Does QM = AIT?

Tariq Khan
Omaha, NE USA

A short philosophical and speculative essay proposing that quantum mechanics (QM) and algorithmic information theory (AIT) are actually equivalent, or reciprocal, means or operations of describing nature. An understanding of this relationship could provide a means to solve complex problems via a formal correspondence between continuous mathematics and discrete mathematics. Subjective attributes are noted between the operations and compared for their similarities as a first step to motivate additional technical research toward a formal and fundamental theorem of the relationship.

> *"Without mathematics we cannot penetrate deeply into philosophy.*
> *Without philosophy we cannot penetrate deeply into mathematics.*
> *Without both we cannot penetrate deeply into anything."*
> *-- Gottfried Wilhelm Leibniz*
>
> *"If it looks like a duck, walks like a duck, quacks like a duck, you got a duck"*
> *-- Dan Rather*

As physicists continued their pursuit of a deep understanding of the fundamental nature of reality, since the early 20th century, the best path forward has appeared to be quantum mechanics (QM). A key to quantum mechanics is the Born Rule (named after Max Born) that provides a probability (a value between 1 and 0) for an experiment's outcome given the complex number value from the Schrödinger wave function. The wave function values are inverted and then squared to produce the final probability.[1] It is commonly used to provide a probability density for finding a particle at a given position. The probability is proportional to the square of the amplitude of the system's wavefunction at that position.[1]

As computer scientists researched the foundations of math and computer science, the field of algorithmic information theory (AIT) developed towards the end of the 20th century. The field progressed form Kurt Gödel to Alan Turing to Gregory Chaitin and continues with others to this day.[2] The concept known as Chaitin's Constant in algorithmic information theory takes a sum of inverted binary string lengths to get a probability (a value between 0 and 1).[3] Chaitin's Constant, represented using the symbol Ω (the Greek letter Omega), is a mathematical number that represents the probability that a randomly generated computer program will eventually stop running (halt) on a theoretical computer with unlimited memory.[3,4,5,6] It is basically a measure of how likely it is for a random sequence of instructions to lead to a defined result, and because of its nature as a truly random number, it cannot be fully calculated but its digits can be approximated.[3] A more formal definition of Chaitin's Constant states:

> The constant Ω is then defined as where |p| denotes the length of a string p. This is an infinite sum which has one summand for every p in the domain of F. …this sum converges to a real number between 0 and 1. … Knowing the first N bits of Ω, one could calculate the halting problem for all programs of a size up to N. Let the program p, for which the halting problem is to be solved, be N bits long. In dovetailing fashion, all programs of all lengths are run, until enough have halted to jointly contribute enough probability to match these first N bits. If the program p has not halted yet, then it never will, since its contribution to the halting probability would affect the first N bits. Thus, the halting problem would be solved for p.[4,5,6]

Starting with just these simplified descriptions, one might already begin to consider if there could exist a relationship between quantum mechanics and algorithmic information theory? Might the action of summing inverses of bit sequences be equivalent to, or the inverse function of, the squaring of wave function amplitudes? At first glance the *locations of particles* and the *halting of programs* do not appear to be related, but neither, at first, were the concepts of derivatives (slopes of tangent lines or rates of change) and integrals (areas under curves or amounts accumulated over an interval) which are actuality inverses of each other as discovered by Newton and Leibniz and represented in the Fundamental Theorem of Calculus.[7]

Fundamentally it is the concept of *randomness* that is the bridge between quantum mechanics (the Born Rule) and algorithmic information theory (Chaitin's Constant). Both operations quantify the probability of an event occurring in a system and both are considered to be fundamentally *uncomputable* (due to inherent complexity) and share a deep connection to the idea of *incompleteness* in mathematics.[3,8,9,10] Note too that both of these operations involve analysis of samples of experiments or programs that run over periods of time. Both of these fields require this "comparison of trials" which, thus, requires the mathematics of *probability* to provide a result.

Quantum mechanics, by definition, shows a world that involves *discreteness*: quanta of energy, photons of light, identical particles, etc. Quantum mechanics also describes a reality, at first considered to contain hidden variables, which contains an impenetrable wall limiting the amount of information (precision) we can ever know about a system, e.g., the Heisenberg Uncertainty Principle where, the more one knows about the position of a particle, the less one can know about its momentum.[11] In a similar fashion, we can never know all the digits of Chaitin's Constant as they are literally proven to be not computable.[3,4,5,6]

Algorithmic information theory keys on the halting of computer programs (the Halting Problem) which is unsolvable.[12] The most difficult problem in quantum mechanics, and some argue in all of science, is the quantum measurement problem or the problem of quantum *decoherence* where a particle collapses into classical existence from a quantum field of nebulous probabilities.[13] Given the numerous arguments for the Simulation Hypothesis, might quantum decoherence actually be the same as the halting of a computer program?[14] Every electron in the universe, incredibly, is identical to every other electron as if created from a preordained recipe i.e., computer code, *before* it is observed or measured.

Also note the mysterious nature of *quantum entanglement* where two entangled particles will instantly collapse into specific states, even if billions of light-years apart, as soon as the state of one of them is measured.[15] This too seems very similar to computer program halting, even if the given program exists outside of, or on the boundary of, our universe, i.e., the Holographic Paradigm.[16]

Algorithmic information theory involves consideration of computers with infinite memory and limits of all programs (lengths in bits) of a given computer language, where quantum mechanics has a Path Integral formulation, from physicist Richard Feynman, which involves the "sum of all histories" to calculate the path of a light beam or particle.[17]

A formal understanding of this possible relationship (equivalent or inverse) could provide a means to solve complex problems by a correspondence between continuous mathematics and discrete mathematics. In mathematics there is infinite precision of real numbers while in computer science precision is finite.[4]

Note how algorithmic information theory and quantum mechanics both invert values to get probabilities. Is this perhaps an indication of a bridge between these fields? Is *randomness* the bridge between the fields of the discrete math of computer science and the continuous math of quantum mechanical wave functions? If so, might we thus discover a sort of *Rosetta Stone* mechanism where complex problems in one field can be transformed into the other field to allow for easier solutions akin to how the ADS/CFT correspondence of modern physics is used to solve problems in quantum field theories with string theories and the converse:

> *It also provides a powerful toolkit… Much of the usefulness of the duality results from the fact that it is a strong–weak duality: when the fields … are strongly interacting, the ones in the ….theory are weakly interacting and thus more mathematically tractable. This fact has been used to study many aspects …by translating problems in those subjects into more mathematically tractable problems.*[18]

Consider how a Rosetta Stone mechanism translating directly from quantum mechanics to binary AIT might show that a true "simulation" of reality may not require the use of a quantum computer.[19,20,21]

Figures

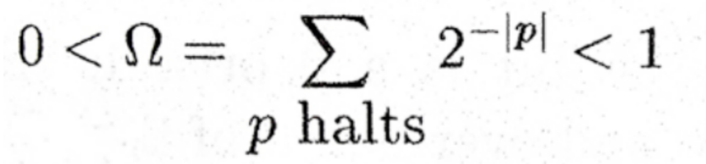
Figure 1.
Formal description of Chaitin's Constant from Gregory Chaitin's book Meta Math: the quest Omega.*3*

Nth approximation to Ω

Run each program up to *N* bits in size for *N* seconds.
Then each *p*-bit program you discover that halts contributes $1/2^p$
To this approximate value for Ω

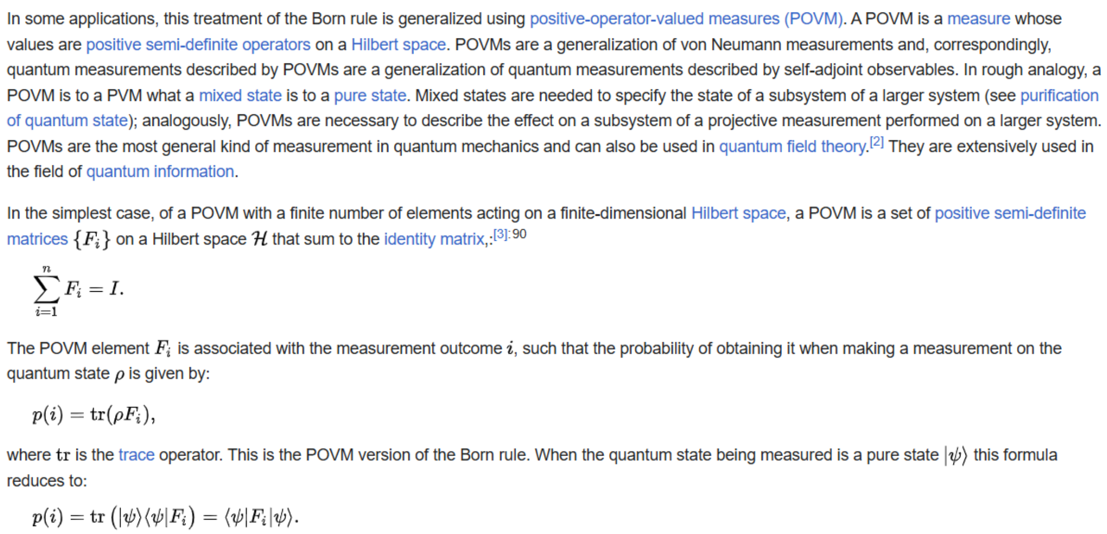
These approximate values get bigger and bigger (slowly!) and they
Approach Ω more and more closely, from below.

1st approx.. ≤ 2nd approx.. ≤ 3rd approx.. ≤ ….. ≤ Ω



$$0 < \Omega = \sum_{p \text{ halts}} 2^{-|p|} < 1$$

Figure 2.
Online elaboration of the Born Rule in quantum mechanics also shows a summation formula.1



In some applications, this treatment of the Born rule is generalized using positive-operator-valued measures (POVM). A POVM is a measure whose values are positive semi-definite operators on a Hilbert space. POVMs are a generalization of von Neumann measurements and, correspondingly, quantum measurements described by POVMs are a generalization of quantum measurements described by self-adjoint observables. In rough analogy, a POVM is to a PVM what a mixed state is to a pure state. Mixed states are needed to specify the state of a subsystem of a larger system (see purification of quantum state); analogously, POVMs are necessary to describe the effect on a subsystem of a projective measurement performed on a larger system. POVMs are the most general kind of measurement in quantum mechanics and can also be used in quantum field theory.[2] They are extensively used in the field of quantum information.

In the simplest case, of a POVM with a finite number of elements acting on a finite-dimensional Hilbert space, a POVM is a set of positive semi-definite matrices $\{F_i\}$ on a Hilbert space $\mathcal{H}$ that sum to the identity matrix,:[3]:90

$$\sum_{i=1}^{n} F_i = I.$$

The POVM element F_i is associated with the measurement outcome i, such that the probability of obtaining it when making a measurement on the quantum state ρ is given by:

$$p(i) = \operatorname{tr}(\rho F_i),$$

where tr is the trace operator. This is the POVM version of the Born rule. When the quantum state being measured is a pure state $|\psi\rangle$ this formula reduces to:

$$p(i) = \operatorname{tr}\left(|\psi\rangle\langle\psi|F_i\right) = \langle\psi|F_i|\psi\rangle.$$

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