			5
																						5
																		17		11	7	
																	19		13			
																					7	5
																23						5
																		17				
																	19			11	7	5
															29						7	5
														31					13			
																				11		
																23						5
																			13		7	5
													37									
																					7	
																		17				5
												41								11		
											43				29							5
														31			19					
																					7	
										47										11		5
																		17	13		7	
																						5
																	19					
									53				37			23			13			5
																					7	
																				11		5
																					7	
								59				41										5
							61				43									11		
																		17				5
																23					7	
																			13			
																					7	5
						67				47					29							
																	19					5
																		17	13	11		
					71									31								
				73																	7	5
																				11	7	5
									53								19					
			79																			5
															29							
																			13		7	5
		83														23						
														31				17		11	7	
								59					37									5
							61												13			
																						5
	89																			11		
																					7	
																		17				5
																23	19				7	5
												41										5
						67																
97											43								13	11		5

The second of these charts shows an even mix of shorter/longer regions optimized.

97																						
	89																					
		83																				
			79																			
				73																		
					71																	
						67																
							61															
								59														
									53													
										47												
											43											
												41										
													37									
														31								
															29							
																23						
																	19					
																		17				
																			13			
																				11		
																					7	
																						5

																23		17		11		5
															29							
										47		41									7	
									53													5
								59											13		7	
					71																	5
		83															19					
	89																			11		
																			13			5
																					7	
																						5
														31				17		11	7	
																	19					
																						5
													37									
																23						5
											43										7	
																		17	13			
																				11	7	5

															29							5
														31					13			
																				11		
																23					7	5
							61															
																	19				7	5
													37									
						67																
																		17				5
				73								41								11		
											43				29				13		7	5
																	19					
			79																		7	
										47										11		5
																		17	13			
																						5

																					7	
									53							23						5
																					7	
																				11		5
97																						
														31								
								59				41							13			5
							61										19			11	7	
																		17				5
																23					7	
																			13			
																						5
						67				47					29							
																	19					5
													37					17		11	7	
					71									31								
				73																	7	5

																				11		5
									53										13			
			79																		7	5
											43				29							
																			13		7	5
		83														23						
																		17		11		
								59					37				19					5

																					7	5
	89																			11		
																					7	
																		17				5
																23	19		13			
												41										5

																					7	
97											43			31					13	11		5

And the third shows longer regions first with optimiztion.

97																						
	89																					
		83																				
			79																			
				73																		
					71																	
						67																
							61															
								59														
									53													
										47												
											43											
												41										
													37									
														31								
															29							
																23						
																	19					
																		17				
																			13			
																				11		
																					7	
																						5
																						5
																					7	
																				11		5
																			13		7	
																		17				
																	19					5
																23				11		
																			13			5
																					7	
															29							
														31				17			7	5
																	19					
																						5
													37							11		
												41				23						
											43										7	5
										47									13			
																				11	7	5
									53						29							
														31					13			5
																		17				
								59													7	5
							61															
																	19			11	7	
													37									5
						67																
					71													17				5
				73								41								11		
											43					23			13		7	
																	19					5
			79																		7	
		83								47												5
																			13			
	89																			11		5
																					7	
									53							23						5
															29			17			7	
																				11		
97																						5
														31								
								59											13			5
							61										19				7	
																		17				
																					7	5
																			13	11		
																						5
						67									29							
																	19					
													37							11	7	5
					71									31								
				73												23					7	5
												41						17	13			5
			79																	11	7	5
											43											
																			13		7	
		83														23						5
																		17		11		
													37				19					5
										47												
																					7	
	89																					5
																					7	
															29							5
																	19		13	11		
												41										
																						5
																					7	
97									53		43			31				17	13	11		5

So we can potentially stop here knowing that no matter what the combination of the combined elimination patterns there will always be at least one in the entire elimination region for any prime. Now lets review actual overlaps regions of a twin prime pair...with no guessing and optimiztion but actual data to show that our combinatorics and optimization hold true in all cases.

Starting at twin prime pair '5, 7' here is the result:

23	25		x	
29	31			
35	37		x	
41	43			
47	49			x
53	55		x	
59	61			
65	67		x	
71	73			
77	79			x
83	85		x	
89	91			x

Here's '11, 13':

119	121			x	x	
125	127		x			
131	133			x		
137	139					
143	145		x		x	
149	151					
155	157		x			
161	163			x		
167	169					x
173	175		x	x		
179	181					
185	187		x		x	
191	193					
197	199					
203	205		x	x		
209	211				x	
215	217		x	x		
221	223					x
227	229					
233	235		x			
239	241					
245	247		x	x		x

And '17, 19':

287	289			x			x	
293	295		x					
299	301			x		x		
305	307		x					
311	313							
317	319				x			
323	325		x			x	x	
329	331			x				
335	337		x					
341	343			x	x			
347	349							
353	355		x					
359	361							x
365	367		x					
371	373			x				
377	379					x		
383	385		x	x	x			
389	391						x	
395	397		x					
401	403					x		
407	409				x			
413	415		x	x				
419	421							
425	427		x	x			x	
431	433							
437	439							x
443	445		x					
449	451				x			
455	457		x	x		x		
461	463							
467	469			x				
473	475		x		x			x

Here's '29, 31' since we skipped over '23, 25' sinced it is not a twin prime:

839	841									×	
845	847		×	×	×	×					
851	853								×		
857	859										
863	865		×								
869	871				×	×					
875	877		×	×							
881	883										
887	889			×							
893	895		×					×			
899	901						×			×	
905	907		×								
911	913				×						
917	919			×							
923	925		×			×					
929	931			×				×			
935	937		×		×		×				
941	943								×		
947	949					×					
953	955		×								
959	961			×							×
965	967		×								
971	973			×							
977	979				×						
983	985		×								
989	991								×		
995	997		×								
1001	1003			×	×	×	×				
1007	1009							×			
1013	1015		×	×						×	
1019	1021										
1025	1027		×			×					
1031	1033										
1037	1039						×				
1043	1045		×	×	×			×			
1049	1051										
1055	1057		×	×							
1061	1063										
1067	1069				×						
1073	1075		×							×	
1079	1081					×			×		
1085	1087		×	×							×
1091	1093										
1097	1099			×							
1103	1105		×			×	×				
1109	1111				×						
1115	1117		×								
1121	1123							×			
1127	1129			×					×		
1133	1135		×		×						
1139	1141			×			×				
1145	1147		×								×

Continuing on to '41, 43' because '35, 37' is not a twin prime:

1679	1681								×				×	
1685	1687		×	×										
1691	1693							×						
1697	1699													
1703	1705		×		×	×					×			
1709	1711									×				
1715	1717		×	×			×							
1721	1723													
1727	1729			×	×	×		×						
1733	1735		×											
1739	1741											×		
1745	1747		×											
1751	1753						×							
1757	1759			×										
1763	1765		×										×	
1769	1771			×	×				×	×				
1775	1777		×											
1781	1783					×								
1787	1789													
1793	1795		×		×									
1799	1801			×										
1805	1807		×			×		×						
1811	1813			×								×		
1817	1819						×		×					
1823	1825		×											
1829	1831										×			
1835	1837		×		×									
1841	1843			×				×						
1847	1849													×
1853	1855		×	×			×							
1859	1861				×	×								
1865	1867		×											
1871	1873													
1877	1879													
1883	1885		×	×		×				×				
1889	1891										×			
1895	1897		×	×										
1901	1903				×									
1907	1909								×					
1913	1915		×											
1919	1921						×	×						
1925	1927		×	×	×								×	
1931	1933													
1937	1939			×		×								
1943	1945		×							×				
1949	1951													
1955	1957		×				×	×	×					
1961	1963					×						×		
1967	1969			×	×									
1973	1975		×											
1979	1981			×										
1985	1987		×											
1991	1993				×									
1997	1999													
2003	2005		×											
2009	2011			×									×	
2015	2017		×			×					×			
2021	2023			×			×							×
2027	2029													
2033	2035		×		×			×				×		
2039	2041					×								
2045	2047		×						×					
2051	2053			×										
2057	2059				×		×			×				
2063	2065		×	×										
2069	2071							×						
2075	2077		×								×			
2081	2083													
2087	2089													
2093	2095		×	×		×			×					
2099	2101				×									
2105	2107		×	×										×

And here is one final to hammer the point home. '59, 61' skipping '47, 47' and '53, 55' both of which are not twin prime pairs:

3479	3481			×													×	
3485	3487		×		×		×						×					
3491	3493			×														
3497	3499					×												
3503	3505		×								×							
3509	3511				×					×								
3515	3517		×					×				×						
3521	3523			×		×												
3527	3529																	
3533	3535		×	×														
3539	3541																	
3545	3547		×															
3551	3553				×		×	×								×		
3557	3559																	
3563	3565		×	×					×		×							
3569	3571													×				
3575	3577		×	×	×	×												
3581	3583																	
3587	3589						×					×						
3593	3595		×															
3599	3601					×											×	
3605	3607		×	×														
3611	3613								×									
3617	3619			×	×										×			
3623	3625		×							×								
3629	3631							×										
3635	3637		×															
3641	3643				×													
3647	3649			×									×					
3653	3655		×			×	×							×				
3659	3661			×														
3665	3667		×					×										
3671	3673																	
3677	3679					×												
3683	3685		×		×					×								
3689	3691			×			×				×							
3695	3697		×															
3701	3703			×					×									
3707	3709				×													
3713	3715		×												×			
3719	3721																	×
3725	3727		×															
3731	3733			×		×							×					
3737	3739											×						
3743	3745		×	×				×										
3749	3751				×				×		×							
3755	3757		×			×	×											
3761	3763															×		
3767	3769																	
3773	3775		×	×	×													
3779	3781							×										
3785	3787		×	×														
3791	3793						×											
3797	3799									×								
3803	3805		×															
3809	3811					×						×						
3815	3817		×	×	×													
3821	3823																	
3827	3829			×										×				
3833	3835		×			×											×	
3839	3841				×				×									
3845	3847		×															
3851	3853																	
3857	3859			×			×	×		×								
3863	3865		×															
3869	3871			×												×		
3875	3877		×								×							
3881	3883				×													
3887	3889					×			×									
3893	3895		×				×	×					×					
3899	3901			×											×			
3905	3907		×		×													
3911	3913			×		×								×				
3917	3919																	
3923	3925		×															
3929	3931																	
3935	3937		×								×							
3941	3943			×														
3947	3949				×													
3953	3955		×	×													×	
3959	3961						×					×						
3965	3967		×			×												×
3971	3973				×			×		×								
3977	3979								×				×					
3983	3985		×	×														
3989	3991					×												
3995	3997		×	×			×								×			
4001	4003																	
4007	4009							×										
4013	4015		×		×													
4019	4021																	
4025	4027		×	×					×									
4031	4033									×		×						
4037	4039			×	×													
4043	4045		×			×												
4049	4051																	
4055	4057		×															
4061	4063						×				×							
4067	4069			×		×												
4073	4075		×															
4079	4081			×	×											×		
4085	4087		×					×						×				×

Again we could have shown the existance of at least one candidate twin prime in the very first skip region of all primes! But to ensure a greater number of potentials I elected to include the combined overlapped regions of both halves of a twin prime pair as shown in the above tables. The following tables prove the same. Here are the actual distributions of the first 'skip' for several prime eliminations:

23	25		x
29	31		
35	37		x

The above is for '5'. You can easily see that the twin primes (29,31) appear in that first skip region.

47	49			x
53	55		x	
59	61			
65	67		x	
71	73			
77	79			x

The above is for '7'. Twin primes (59,61) & (71,73) appear in that first skip region.

119	121			x	x
125	127		x		
131	133			x	
137	139				
143	145		x		x

For '11' there is only one twin prime (137,139) in it's first skip region.

167	169					x
173	175		x	x		
179	181					
185	187		x		x	
191	193					
197	199					
203	205		x	x		
209	211				x	
215	217		x	x		
221	223					x

Prime 13

287	289			x			x
293	295		x				
299	301			x		x	
305	307		x				
311	313						
317	319				x		
323	325		x			x	x

Prime 17

359	361							x
365	367		x					
371	373			x				
377	379					x		
383	385		x	x	x			
389	391						x	
395	397		x					
401	403					x		
407	409				x			
413	415		x	x				
419	421							
425	427		x	x			x	
431	433							
437	439							x

Prime 19

527	529						x		x
533	535		x			x			
539	541			x	x				
545	547		x						
551	553			x				x	
557	559					x			
563	565		x						
569	571								
575	577		x						x

Prime 23

839	841									x
845	847		x	x	x	x				
851	853								x	
857	859									
863	865		x							
869	871				x	x				
875	877		x	x						
881	883									
887	889			x						
893	895		x					x		
899	901						x			x

Prime 29

959	961			x							x
965	967		x								
971	973			x							
977	979				x						
983	985		x								
989	991								x		
995	997		x								
1001	1003			x	x	x	x				
1007	1009							x			
1013	1015		x	x						x	
1019	1021										
1025	1027		x			x					
1031	1033										
1037	1039						x				
1043	1045		x	x	x			x			
1049	1051										
1055	1057		x	x							
1061	1063										
1067	1069				x						
1073	1075		x							x	
1079	1081					x			x		
1085	1087		x	x							x

Prime 31

1367	1369											x
1373	1375		x		x							
1379	1381			x								
1385	1387		x					x				
1391	1393			x		x						
1397	1399				x							
1403	1405		x						x			
1409	1411						x					
1415	1417		x			x						
1421	1423			x						x		
1427	1429											
1433	1435		x	x								
1439	1441				x							
1445	1447		x				x					
1451	1453											
1457	1459										x	
1463	1465		x	x	x			x				
1469	1471					x						
1475	1477		x	x								
1481	1483											
1487	1489											
1493	1495		x			x			x			
1499	1501							x				
1505	1507		x	x	x							
1511	1513						x					
1517	1519			x							x	x

Prime 37

1679	1681								x				x
1685	1687		x	x									
1691	1693							x					
1697	1699												
1703	1705		x		x	x					x		
1709	1711									x			
1715	1717		x	x			x						
1721	1723												
1727	1729			x	x	x		x					
1733	1735		x										
1739	1741											x	
1745	1747		x										
1751	1753						x						
1757	1759			x									
1763	1765		x										x

Prime 41

1847	1849													x
1853	1855		x	x			x							
1859	1861				x	x								
1865	1867		x											
1871	1873													
1877	1879													
1883	1885		x	x		x				x				
1889	1891										x			
1895	1897		x	x										
1901	1903				x									
1907	1909								x					
1913	1915		x											
1919	1921						x	x						
1925	1927		x	x	x								x	
1931	1933													
1937	1939			x		x								
1943	1945		x							x				
1949	1951													
1955	1957		x				x	x	x					
1961	1963					x						x		
1967	1969			x	x									
1973	1975		x											
1979	1981			x										
1985	1987		x											
1991	1993				x									
1997	1999													
2003	2005		x											
2009	2011			x									x	
2015	2017		x			x					x			
2021	2023			x			x							x

Prime 43

2207	2209														x
2213	2215		x												
2219	2221			x											
2225	2227		x				x								
2231	2233			x	x				x	x					
2237	2239														
2243	2245		x												
2249	2251					x									
2255	2257		x		x							x	x		
2261	2263			x			x	x			x				
2267	2269														
2273	2275		x	x		x									
2279	2281													x	
2285	2287		x												
2291	2293									x					
2297	2299				x			x							
2303	2305		x	x											x

Prime 47

Showing the primes up to 50 should be quite sufficient to show that there will always be at least 1 twin prime in the first skip region of any and all prime numbers. As the primes grow larger their initial skip regions spread out considerably.

This is the basis of my proof. In essence one can say I contradicted myself in assuming that there would be no twin primes left available in the primary skip(n) regions of all prime numbers to infinity. With some easy manipulation I easily show that there will always be at least one twin prime in every such region. And to guarantee a greater number I expanded my proof to show the overlapped regions of both halves of the twin prime pair.

Conclusion

There are a large number of aspects I considered before writing this report but there was no need to include them. They would only complicate this simplified final approach. After all, the purpose of this paper is to prove that there will be an infinite supply of twin primes. This is by far the easiest aproach I could find.

The basis of this inductive proof is through contradiction. In essence I assume that there are a finite number of twin prime candidates. I then proceed to take that last known twin prime pair and pass both of them combined through elimination matricies to show that there are at a minimum one candidate pair in that new region. Without doubt there will always be a minimum of one and thus through contradiction the proof jumps out.

It worked for Euclid's proof of infinitely many primes! It works as well for this proof of infinitely many twin primes.

Isolating a method that would lend itself to such an approach was the challenge. I believe I have accepted and defeated that challenge. Please enjoy this proof.