Proving Goldbach's strong conjecture by analyzing gaps between prime numbers and their digits

─ *Setting up new algorithms for converting an even number into the sum of two primes*─

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Abstract

The main idea of this article lies in the fact that Goldbach's strong conjecture is associated with the progression of natural integers from 0 to infinity, which results in precise gaps between prime numbers. The gap of 6 is the most regular between primes 6x + 1 on the one hand and primes 6x – 1 on the other. In this article, using the equations 3x ± 5 and analyzing the 6-based gaps between primes while determining the initial conditions that make a prime appear after or before an integer, this article argues for the truth of Goldbach's strong conjecture. Two new concepts are introduced for the first time : Goldbach's gap and Goldbach's transposition. By analyzing its key digits (units and tens), a prime number itself can lead to the conversion of an even number into two primes. A new algorithm is deduced from these results, enabling us to locate prime numbers located at equal distance from any integer, even or odd, prime or composite. This constitutes a decisive proof of Goldbach's strong conjecture, since it means that any even number can be converted into the sum of two prime numbers.

Keywords. Goldbach. Conjecture. Even. Primes. Gaps. Transposition. Algorithm. Conversion. Prime digits. Equidistance.

Introduction

One of the best-known unsolved problems in number theory is Goldbach's conjecture, which appeared in a correspondence between Christian Goldbach and Leonhard Euler in 1742 (Golabach, 1742). The Goldbach's strong conjecture states that every even number larger then 2 can be expressed as the sum of two primes. As the Goldbach's conjecture lies in the eld of number theory and its very core is prime numbers, the distribution of such numbers may be an integral part of any attempted solution to the conjecture. The prime number theorem gives an asymptotic form for the prime counting function, which counts the number of primes less than some integer n. The set of numbers primes < an integer n are denoted π(n) while the asymptotic law for the distribution of prime numbers asserts that π(n) = n/ln(n) (Atle, 1950).

As the proof of Goldbach conjecture is still out of reach, the conjecture has been extensively verified computationally, with the most recent efforts pushing the boundaries of numerical verification to unprecedented levels (Daniel, 2023; Oliveira e Silva, 2014; Sinisalo, 2013). Despite this, the formal proof of the Goldbach Conjecture remains elusive. Apart from empirical verification, countless reports of research provide pure mathematical verification of Goldbach's strong conjecture by different logical propositions (Daniel, 2024; Farkas, 2017; Hardy and Littlewood, 1923). Websites such https://www.dcode.fr/conjecture-goldbach allow conversion of evens in sums of two primes to a given limit.

This article is a continuation of efforts to understand the mathematical rules governing Goldbach's strong conjecture. It is essentially based on the analysis of the gaps between primes and the proposal of a new algorithm for the conversion of any even number > 4 into the sum of two primes.

It introduces two new concepts for the understanding of Goldbach's strong conjecture including the Goldbach's gap and Goldbach's transposition. The article also shows that the digits of prime numbers and the primes numbers themselves can be used to prove Goldbach's strong conjecture by using elementary rules of calculation. In addition, it provides a logical and infinitely reproducible line of reasoning that argues for the truth of Goldbach's strong conjecture.

1. Logical propositions on Goldbach strong conjecture and gaps between primes

1a. Meaning of Goldbach's transposition.

Lemma a. « If we calculate the Goldbach gaps for a given integer, then the same gaps will give prime numbers with another integer of the same kind (for even or ; odd multiple of 3 or not). This is what is called here the Goldbach transposition ».

Tables 1A+1B show examples of the Goldbach transposition (a concept used in this article). If E is any even > 4 and p and p' two equidistant primes such that p < E/2 and p' > E/2 then E/2 – p = p' – E/2. Note that E/2 is any integer n located between the two equidistant primes p and p' such that p — E/2 or n — p' ↔ 2 x E/2 = E = 2n = p + p'. *The Goldbach gap is any value of* ***t = E/2 – p = p' – E/2***. Do not confuse the Goldbach gap with the classical gap between two consecutive prime numbers, it represents the gap between two prime numbers p and p' equidistant at E/2 whose sum is E. Remember that equidistant primes are located at equal distance from an integer before and after. They are essential for Goldbach's strong conjecture to be true. The question is, how far apart are they from an integer? Tables 1 and 2 show that their location depends on the number: is it even or odd ? 3n or not ?

E/2 represents any integer > 2 and is either even or odd. To obtain a prime number from an even E/2, we add odd values of **t**, i.e. either values corresponding to primes in ascending order, or odd values of 3n (O3n). For an odd E/2, we obviously add 2n values in ascending order. Table 1A+1B shows the case of an even E/2 to which we add values t = primes or t = 3n (O3n). We can see that an even number 3n always gives equidistant prime numbers when t values of prime numbers are added to it, whereas a non-3n even number needs t = O3n (odd 3n) values to give prime numbers. *This is the Goldbach transposition*. Let us recall that Goldbach's transposition is symmetrical and that an addition is accompanied by a subtraction of the same quantity. For two equidistant primes result from the subtraction and addition of the same value to or from E/2.

Indeed, if the t gaps between E/2 and the equidistant primes have prime values in increasing order, all 3n numbers give equidistant primes and therefore verify the strong Goldbach conjecture (Tables 1A+1B). On the other hand, for numbers that are not multiples of 3 (or non-3n), gaps t that have 3n values are needed to obtain equidistant primes. This fact is true for all numbers that can be tested, even those with up to 500 equidistant primes or more. *The Goldbach transposition means that even numbers of the same type (3n or not) give equidistant primes with the same spacings t*. This means that the strong Goldbach conjecture is a function of the spacings between E/2 and p or p' for every even number at infinity. Given that all primes numbers are 6x ± 1 and 6n is the most frequent gap between primes that follow each other, Goldbach's transposition is occurring to infinity so that the same distribution of primes takes place with the same kind of numbers.

Table 1A. T-values of gaps between E/2 and equidistant primes. If E is any even > 4 and p and p' two equidistant primes such that p < E/2 and p' > E/2 then E/2 – p = p' – E/2 with p = E/2 – **t** and p' = E/2 + **t**. We add to an even integer either t = prime (p) or t = odd3n (O3n) to get a prime number. Equidistant primes are in bold-italic. Note that the sum of two equidistant primes = 2 x E/2 (2 x 40 ; 2 x 36 ; 2 x 50). Note E = 80 and E/2 = 40 does not give equidistant primes with t = primes except with 3 but rather it generates them with t = 3n. if t-gaps = odd 3n values, the non-3n numbers 80 and 100 do give equidistant primes.

	80			**72**			**80**				**100**		
t = p	**- ←**	**40**	**→ +**	**- ←**	**36**	**→ +**	**t = 03n**	**- ←**	**40**	**→ +**	**- ←**	**50**	**→ +**
3	***37***		***43***	33		39	3	***37***		***43***	***47***		***53***
7	33		47	***29***		***43***	9	31		49	***41***		***59***
11	29		51	25		47	21	***19***		***61***	***29***		***71***
13	27		53	23		49	27	***13***		***67***	23		77
17	23		57	***19***		***53***	33	***7***		***73***	***17***		***83***
19	21		59	17		55	39	1		79	***11***		***89***
23	17		63	***13***		***59***							
29	11		69	7		65							
31	9		71	5		77							
37	3		77										

Table 1B. Comparison between a 3n number (E = 240) and a non-3 (E = 188). We can clearly see that the 3n only give equidistant prime numbers if the t gaps have prime number values while the non-3n give them with t which have odd 3n values. Equidistant primes are in bold italic.

	188			240			188			
t = p	**- ←**	**94**	**→ +**	**- ←**	**120**	**→ +**	**t = O3N**	**- ←**	**94**	**→ +**
3	91		97	117		123	3	91		97
7	87		101	***113***		***127***	9	85		103
11	83		105	***109***		***131***	15	***79***		***109***
13	81		107	107		133	21	73		115
17	77		111	***103***		***137***	27	67		121
19	75		113	***101***		***139***	33	***61***		***127***
23	71		117	***97***		***143***	39	55		133
29	65		123	91		149	45	49		139
31	63		125	***89***		***151***	51	43		145
37	57		131	***83***		***157***	57	***37***		***151***
41	53		135	79		161	63	***31***		***157***
43	51		137	77		163	69	25		163
47	47		141	***73***		***167***	75	19		169
51	43		145	69		171	81	13		175
53	41		147	***67***		***173***	87	***7***		***181***
59	35		153	***61***		***179***	93			

The Table 2 shows the different ways to obtain Goldbach gaps and equidistant primes depending on the type of the number. Primes < E (E any even > 4) within π(E) will always give at least one couple of equidistant primes when added to or substracted of an even E that is 3n; odd 3n numbers will give them as well with non-3n numbers; while 2n are required to obtain them with odd primes or composite numbers.

These rules apply to all integers to infinity and are therefore responsible for what is called here the Goldbach transposition. Goldbach gaps are therefore produced in the same ways even if in an irregular and non-linear way. We can see that the larger the number, the more equidistant prime numbers it will generate. If we assume an infinite number, then there will be an infinity of possible gaps with prime numbers < E/2 and it only takes a few prime numbers > E/2 for Goldbach gaps to appear. **Table 2** shows that each integer has its own configuration of equidistant primes as a kind of specific trace or pattern. If we move by one unit, the distribution of equidistant primes varies. For example, **35** - 6 = 29 and **35** + 6 = 41 while **36** - 5 = 31 and **36** + 5 = 41. This shows that the Goldbach's gaps vary infinitely with each integer.

Table 2. The transposition ways to get Goldbach's gaps and equidistant primes relatively to an integer n of different types. Either using p of π(n)to substract p from n or using odd 3n numbers with evens non-3n and 2n with odd numbers of all kinds. This shows that Goldbach's gaps are natural gaps between primes but always occur at least one for any integer. The data shown can be reproduced for whatever number > 4. Be E an even, and p and p' (p' > p) two equidistant primes at E/2 then t = E/2 – p = p' – E/2, the table shows t values to get equidistant primes from diferent types of numbers. O3n means odds numbers 3n. Equidistant primes are highlighted.

t	Way 1 : adding or substracting p from π(E) to or of E/2				t	Way 2 : adding or substractig O3n to or of E/2		t	Way 3 : adding or substractig 2n to or of E/2			
p	**48 - p**	**48 + p**	**76 - p**	**76 + p**	**O3n**	**76 - 3n**	**76 + 3n**	**2n**	**45 - 2n**	**45 + 2n**	**39 - 2n**	**39 + 2n**
3	45	51	***73***	***79***	3	***73***	***79***	2	***43***	***47***	***37***	***41***
5	***43***	***53***	71	81	9	67	85	4	41	49	35	43
7	41	55	69	83	15	61	91	6	39	51	33	45
11	***37***	***59***	65	87	21	55	97	8	*37*	*53*	*31*	*47*
13	35	61	63	89	27	49	103	10	35	55	29	49
17	31	65	59	93	33	***43***	***109***	12	33	57	27	51
19	***29***	***67***	57	95	39	37	115	14	***31***	***59***	25	53
23	25	71	53	99	45	31	121	16	***29***	***61***	23	55
29	19	77	47	105	51	25	127	18	27	63	21	57
31	***17***	***79***	45	107	57	19	133	20	25	65	***19***	***59***
37	***13***	***73***	39	113	63	***13***	***139***	22	***23***	***67***	***17***	***61***
41	***7***	***89***	35	117	69	7	145	24	21	69	15	63
43	5	91	33	119	75			26	***19***	***71***	13	65
47			29	123	81			28	***17***	***73***	***11***	***67***
53			23	129	87			30	15	75	9	69
59			17	135				32	13	77	*7*	***71***
61			15	137				34	***11***	***79***	*5*	*73*
67			9	143				36	9	81	3	75
71			5	147				38	***7***	***83***		
73			***3***	***149***				40	5	85		
79								42	3	87		
89								44				
97								46				
101								48				

1b. A reasoned example of proposition b (lemma b).

- Lemma b**.** « *E is any even > 4 et E/2 is any integer > 2. Let us consider π(E) and let us denote P1, P2,...Pn any prime of π(E) < E/2 and Q1, Q2,...Qm any prime of π(E) > E/2. Then there is at least one value of P and one value of Q such that E - P = Q and E - Q = P which is a Goldbach's gap. This proposition must be verified for Goldbach's strong conjecture to be true.*».

Here we see (Table 3) that if we subtract a prime number < E/2 from E we get a prime number > E/2 in some cases. This means E/2 – t and E/2 + t are primes or p + 2t = q. *This means that a minus gap that generates a prime number can generate it too if it is added*. This antisymmetry is common in natural numbers. All prime numbers have an integer n = E/2 in the middle separated from them by the same distance. For any integer n, there exists at least one Goldbach's gap t such that n – t and n + t are primes. In Table 3 two numbers are shown. For example, 11 + 109 = 120 and therefore 60 **–** 11 = 109 **–** 60. And so **11 = 60 – 49** and **109 = 60 + 49**. The difference 49 allows us to obtain two prime numbers equidistant from 60 which are 11 and 109. Otherwise, 11 + 98 gives 109 and 98: 2 = 49 and thus 11 + 49 = 60 which is therefore the integer in the middle between 11 and 109. Other differences in Table 3 generate other equidistant prime numbers according to the same rule. All these gaps are called here Goldbac's gaps. These rules are true to infinity.

Let us take another example such 31 + 157 = 188. The integer in the middle is (31 + 157)/ 2 = 94 and thus 31 + (63 x 2) = 157 and so **31 = 94 – 63 and 157 = 94 + 63**. Again E/2 – t and E/2 + t must be primes for the Goldbach's conjecture to hold true and that what happens in the set of integers. In fact, there is always any integer n between any two primes p and p' such that n – t = p and n + t = p'.

To find Goldbach's gaps it is needed to determine π(E) (E any even > 4) containing primes p < E. Then determine the gaps t such that E/2 – t and E/2 + t are primes and equidistant to E/2. This also represents a method that can be programmed to verify the strong Goldbach conjecture and generate all possible sums of equidistant prime numbers.

Table 3. Goldbach's gaps. Table show π(120) (first column) and π(188) (third column). Any prime of π(120) or π(188) is denoted p. We see that 120 – p or 188 – p give equidistant prime numbers > E/2 with Goldbach's gaps. These gaps are the distance between p and 120/2 = 60 or 188/2 = 94. This shows that for any even E > 4 there exists a Goldbach's gap denoted t such that E/2 – t and E/2 + t are primes. Equidistant primes are highlighted in bold.

p	***120 - p***	***p***	***188 - p***
3	117	**7**	**181**
7	**113**	11	177
11	**109**	13	175
13	**107**	17	171
17	**103**	19	169
19	**101**	23	165
23	**97**	29	159
29	91	**31**	**157**
31	**89**	**37**	**151**
37	**83**	41	147
41	**79**	43	145
43	77	47	141
47	**73**	53	135
53	**67**	59	129
59	**61**	**61**	**127**
61	**59**	67	121
67	**43**	71	117
71	49	73	115
73	**47**	**79**	**109**
79	**41**	83	105
89	**31**	89	99
97	**23**	97	91
101	**19**	101	87
103	**17**	103	85
107	**13**	107	81
109	**11**	**109**	**79**
113	**7**	**113**	**71**
		127	**61**
		133	55
		137	**47**
		139	49
		149	39
		151	**37**
		157	**31**
		163	25
		167	21
		173	15
		179	9
		181	**7**

Let us suppose an even number denoted E and determine π(E). Then π(E) will contain the prime numbers which are all < E (prime number theorem). Let us take E/2 and then we will have the prime numbers of π(E) which are < E/2 and those greater than E/2. We will then subdivide π(E) into P1, P2, P3, ...PN which are < E/2 on the one hand, and on the other hand Q1, Q2, Q3,...Qm which are > E/2. Then E/2 – P1 = t1; E/2 – P2 = t2; E/2 – P3 = t3;...E/2 – Pn = tn are calculated. Then there is a least one value of t (t1 to tn) such that E/2 + t = Q1 or Q2...or Qm (Table 4). Since Q – P = 2n and given Q > P then (Q + P)/2 = E/2 which means that there is always an integer E/2 at equal distance from P and Q. If (Q + P)/2 = E/2 then Q + P = E and Q and P are equidistant relative to E/2. This proposition postulates that Goldbach gaps before E/2 repeat at least once after E/2 because prime numbers are formed in a symmetrical manner from 0 to infinity and from infinity to 0. For example, if n – t is prime then it is likely that n + t is also prime.
Similarly and symmetrically, the gaps Q1 – E/2 = z1; Q2 – E/2 = z2; Q3 – E/2 = z3;...Qm – E/2 = zm are opposite gaps. And so there exists at least one value of z (z1 to zm) such that E/2 – z = P1 or P2 or P3 or Pn (Table 4). Any even number E > 4 has both types of sets of Goldbach gaps on either side of E/2.

Table 4 : Goldbach's gaps with E/2 + t and E/2 – t numbers with E = 120 (3n) and E/2 = 60 and t = prime value in an increasing order. This can be reproduced for any number but it must be taken into account whether the number is even or odd, 3n or not (see above). If it is odd, then it is necessary to subtract and add 2n numbers in ascending order. Equidistant primes are highlighted in italic and there are 10 Goldbach's gaps. The table show that there is at least one value of t such that E/2 + t and E/2 – t numbers are both primes which represents an initial condition essential for Goldbach's strong conjecture to be true. The Table shows that there are 10 goldbach's gaps and 10 couples of E/2 + t and E/2 – t prime numbers obtained with the number E = 120 (E/2 =60). This will happen with any even E > 4 of same type (3n), only the values and positions of primes changes between 0 and E/2 and between E/2 and E.

t	**60 – t**	**60 + t**	**Goldbach gap**
3	57	63	
5	55	65	
7	***53***	***67***	1
11	49	71	
13	***47***	***73***	2
17	43	77	
19	***41***	***79***	3
23	***37***	***83***	4
29	***31***	***89***	5
31	29	91	
37	***23***	***97***	6
41	***19***	***101***	7
43	***17***	***103***	8
47	***13***	***107***	9
53	***7***	***113***	10
59	1	119	

1c. Specific case of an even number denoted E which tends to infinity

If E tends to infinity, the Goldbach'gaps tend to infinity. π(E) will contain an infinity of prime numbers P < E/2 and Q > E/2. And so for a prime number Q > E/2, there must exist a Goldbach gap such that Q – E/2 = E/2 – P. Even if there is a long empty gap of prime numbers after E/2 and the number of prime numbers Q > E/2 are rarer and more dispersed, there will still exist a Goldbach gap such that Q – E/2 = E/2 – P because all possible Goldbach gaps exist before E/2 for a number E that tends to infinity. For example, there are infinitely many gaps between 3 and all other primes (7 – 3; 11 – 3; 17 – 3;...to infinity) and this is true for every prime number and therefore all possible Goldbach gaps exist at infinity to the point that any prime number Q > E/2 will find a number P < E/2 which is symmetric to it such that Q – E/2 = E/2 – P. Goldbach's conjecture is true because there are infinitely many possible gaps between known or possibly known prime numbers at infinity and any prime E/2 + t whatever its value and > E/2 would find a symmetric prime E/2 – t < E/2. This rule of symmetry can be more easily seen with twin prime numbers to infinity.

Let us suppose an even number E such that E/2 tends to infinity, so [E/2 → E] is in infinity. On the other hand, from 0 to E/2 we have all the prime numbers that we could discover by the means available to date or which are known at the maximum limit. So if we take any prime number p and any prime number q that we know such that q > p, then q – p can take any possible value 2n and all possible 2n gaps between any two prime numbers (not just twins but distant from each other by any gap) exist from 0 to E/2. We know well today that prime numbers are infinite and probably at a much higher density than we imagine. Let us then suppose that between E/2 and E we have two prime numbers among others s and t such that **s – t = 2n'**. Then, there must exist two other prime numbers u and v < E/2 such that **u – v = 2n'**. Therefore s – t = u – v and s + v = t + u and therefore if u and t are equidistant at E/2 then s and v are also equidistant. Since s and t > E/2 and u and v < E/2 *with u et t, and s and v, equidistant from E/2*, then 2 x E/2 = s + v = t + u and therefore E = s + v = t + u. Thus Goldbach's conjecture can be verified at infinity knowing that any even number that we know or can imagine will be the sum of two prime numbers. Indeed for a number which tends to infinity, all possible gaps 2n between any two primes are limitless.
Let us remember that if P1 ← E/2 →P2 means P1 and P2 are equidistant to E/2 then E/2 – t = P1 ad E/2 + t = P2 (t any integer < E/2). Therefore E = E/2 + E/2 = (E/2 – t) + (E/2 + t) = P1 + P2. Any time there are two equidistant primes relatively to E/2 the strong Goldbach conjetcure is verified and holds true. The t is the Goldbach's gap and is also limitless as the number tends to infinity which means all primes at any t gap are possible. Note here that two initial conditions are necessary s – t = u – v and s, v on one hand and t, u on the other are equidistant to E/2. This means that all equidistant primes to E/2 are related to each other by 2n gaps and related to E/2 by t Goldbach's gaps. The two gaps overlap in all possible ratios. If E and E/2 extend to infinity, and even if the density of primes > E/2 is lower or there are long sequences devoid of primes, the distances between primes close to E/2 and 0 would be as infinite as the distances between these distant primes and E/2. Moreover, given their higher density, more possible gaps occur before E/2. It is therefore quite possible that these distant primes are equidistant to primes < E/2.
To illustrate this with an example, we limit ourselves to a small number like 100 knowing that what is shown applies to infinity (Table 5A+B). These tables show the large number of possibilities for a small number and a fortiori for a number which extends to infinity.
Table 5A. E is any even and be E = 100. Let us calculate π(100) and calculate u – v such that u is any prime > v in primes π(100) < E/2 = 50.

v → **u↓**	3	5	7	11	13	17	19	23	29	31	37	41	43	47
47	44	42	40	36	34	30	28	24	18	16	10	6	4	
43	40	38	36	32	30	26	24	20	14	12	6	2		
41	38	36	34	30	28	24	22	18	12	10	4			
37	34	32	30	26	24	20	18	14	8	6				
31	28	26	24	20	18	14	12	8	2					
29	26	24	22	18	16	12	10	6						
23	20	18	16	12	10	6	4							
19	16	14	12	8	6	2								
17	14	12	10	6	4									
13	10	8	6	2										
11	8	6	4											
7	4	2												

Table 5B. Let us calculate π(100) and calculate s – t such that s is any prime > t in primes > E/2 = 50.

t→ **s↓**	53	59	61	67	71	73	79	83	89	97
97	44	38	36	30	26	24	18	14	8	
89	36	30	28	22	18	16	10	6		
83	30	24	22	16	12	10	4			
79	26	20	18	12	8	6				
73	20	14	12	6	2					
71	18	12	10	4						
67	14	8	6							
61	8	2								
59	6									

Let determine any u – v = s – t such that u + t or s + v = 100.
Examples :

47 – 41 = 59 – 53	→	47 + 53 = 59 + 41 = 100
~~47 – 31 = 69 – 53~~	→	~~47 + 53 = 69 + 31 = 100~~
47 – 3 = 97 – 53	→	47 + 53 = 97 + 3 = 100
47 – 17 = 83 – 53	→	47 + 53 = 17 + 83 = 100
41 – 17 = 83 – 59	→	41 + 59 = 17 + 83 = 100
41 – 11 = 89 – 59	→	41 + 59 = 89 + 11 = 100
17 – 3 = 97 – 83	→	17 + 83 = 97 + 3 = 100

2. Demonstrate and prove the correctness of the Goldbach's strong conjecture by a table using 3x + 5 and 3x – 5 equations while following the remainders of Euclidean divisions

2a. Goldbach's strong conjecture and the gaps that separate prime numbers using 3x ± 5 equations

Prime numbers and their multiples except multiples of 2 and 3 are all 6x ± 1 (Bahbouhi[1], 2024). Here the equations **3x ± 5** reconstruct all prime numbers and their multiples in their natural order (Table 6A+B). However, using Euclidean division, one can know why a number is prime or not by examining the remainders of the Euclidean division of **3x** by the prime numbers below it. This is possible with the equations 3x ± 5. Let's take an example, the number 35 has 5 as a factor and therefore cannot be prime because 35 = 3 x 10 + 5. But if we take 77, we have 77 = 3 x 24 + 5 and in fact 77 – 5 = 72 and 72 : 11 = 6 and the remainder (denoted r in the table) = 6 and therefore if we add 5, we will have a new factor 11 = 5 + 6 or 5 = 11 – 6 and therefore 77 is not prime. In general for the equation 3x + 5 if **5 = prime factor – remainder** (the factor is denoted q in the table) then the number obtained is not prime. This could be used as a factorization method by examining each time the remainder of the division of the number 3x by the prime factors q which are less than its square root, if 5 = q – r then the number obtained with the equation 3x + 5 is composite. *Here are the steps, take the number 3x + 5 and substract 5, you get 3x, then divide it by primes q < its square root. See the remainders and determine if 5 = q – r. If it is the case, then the number is composite. You get the prime factor by adding 5 to the remainder. This method can work well to decompose a number in product of prime factors . If 5 ≠ q – r, then the number is prime*. Factoring an integer into a product of prime factors is still a topic of primary importance in mathematics (Bahbouhi[2], 2024).
Let us take another example, 119 = 3 x 38 + 5. We have 3 x 38 = 114 and 114 : 17 = 6 and r = 12. Therefore 5 = 17 – 12 and thus 119 is composite. By contrast, if the number is prime, 5 ≠ q – r in all euclidean divisions.
On the other hand, for 3x – 5 equation, if 5 = r of the euclidean divisions then the number is composite.
For the rest, let us denote any prime number **P** and any composite number **C**. The prime numbers obtained by 3x + 5 are the **P+** and those of the equation 3x – 5 are **P-**. Similarly, we have the **C+** and **C-**. Note that multiples of 3 are excluded from the tables. We see that there are gaps of 6n between the P+ primes on the one hand and between the P- on the other hand. Each line break = gap of 6 by going down or up. If we go up, we have gaps of -6n and if we go down we have +6n. On the other hand, we have variable gaps of 2n between the P+ and the P- primes. We have the same pattern of gaps between the C+ and the C-. Let us note that 3 x + 5 corresponds to 6x – 1 primes and 3x – 5 to the 6x + 1 equation. In fact, there are two types of primes of which the former are 6x – 1 and the latter 6x + 1. Not only primes, but all their multiples except those of 2 and 3 can be written as 6x + 1 or 6x – 1.
Since all composite or prime numbers can be written as the equation 3x ± 5, we can then develop a method for their factorization by applying the rules 5 = q – r or 5 = r for the equations 3x + 5 and 3x – 5, respectively. We can clearly see that the integers, during their progression to infinity, give prime or composite numbers, depending on the remainders of their Euclidean divisions by the prime numbers that are less than their square roots. Conversely and by the same process, a number P or C comes from a natural integer P or C which precedes it.
This progression of integers in tables-6 by 3x ± 5 equations can be used to demonstrate and prove the Goldbach's strong conjecture. In fact, it appears clear that this conjecture is a result of the progression of natural numbers into primes or composite numbers. The proof of this conjecture lies in this progression itself.
Here are the key arguments to prove Goldbach's strong conjecture:

- Any odd number P or C is preceded and followed by prime numbers at regular intervals of 6n. Therefore
  P' (+6n) ▬ P ▬ P'' (-6n) ↔ 2 x P = 2n = P' + P''.
  P''' (+6n) ▬ C ▬ P'''' (-6n) ↔ 2 x C = 2n = P''' + P''''. Take any umber P or C in the tables-6 using the equation 3x + 5 or 3x – 5, go up and down till you get the two equidistant primes then Goldbach's strong conjecture is demonstrated this way.
  For example 7(+6 x 5) ▬ **37** ▬ 67 (-6 x 5) ↔ 2 x 37 = 7 + 67 = 74.
  71 (+6 x 1) ▬ 77 ▬ 83 (-6 x 1) ↔ 2 x 77 = 71 + 83 = 154.
  This table shows that since all numbers P and C are equidistant from prime numbers preceding and following them, even numbers are sums of two prime numbers because if a number n is equidistant from two prime numbers P1 and P2 then 2n = P1 + P2. Goldbach's strong conjecture can thus be demostrated by 6n gaps.

- However, we are missing the even numbers in the tables-6 (multiples of 3 or not) and the odd multiples of 3. We just need to convert them first to P or C of the table and then move in gaps of 6. *The numbers P and C in tables-6 are converters because they are used to transform any number > 4 into the sum of two prime numbers*.
- Note that we,can also use the gaps between the prime numbers 3x + 5 and 3x – 5 but they are = 2n and are variable and just require more attention and calculation but the table demonstrates Goldbach's strong conjecture in all directions and cases.
- For example :
  23(+6)▬ 29 (+1)▬ **30** ▬ 31(-1) ▬37(-6) ↔ 2 x 30 = 60 = 31 + 29 = 23 + 37
  37 (+6 x 5)▬ 67 (+2)▬ **69** ▬ 71(-2) ▬101(-6x 5)↔ 2 x 69 = 138 = 67 + 71 = 37 + 101.
  37 (+6 x 5)▬ 67 (+3)▬ **70** ▬ 73(-3) ▬103(-6x 5)↔ 2 x 70 = 140 = 67 + 73 = 37 + 103.
- It is important to note that if we start with a non-3n even number like 56 or 88, we should not convert it to a 3n number because we will no longer be able to find prime numbers with steps of 6.
  Example if we do 56 – 2 = 54, we cannot have prime numbers below or above 54. Rather 56 – 3 = 53 or even 56 – 1 = 55 because 55 – 6 = 49 – 6 = 43 or 55 + 6 = 61 + 6 = 67 and so on.
- According to this table here is the demonstration of Goldbach's strong conjecture: Be E any even >4, C is composite and P is prime.
  E = C 1 + C2 = (C1 + 6n) + (C2 – 6n) = P1 + P2.
  E = C1 + P = (C1 + 6n) + (P – 6n) = P3 + P4.
  E = P' + P'' = (P'+ 6n) + (P'' – 6n) = P5 + P6
- Because there is always a 2n gap between any two prime p and q such that q > p, then there is always an integer in the middle. Be q > p and q = p + 2t ↔ ∃n such that p + t = n and q – t = n ↔ 2n = p + q. All prime numbers taken to infinity will generate all possible evens 2n and therefore 2n would always be the sum of two primes.
- Conversely, be 2n ↔ n is in the middle of two primes p and q ↔ 2n = p + q.
- 2n ↔ 2n/2 = n ↔ n= 3x + 5 or n = 3x – 5 ↔ n is P or C ↔ n – 6n and n + 6n ↔ p + q.
- There are four possible equalities or Goldbach equations to complete to convert any even number into the sum of two prime numbers P and P' using tables-6 and 3x ± 5 to infinity :
  P' (+6n) ▬ P ▬ P'' (-6n) (Even = 2n = **2 x P** = P + P').
  P' (+6n) ▬ C ▬ P'' (-6n) (Even = 2n = **2 x C** = P + P').
  P' (+6n) ▬C or P▬ 2n + x ▬ **2n** ▬ 2n – x ▬ C or P▬ P'' (-6n) (Even = 2n = **2 x 2n** = P + P').
  P' (+6n)▬ C or P ▬ 3n + x ▬ **O3n** ▬ 3n – x ▬ C or P▬ P'' (-6n) (Even = 2n = **2 x O3n** = P + P').
  Note 2n might be 3n or not while O3n is odd 3n in the last equation.
  Note that x is used to preconvert the even or O3n into the converters P or C before searching for the P by the jumps of 6n (see examples above). Or we can use any gap between primes to complete these equations but 6n is the most regular.

Table 6A+B : The equations 3x ± 5 reconstruct all the prime numbers and odd multiples of prime numbers except those of 2 and 3. Depending on the remainders (r) of the Euclidean division of a number 3x by the primes < its square root denoted q, the number is prime (P) or composite (C). The table shows that all natural numbers (primes, odd multiples of primes, evens and multiples of 3) can be in the middle of two primes by making 6n jumps in two opposite directions (up down ↓**N**↑) or left-right by 2n gaps. In the table P is prime and C is composite while r is the remainder of euclidean division of numbers 3x in 3x + 5 or 3x – 5 by q which is any prime less than their square roots. If 5 = q – r in the case of 3x + 5 or 5 = r in the case of 3x – 5, the number is not prime (C).

6A

x = 2n	**3x + 5**	**P *or* C (5 = q – r)**	**x = 2n**	**3x – 5**	**P *or* C (*r* = *5)***
2	**11**	P	4	**7**	P
4	**17**	P	6	**13**	P
6	**23**	P	8	**19**	P
8	**29**	P	10	↓**25**↑	q = 5
10	↓**35**↑	q = 5	12	**31**	P
12	**41**	P	14	**37**	P
14	**47**	P	16	**43**	P
16	**53**	P	18	↓**49**↑	**r = 5**
18	**59**	P	20	↓**55**↑	q = 5
20	↓**65**↑	q = 5	22	**61**	P
22	**71**	P	24	**67**	P
24	↓**77**↑	5 = 11 – 6 5 = 11 – 6	26	**73**	P
26	**83**	P	28	**79**	P
28	**89**	P	30	↓**85**↑	q = 5
30	↓**95**↑	q = 5	32	↓**91**↑	**r = 5**
32	**101**	P	34	**97**	P
34	**107**	P	36	**103**	P
36	**113**	P	38	**109**	P
38	↓**119**↑	5 = 17 – 2 5 = 7 – 2	40	↓**115**↑	q = 5
40	↓**125**↑	q = 5	42	↓**121**↑	**r = 5**
42	**131**	P	44	**127**	P
44	**137**	P	46	**133**	**r = 5**
46	↓**143**↑	5 = 11 – 6 5 = 13 – 8	48	**139**	P
48	**149**	P	50	↓**145**↑	q = 5
50	↓**155**↑	q = 5	52	**151**	P
52	↓**161**↑	5 = 7 – 2 5 = 23 – 18	54	**157**	P
54	**167**	P	56	**163**	P
56	**173**	P	58	↓**169**↑	**r = 5**
58	**179**	P	60	↓**175**↑	q = 5
60	↓**185**↑	q = 5	62	**181**	P
62	**191**	P	64	↓**187**↑	**r = 5**
64	**197**	P	66	**193**	P
66	↓**203**↑	5 = 29 – 24 5 = 11 – 6	68	**199**	P
68	↓**209**↑	5 = 19 – 14 5 = 11 – 6	70	↑**205**↑	q = 5
70	↓**215**↑	P	72	**211**	P

6B

x = 2n	**3x + 5**	**P *or* C (5 = q – r)**	**x = 2n**	**3x - 5**	**P *or* C (*r = 5)***
72	**↓221↑**	5 = 13 – 8 5 = 17 – 12	74	**↓217↑**	**r = 5**
74	**227**	P	76	**223**	P
76	**233**	P	78	**229**	P
78	**239**	P	80	**↓235↑**	q = 5
80	**↓245↑**	q = 5	82	**241**	P
82	**251**	P	84	**↓247↑**	**r = 5**
84	**257**	P	86	**↓253↑**	**r = 5**
86	**263**	P	88	**↓259↑**	**r = 5**
88	**269**	P	90	**↓265↑**	q = 5
90	**275**	q = 5	92	271	P
92	**281**	P	94	277	P
94	**↓287↑**	5 = 41 – 36 5 = 7 – 2	96	**283**	P
96	**293**		98	**↓289↑**	**r = 5**
98	**↓299↑**	5 = 23 – 18 5 = 13 – 8	100	**↓295↑**	q = 5
100	**↓305↑**	q = 5	102	**↓301↑**	**r = 5**
102	**311**	P	104	**307**	P
104	**317**	P	106	**313**	P
106	**↓323↑**	5 = 17 – 12 5 = 19 – 14	108	**↓319↑**	**r = 5**
108	**↓329↑**	5 = 7 – 2 5 = 47 – 42	110	**↓325↑**	q = 5
110	**↓335↑**	q = 5	112	**331**	P
112	**↓341↑**	5 = 31 – 26 5 = 11 – 6	114	**337**	P
114	**347**	P	116	**↓343↑**	**r = 5**
116	**353**	P	118	**349**	P
118	**359**	P	120	**↓355↑**	q = 5
120	**↓365↑**	q = 5	122	**↓361↑**	**r = 5**
122	**↓371↑**	5 = 7 – 2 5 = 53 – 48	124	**367**	P
124	**↓377↑**	5 = 13 – 8 5 = 29 – 24	126	**373**	P
126	**383**	P	138	**379**	P
138	**389**	P	140	**↓385↑**	q = 5
140	**↓395↑**	q = 5	142	**↓391↑**	**r = 5**
142	401	P	144	**397**	P
144	**↓407↑**	5 = 11 – 3 5 = 37 – 32	146	**↓403↑**	**r = 5**
146	**↓413↑**	5 = 7 – 2 5 = 59 – 54	148	**409**	P
148	419	P	150	↓415↑	**r = 5**
150	**↓425↑**	q = 5	152	**421**	P

2b. A new algorithm-I to convert an even in sum of two primes

Be E an even. Calculate E/2.
If E/2 is P or C
Follow table-6 lines (to infinity).
Perform conversion
P'(+6n) ▬P or C ▬P''(-6n)
if P or C = P' + 6n ad P'' = P or C + 6n
E = 2x P or 2 x 2C = P' + P''.
If E/2 is even (2n) non-3n
Covert it to P or C. Calculate E/2 – 1 and E/2 + 1
E/2 – 1 and E/2 + 1 should be ≠ odd 3n (O3n)
If O3n calculate O3n – 2 and O3n + 2
If the result is C or P proceed as above
P'(+6n)▬P or C ▬2n ▬P or C ▬P''(-6n)
if 2n = P' + 6n and P'' = 2n + 6n
E = 2 x 2n = P' + P''.
If E/2 is even 3n
Covert it to P or C
E/2 – 1 and E/2 + 1
If the result is C or P proceed as above
P'(+6n) ▬P or C▬2n ▬P or C ▬P''(-6n)
if 2n = P' + 6n and P'' = 2n + 6n
E = 2 x 2n = P' + P''.

If E/2 is odd 3n
calculate E/2 – 2 and E/2 + 2
If the result is C or P proceed as above
P'(+6n) ▬P or C▬O3n ▬P or C ▬P''(-6n)
if O3n = P' + 6n ad P'' = O3n + 6n
E = 2 x O3n = P' + P''.

Note the method works with other gaps = 2n or any gap N (N any integer >0)
From 3x + 5 to 3x – 5 primes or vice versa
P'(+N) ▬P or C ▬P''(-N)
E = 2 x P or C = P' + P''.

2c. Corresponding examples of the algorithm-I application

E = 170 E/2 = 85
E/2 is C
67(+18) ▬ 85 ▬103(-18)
170 = 2 x 85 = 67 + 103.

E = 194 and E/2 = 97
E/2 is P
67(+30) ▬ 97 ▬127(-30)
194 = 2 x 97 = 67 + 127.

E = 320 E/2 = 160
E/2 is even non-3n
160 – 1= 159 and 160 + 1 = 161
159 is odd 3n to discard
160 – 3 = 157 and 160 + 3 = 163
157 ▬160 ▬163 or
139 (+6x3)▬157 ▬160 ▬163▬181(-6x3) or
139 (+21) ▬160 ▬181(-21)
320 = 2 x 160 = 157 + 163
320 = 2 x 160 = 139 + 181

E = 660 and E/2 = 330
E/2 is even 3n
330 – 1 = 329 and 330 + 1 = 331
329 ▬330 ▬331
311(+18) ▬329▬330 ▬331▬349(-18)
or 313(+17) ▬330 ▬347(-17)
660 = 2 x 330 = 311 + 349 or 660 = 2 x 330 = 313 + 347

E = 3006 and E/2 = 1503
E/2 is O3n
1503 – 2 = 1501 and 1503 + 2 = 1505
1501 ▬1503 ▬1505
1483(+18)▬1501▬1503 ▬1505▬1523(-18)
3006 = 2 x 1503 = 1483 + 1523

3. Key Digits of primes numbers and equidistance to verify Goldbach's Strong Conjecture

3a. The goal behind the use of prime digits

*B*etween two primes there is an integer in the middle the double of which is the even equal to the sum of the two primes.
q > p → q – p = 2n → p ← E/2 → q → 2 x E/2 = 2n = p + q.
p = E/2 – **t** and q = E/2 + **t** and p + 2**t** = q.
The integer in the middle is the mean = (p +q)/2. The unit digit of this average value always depends on the units or other key digits of the prime numbers p and q. Here we demonstrate that starting from these digits, we can convert an even into the sum of two prime numbers.

First, formulas are given following this article that are based on the location of the digits of prime numbers. Secondly, how to apply them to convert an even number into a sum of two prime numbers. Note that these formulas do not cover all the rules that we could know but only a part and the purpose of this paper is mainly to show that they exist. Here examples of calculation are given for illustration purposes.

3b. Unit Digits in case q – p = 6 or q – p = 10 or q – p = 42 and therefore t = 3 or t = 5 or t = 21.
There are very common gaps between prime numbers that can be deduced from their unit digits in case of all primes (see below). There is always a 2n gap betwee two primes and a integer in the middle between them. Hence Omnipresence of the equidistance between primes in all natural numbers.
Here gaps of 3, 10 and 21 are used. The gap used is indicated in parentheses (see below). In case of gap = 6 and t = 3, the difference between the unit digits is 6 example 11 and 17 (7 – 1). However if the unit of the larger prime is < that of the smaller prime we have to add 10 to see the gap. For example 17 and 23, we do not do 3 – 7 but 13 – 7 = 6. See in those having three digits we can see the difference of 6 between the last two digits example 103 and 109 (9 – 3 = 6) or 131 and 137 (37 – 31 = 6). Here are other examples not shown. The primes numbers 1187 and 1193 such that 193 – 187 = 6 or 1117 and 1123 such that 123 – 117 = 6. In case of gap = 10 the larger and smaller primes have the same unit digit. In case of gap = 42 = 2x 21, the unit digit of the larger is = that of the smaller + 2. For example 11 and 53 (3 = 1 + 2). Again we add 10 if the unit digit of the larger prime < that of the smaller example 19 and 61 such that 1 of 61 + 10 – 9 = 2. All this show that digits of primes are the result of the gaps that are in betwee them. In addition, the digits of the evens that are sum of two primes are also the result of those gaps. In case of gap = 6, t= 3, we see that the difference between the unit digit of the average value (the integer in the middle) and that of the smaller prime = 3. Again we add 10 in case it is smaller example in case 5-8-11 we have 8 – 5 = 3 or 11-14-17 we have 4 – 1 = 3 but in case of 37-40-43 we do 10 (0 of 40) – 7 (of 37) = 3. We see that the same difference can be deduced from the unit digit of the larger prime and that of the integer in between example 11-14-17 (7 – 4 = 3) ; 47-50-53 (3 – 0 = 3).

5	→ 8	→ 11 (+ 6)	3	→ 8	→ 13 (+10)
7	→ 10	→ 13	7	→ 12	→ 17
11	→ 14	→ 17	13	→ 18	→ 23
13	→ 16	→ 19	19	→ 24	→ 29
17	→ 20	→ 23	31	→ 36	→ 41
23	→ 26	→ 29	37	→ 42	→ 47
31	→ 34	→ 37	43	→ 48	→ 53
37	→ 40	→ 43	61	→ 66	→ 71
41	→ 44	→ 47	73	→ 78	→ 83
47	→ 50	→ 53	79	→ 84	→ 89
53	→ 56	→ 59	97	→ 102	→107
61	→ 64	→ 67	127	→ 132	→137
			139	→ 144	→149
67	→ 70	→ 73	11	→ 32	→ 53 (+ 42)
73	→ 76	→ 79	17	→ 38	→ 59
97	→ 100	→ 103	19	→ 40	→ 61
103	→ 106	→ 109	29	→ 50	→ 71
107	→ 110	→ 113	31	→ 52	→ 73
131	→ 134	→ 137	37	→ 58	→ 79

3c. Algorithm-II for the conversion of an even number into the sum of two prime numbers by their units digits.

Even numbers have 0, 2, 4, 6, and 8 as unit digits and so if an even number is the sum of two prime numbers, the unit digits of the latter count to deduce that of the even number of which they are the sum. The partitions of 0, 2, 4, 6, and 8 are therefore determinant in Goldbach's conjecture excluding the number 5 because we know that there is no prime number ending with this digit. For example evens ending with 8 are sums of two primes having units digits such 1 and 7 or 9 twice and those with 4 are sums of primes with 1 and 3 or 7 twice while 6 is either sum of two primes having both 3 as units digits or 7 and 9.
This counts for converting an even number into the sum of two primes. Let E be an even number and calculate π(E). We separate the primes < E/2 and those > E/2. Knowing that the even number is the sum of a prime < E/2 and another > E/2, we will then sort the prime numbers of π(E) < E/2 and > E/2 according to their unit digits. Note that this process is symmetrical for example if we take prime numbers < E/2 ending with a digit like 1 and those >E/2 ending with a digit like 7, we must also do the inverse or the reciprocal i.e. those < E/2 having 7 as the unit digit and those >E/2 having 1 as the unit digit. *In all cases and each time we convert an even > 4 into a sum of two prime numbers, we apply these rules whether we realize it or not. This article seeks, however, to state them*.
This selection of primes by their unit digits therefore results in a method or a **new algorithm-II** that is simple to execute in computer science and which eliminates all other useless prime numbers. For example, an even number ending in 4 will not be affected by all the prime numbers ending in 9 since there are no prime numbers ending in 5. This is also the case for evens with unit digit = 8 with primes ending in 3. This exclusion accelerates the process of converting an even number into the sum of two prime numbers. There is also the total number of digits. For example, an even number of 4 digits will be the sum of a prime number of 2 digits and another of not less than 3 digits. A three-digit prime number depending on its value relative to the even number of 4 digits to be converted will add either another three-digit prime or a 2-digit prime or even primes of one digit. The two selections can be superimposed, those based on the unit digit and those based on the total number of digits, which will further speed up the process of converting an even number into the sum of two prime numbers. In this article, we restrict ourselves to safer and simpler rules that go in one direction only. For example, eliminate prime numbers with 9 as the unit digit as soon as we convert an even number ending in 4 or those with 3 with even numbers ending in 8. The algorithm-II is more robust with only one paramater which is the unit digits.

Here we give a single example of E = 580 and E/2 = 290 by using unit digit only and the rule applies in the same way in the case of other unit digits and for any even number. Indeed, a prime number has 1, 3, 7 and 9 as unit digits. Therefore, an even number that ends in 0 is either the sum of two prime numbers having 3 and 7 as unit digits or 9 and 1. We will then look for them before E/2 and after E/2 and identify those that are equidistant. We can do it in a reciprocal manner, that is to say, look for those having 1 before E/2 and 9 after E/2 and vice versa (same for 3 and 7). ***Only those prime numbers that satisfy the partition rule of the unit digit of the even number count to convert it into a sum of two prime numbers*** (Tables 7A+7B).

Table 7. Algorithm-II application. Conversion of an even (E = 580 and E/2 = 290) following the partition in sum of 0, let there be prime numbers with digits 1 and 9 (table 7A) or 3 and 7 (Table 7B). The equidistant prime numbers P and P' make a sum = 2 x E/2 = 2 x 290 = 580. Note that this method based on digits operates in the two directions (1,9) and (9,1) (table 7A) or (3,7) and (7,3) in table 7B.

Table 7A

< E/2 = 290	> E/2 = 290	P	P'	< E/2 = 290	> E/2 = 290	P	P'
11	269	**11**	**569**	19	311	**59**	**521**
31	349	**71**	**509**	29	331	**89**	**491**
41	359	**101**	**479**	59	401	**149**	**431**
61	379	**131**	**449**	79	421	**179**	**401**
71	389	**191**	**389**	89	431	**269**	**311**
101	409			109	461		
131	419			139	491		
151	439			149	521		
181	449			179	541		
191	479			199	571		
211	499			229			
241	509			239			
251	569			269			
271							
281							

Table 7B

< E/2 = 290	> E/2 = 290	P	P'	< E/2 = 290	> E/2 = 290	P	P'
7	293	**17**	**563**	3	307	**3**	**577**
17	313	**137**	**443**	13	317	**23**	**557**
37	353	**197**	**383**	23	337	**113**	**467**
47	373	**227**	**353**	43	347	**233**	**347**
67	383			53	367	**263**	**317**
97	433			73	397		
107	443			83	457		
127	463			103	467		
137	503			113	487		
157	523			163	557		
167	563			173	577		
197				193			
227				223			
257				233			
277				263			
				283			

3d. Digits in case XX and X0X or XX and X00X.

The remainder of this article focuses on the relationship that exists between numbers which have in common digits placed in key positions. When applied to prime numbers, we find the average or integer that is equally distant from two prime numbers, therefore confirming again that prime equidistance is omnipresent among integers. Finally, these rules will be used to convert any even into prime numbers with some examples.

- Let note digits by X except the key digit. Example XX and X0X. Here 0 is a key digit and so **X0X – XX = 90**.

11 and 1**0**1 → 101 – 11 = 90 → 11 + 45 = 56 and 101 + 45 = 146 → 56 + 146 = 202 = 101 x 2.
Hence 11 ← **56** → 101 so that 56 – 11 = 101 – 56 = 45.
17 ad 1**0**7 → 107 – 17 = 90 → 17 + 45 = 62 and 107 + 45 = 152 → 62 + 152 = 214 = 107 x 2.
Hence 17 ← **62** → 107 so that 62 – 17 = 107 – 62 = 45.

- **X00X – XX = 990.**

13 and 1**00**3 → 1**00**3 – 13 and → 1000 – 10 = 990. Then, 990 : 2 = 495 → 13 + 495 = 508 and 1003 + 495 = 1498 → 508 + 1498 = 2006 = 2 x 1003. Hence 13 ← **508** → 1003.

X000n...X – XX = 9999n...0. Example 1**00000**3 – 13 = **99999**0.

3d. Digits in case XXY and XX.

XXY – XX = XX (10 – 1) + Y.

139 – 13. Given that (139) – 13 = (13 x 10 + 9) – 13 = **13 x (10 – 1) + 9** = 13 x 9 + 9 = 126.
126 : 2 = 63 → 13 + 63 = 76 and 139 + 63 = 202 → 76 + 202 = 278 : 2 = 139. Therefore
13 ← **76** →139.
173 – 17 = 17 x 9 + 3 = 156 → 156 : 2 = 78. Therefore 17 + 78 = 95. And 95 + 78 = 173.
17 ← **95** →173.

*3e. Digits in case YXX and XX → **YXX – XX = Y00 or Yn...XX – XX = Yn...00**.*

661 – 61 = 600. Given 600 : 2 = 300 we have 61 + 300 = 361 and 361 + 300 = 661.
61 ← **361** → 661 → 361 x 2 = 61 + 661 → **722 = 61 + 661**.

673 – 73 = 600. Given 600 : 2 = 300 we have 73 + 300 = 373 and 373 + 300 = 673.
73 ← **373** → 673 → 373 x 2 = 73 + 673 → **746 = 73 + 673**.

3f. Common digits in primes XX...ZZ (ZZ are digits of primes p such that $3 \leq p \leq 97$)

The digits of primes ≤ 97 are often at the end of prime numbers of three digits and more to infinity.

Examples :

101	1**03**	1**07**	109	1**13**	127	1**31**	1**37**	139	149
151	157	163	1**67**	1**73**	1**79**	181	191	193	1**97**
199	2**11**	2**23**	227	2**29**	233	239	241	251	257
263	269	2**71**	277	281	2**83**	293	307	3**11**	3**13**
3**17**	331	3**37**	3**47**	349	3**53**	3**59**	3**67**	3**73**	3**79**
3**83**	3**89**	3**97**							

Note this is not limited to numbers with three digits but to infinity as show below with numbers of 9 digits.

85412401	785412409	785412449	785412469	7854124**79**	785412491	7854125**03**	7854125**17**
7854125**53**	785412569	7854125**71**	785412581	7854125**83**	785412601	7854126**13**	7854126**19**
7854126**97**	785412701	7854127**31**	7854127**37**	785412751	785412781	785412791	7854128**11**
7854128**53**	785412877	785412893	7854129**19**	785413021	7854130**37**	785413049	7854130**81**
785413151	7854132**07**	7854132**17**	7854132**29**	785413249	7854132**73**	7854132**97**	785413309
7854134**23**	7854134**59**	785413477	7854134**83**	785413493	7854135**43**	7854135**53**	785413609
7854136**79**	785413691	7854137**03**	785413721	785413751	7854137**71**	785413781	785413793
7854138**29**	7854138**83**	785413901	7854139**07**	785413921	785413927	785413949	7854139**61**
7854140**83**	7854141**07**	7854141**11**	7854141**59**	785414177	785414209	7854142**13**	7854142**37**

In all cases XXXn...YY – YY = XXXn... x 10^2 (n is the total number of the digits of the number reduced by 1).
Example 107 has three digits and so n = 2 and therefore 1**07** – **07** = 01 x 10^2 = 100. Given that 100 : 2 = 50 we have 7 + 50 = 57 and 57 + 50 = 107. Therefore 7 ← **57** → 107 and so 57 x 2 = 114 = 7 + 107. Note here that we use this rule of digits to find the even which makes the sum of the two primes XnYY and YY.
7854126**13** – **13** = 785412600 : 2 = 392706300. And we have 13 + 392706300 = 392706313 and 392706313 + 392706300 = 785412613. Therefore, 13 ← **392706313** → 785412613.
If we do not have a prime at YY such the case of the prime number 785412791, we can still perform the calculation.
7854127**91 + 6** → 7854127**97** – **97** = 785412700 and 785412700 : 2 = 392706350 then 392706350 + 97 = 392706447 and 392706447 + 392706350 = 785412797. We have 97 ← 392706447 → 785412797 and therefore (97 + 6) ← 392706447 → (785412797 – 6). Then **(103) ← 392706447 → (785412791)**.

3g. Convert an even in sum of two primes by starting with close or neighboring prime numbers :

These rules will now be used to put Goldbach's strong conjecture into practice and thus convert an even number into the sum of two prime numbers. Although some specific numbers are used here, the method applies to any even > 4 as described.
The method is to look for two prime numbers around the even to convert into the sum of two prime numbers and put them in the form X**n...YY – YY** and from there convert it. Here is a detailed example but we will limit ourselves to a single coversion afterwards.
83 – 3 = 80 : 2 = 40 then 3 + 40 = 43 and 43 + 40 = 83.
3 ← 43 → 83 ↔ 3(+2) ← 43 (-1) → 83(- 4) ↔ (- 5) + (- 2) = -3. If subtracting **x** from the terms of the equation on the right or left is the same as subtracting **x/2** of the center. However the center remains unchanged if substracting and adding a same quantity from the two terms. In this example, we added 2 to 3 and substracted 4 from 83 which means substraction of 2 then we substract 1 from the center such that 3(**+2**) ← 43(**-1**) → 83(**- 4**) ↔ **5** ← **42** → **79**. We continue by the addition and substraction of equal 6n gaps on terms on left and right.
3(+2) ← 43(-1) → 83(- 4) ↔ **5** ← **42** → **79** ↔ 11 ← 42 → 73 ↔ 17 ← 42 → 67 ↔
23(+24) ← 42 → 61(-24) ↔ 47 ← 42 → 37 ↔ 53(+8) ← 42 → 31(-8) ↔ 61 ← 42 → 23
42 x 2 = 84 = 5 + 79 = 11 + 73 = 17 + 67 = 23 + 61 = 47 + 37 = 53 + 31 = 61 + 23.

Let us convert **240** in sum of two primes following the same method. First we put 240 in the form of **YXX and XX** so we can apply one of the rules shown above such like 240 = 70 + 170. The sum chosen must give numbers close to primes such that 67 and 73 for 70 or 173 for 170. The numbers 173 and 73 are in the form YXX and XX and can be used in a substraction like seen above.
173 – 73 = 100 : 2 = 50 and so we have 73 + 50 = 123 and 123 + 50 = 173. Therefore 73 ← **123** → 173 but
123 x 2 = 246 = 73 + 173. We therefore make (73 – 6) ← **123(-3)** → 173(-0) → (67) ← **120** → 173 →
240 = 67 + 173. Or 73 ← **123(-0)** → (173 – 6) → **240 = 73 + 167**.
In both (73 – 6) ← **123** → 173 and 73 ← **123** → (173 – 6) it is needed to equilibrate this way :
(73 – 6) ← **123 - 3** → 173 and 73 ← ***123 - 3*** → (173 – 6).
Let us convert a number like **1268**. Then using the same calculation as above let us pose
1268 = 38 + **1230**. We take 1268 + 6 = 1274 = 37 + 1237.
Then let take a close prime **1237**. Then 1237 – 37 = 1200 : 2 = 600.
We have 37 + 600 = 637 and 637 + 600 = 1237. We then have 37 ← 637 → 1237. But
637 x 2 = 1274 = 37 + 1237 = 1274 and 1274 – 1268 = 6. Therefore (37 – 6) ← 637(-3) → 1237 ↔ 31 ← 634 → 1237 and then **31 + 1237 = 1268**. Otherwise 37 ← 637 → (1237 – 6) → **37 + 1231 = 1268**. Or
37 (– 18) ← 637 → (1237 + 12) → **19 + 1249 = 1268**. In all these cases we have to substract 3 from the middle namely 637 – 3 = 634.

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1) Note that in both two examples cited above the conversion of 240 and 1268 we have a difference = 6n (n = 1) which is the best way to find out new primes and increase the combinations of primes in sums. In the case of 340 we start with 346 = 73 + 173 and in the latter we start with 1274 = 37 + 1237.
2) N**1** ← **M** → **N2** is an equation or a balance that we can modify by adding and subtracting the same quantity from both sides (N1 and N2). If we add and subtract a quantity x for from N1 and N2, we must add or subtract x/2 to or of M.

Let convert **18985474.** We add 6n to it like 18985474 + 6 = 18985480. Then examine the list of prime numbers so that we have 18985480 = 29 + 18985451. We then have 18985451 – 51 = 18985400 : 2 = 9492700 + 51 = 9492751 and then 9492751 + 9492700 = 18985451. Therefore 51 ← 9492751 → 18985451.
But 9492751 x 2 = 18985502 – **18985474** = 28. We then have to search in primes list to find out how to add them by standing close to 18985451. Then we have 51 (**- 28**) ← 9492751 (– **14**) → 18985451 and finally **23** ← **9492737** → **18985451**. Therefore, 9492737 x 2 = 18985474 = 23 + 18985451.

We will do the same method again but this time by drawing up a table and thus writing the step-by-step instructions to follow to apply this method. Let us see this with a number like **489776**. First 489776 + 6 = 489782 which is converted into the sum of two terms, the larger of which is prime such that for example 489782 = 15 + 489761. We then establish the balance equation between the two terms of the addition and the average at the center as shown above.
489761 – 61 = 489760 : 2 = 244880 + **61** = 244941, and then 244941 + 244880 = 489821.
Then **61** ← **244941** → **489821 (not prime)**. However, 244941 x 2 = 489882 as expected. We have
489882 – **489776** = 106. We will have to remove 106 units from this number to find our initial number **489776** while converting it into sums of two prime numbers.

- n	489821↓	Prime or not	+ n	61↑	Prime or not
106	489715	not	0	**61**	prime
6	489709	not	6	67	prime
6	489703	not	6	73	prime
6	489697	not	6	79	prime
6	489691	prime	6	85	not
12	***489679***	***prime***	***12***	***97***	***prime***
6	***489673***	***prime***	***6***	***103***	***prime***
60	***489613***	***prime***	***60***	***163***	***prime***
60	***489553***	***prime***	***60***	***223***	***prime***

Therefore we get 4 conversion of the number **489776** = 489679 + 97 ; **489776** = 489673 + 103 ; **489776** = 489613 + 163 ; and **489776** = 489553 + 223. This can be continued further.

Let take a number like 890 and then convert it in sum of two primes by manipulation of digits. We have 890 = 880 + 10 → 10 is either 7 + 3 or 9 + 1 to get a prime. We then pose the possible combinations either 883 + 7 or 881 + 9.
Therefore 880 = 883 + 7 but in the second case 9 is not prime, but we can transfer units from 881 to 9 and so :
881 – 4 = 877 and 9 + 4 = 13 → 890 = 13 + 877
881 – 22 = 859 and 9 + 22 = 31 → 890 = 31 + 859
Note we can not use a number with unit digit = 6 to add to 9 because we will get 5 ad thus not prime.
881 – 853 = 28 and 9 + 28 = 37 → 890 = 37 + 853
881 – 52 = 829 and 9 + 52 = 61 → 890 = 61 + 829
and so on.
Let us take another example. An even number has a unit digit = 0, 2, 4, 6, 8. The number 78956. Here we start with 78956 = 78950 + 6 with 6 = 1 + 5 or 6 = 3 + 3. Then we make 78956 = 7895**3** + **3** or 78956 = 7895**1** + **5**. However neither 78953 nor 78951 is prime. We can this time pose 78956 = 78940 + 16 with 16 = 15 + 1 ; 13 + 3 ; 11 + 5 ; and 9 + 7. Then 78956 = 7894**1** + **15** (not primes) ; 78956 = 7894**3** + **13** ; 78956 = 7894**9** + 7 but none are primes. However, all those sums give us chances to find out two primes that sum up. For instance, 78956 = 78953 + 3 = (78919 + 34) + 3 = 78919 + 37 (both primes).
78956 = 78953 + 3 = (78889 + 64) + 3 = 78889 + 67 (both primes).
78956 = 78953 + 3 = (78877 + 76) + 3 = 78877 + 79 (both primes) and so on.

Let E be an even number such that E = (A↓) + (1↑) or E = (A↓) + (7↑). A is an odd number that can be prime or composite but not 3n. So E = (A↓ **– 6n**) + (1↑ **+ 6n**) or E = (A↓ **– 6n**) + (7↑ **+ 6n**) such that A - 6n and 1 + 6n or 7 + 6n ($1 \leq n \leq +\infty$) will produce other odd numbers that are either composite or prime. Let us assume this time that A is in infinity and therefore (A - 6n↓) will tend to 0. Conversely, (1 + 6n↑) or (7 + 6n↑) will tend to infinity. We will therefore admit that A - 6n↓ or 1 + 6n↑ or 7 + 6n↑ will produce all the prime numbers that we know or that exist. Whether we start from infinity to 0 or from 0 to infinity, we will see the same prime numbers with the same gaps in opposite directions. Since every prime number occupies a position, we can predict with certainty that at times both A - 6n↓ and 1 + 6n↑ or 7 + 6n↑ are primes.
During this process, the equidistant prime numbers continue to add up, however large they may be. The gaps devoid of primes are compensated by the infinite inter-prime gaps that exist between the prime numbers taken two by two to infinity (Figure 1).

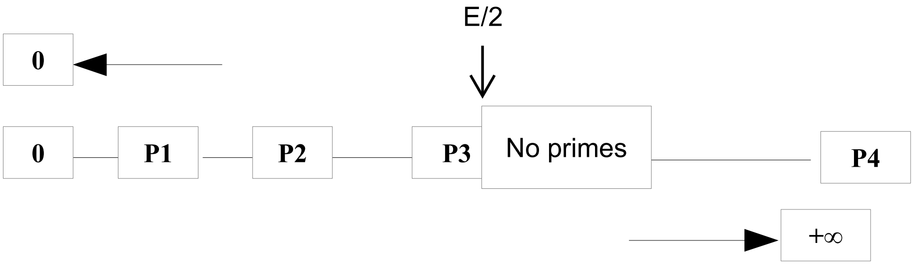

Figure 1 : It shows that while P4 **–** P3 might be relatively large because of the empty sequence of primes, the difference between P3 and P2 or P1 (which are close to 0) would be as large or even larger. Let us call P0 all primes close to 0. The very higher density of primes P0 increases the chances that a difference P3 – P0 = P4 – P3 which is a correct verification of the strong Goldbach conjecture. Assumig that P3 is very close to E/2 then P4 - E/2 = E/2 - P0 and thereofre P0 and P4 are equidistant P4 + P0 =E.

Discussion

The main idea of this article lies in the fact that Goldbach's strong conjecture is associated with the progression of natural integers from 0 to infinity, which results in precise gaps between prime numbers. The gap of 6 is the most regular between primes 6x + 1 on the one hand and primes 6x – 1 on the other. This progression means that any integer is preceded and followed by a prime number. Since all 2n distances are possible between the primes 6x + 1 and 6x – 1, it follows that a natural number is always located at an equal distance from two primes of which one precedes it and the other follows it. This paper shows that prime equidistance is omnipresent among integers.
Two new concepts are introduced in this article for the first time. The first is the Goldbach gap, noted here as t, which for any even number noted E > 4 means the gap separating its half E/2 from two symmetrically equidistant primes p and p' such that p' > p and such that E/2 - t = p and E/2 + t = p'. E/2 represents any natural number > 2 and since p ─E/2─p' then 2 x E/2 = E = p + p'. Without Goldbach's gaps, an even number would not be the sum of two primes. The second concept is the Goldbach transposition, which means that adding a given quantity to a number will result in a prime number, not necessarily in the same position, but somewhere between 0 and E, or in π(E). If an integer is an even 3n, we should add to it values of prime numbers in an increasing order like for example (E = 120, E/2 = 60, to get primes we do 60 + 7 ; 60 + 11 ; 60 + 13 ; 60 + 17 ; 60 + 19 ; 60 + 23 ;...). If it is non-3n, then odd 3 values should be added to it (like for example 80 + 3 ; 80 + 9 ; 80 + 15 ; 80 + 21 ; 80 + 27,...). Even numbers obviously differ in their values, but Goldbach's transposition means that there are common rules governing the appearance of equidistant primes.
Using 3x + 5 and 3x– 5 equations ad by examining the remainders (r) of euclidean divisions of 3x by prime factors q < its square root, if 5 = q – r, we can understand why a number 3x + 5 is prime or composite and in a similar way if 5 = r the 3x – 5 number is composite. Both equations show that any integer > 4 is bounded by two equidistant primes. This led to the development of a new algorithm for converting even numbers into the sum of two primes, whatever the even number in question. Given the equality or balance X─E/2─Y, we do (X +6n)-E/2─(Y -6n) until we obtain p'─E/2─p, which means 2 x E/2 = E = p' + p. This article is the first to offer this conversion algorithm. Others have used the equation 6x ± 1 to prove Goldbach's strong conjecture (M arkaris, 2013) or to locate the equidistant primes around composite numbers (Guiaso, 2019). However, the present article is original in that it uses a different equation 3x ± 5 by examining the remainders of Euclidean divisions, and furthermore shows that any integer > 4 is surrounded by equidistant primes and not just composites. In addition, this article is of practical interest, as it proposes two algorithms that could be easily programmed and links the key digits of the prime numbers to the gaps that separate them and thus to Goldbach's strong conjecture. Prime numbers digits can also be used to predict gaps between primes, and even to use a near or neighboring prime to convert a even into the sum of two primes. This approach is also original in this article. Using unit digits of primes, the proposed algorithm of even conversion in sum of two primes can be greatly improved by only focusing on primes whose unit digits are suitable to form the even.
All the data in this article argue strongly in favor of the truth of Goldbach's strong conjecture at infinity.

The other idea that prevails in mathematics is the following question: if we start from evens (and not with prime numbers as usually done), how many prime numbers will they be the sum of? However let's be precise, Goldbach did not rule that every *even* ***is only*** *the sum of two prime numbers*, but he just said that every even is the sum of two prime numbers. In mathematics, we sometimes emphasize propositions or conjectures with words like *only* or *if only if* to set the context, but Goldbach used no such emphasis. An even number can therefore be the sum of more than two prime numbers; the larger it is, the more it is the sum of several prime numbers, or even an infinity. An infinite even number is the sum of an infinity of even numbers, each of which is the sum of two prime numbers or even much more. But what Goldbach's conjecture means is that the sum of two prime numbers is the most common and invariable form of all evens. While two evens of different values can be sums of variable numbers of primes, Goldbach's conjecture tries to find a common and invariant and minimalist property, i.e. every even is the sum of TWO prime numbers.
Goldbach wanted an invariable law for all evens which unites them all and any law is defined in minimal conditions otherwise it would be subject to exceptions. Therefore the sum of two prime numbers would be this law common and true to all the even numbers (which nevertheless continue to vary by the possible number of prime numbers of which they can be the sum). This is undoubtedly the most decisive point and the deepest meaning of this conjecture. It would rather be wise to know if all the even numbers are the sum of two prime numbers at the same time as they are the sum of several prime numbers. However, this article shows that an integer can only be surrounded by two equidistant prime numbers at a time and therefore Goldbach's conjecture is correct. This article sets up an algorithm that starts with an even to convert in sums of two primes.
What this article shows is that any integer (E/2 or n) > 2 is in the middle of two or more equidistant prime numbers whose sum always gives the same even number (E or 2n) > 4. This configuration of numbers is natural, it is in this way that the natural numbers progress unit by unit to infinity. The natural numbers form a single set and therefore equidistant prime numbers will go to infinity.

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