The Natural Laws of Compressed Euler Wave Equations

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Abstract

Mathematical Functions exhibiting compression behavior under exponential transformations hold significant theoretical and practical implications. The functions $sin(e^x),\ cos(e^x),\ csc(e^x),\ sec(e^x),\ tan(e^x),\ and\ cot(e^x)$ demonstrate a unique form of contraction along the cartesian plane, compressing their oscillatory nature as $x \rightarrow \infty$. Through rigorous mathematical analysis (as well as Graphs), it is established that the peak points of $sin(e^x)$ and $cos(e^x)$ remain strictly within the set $\{1, -1, 0\}$, while $csc(e^x)$ and $sec(e^x)$ achieve local extrema solely at $y = \pm 1$ without ever reaching zero. Additionally, $tan(e^x)$ and $cot(e^x)$ lack local extrema entirely and exhibit vertical compression, approaching the form of asymptotic vertical structures as $x \rightarrow \infty$. These findings establish fundamental laws governing the behavior of trigonometric-exponential compositions, contributing to a deeper understanding of their mathematical properties and potential applications in physics and other potential applications.

Introduction

Exponential trigonometric functions exhibit a unique compression effect in their oscillatory behavior. The functions $sin(e^x)$ and $cos(e^x)$ do not follow the typical wave patterns of standard sine and cosine functions. Instead as x increases, their oscillations become increasingly compressed along the positive x-axis. Through careful analysis, I established three fundamental laws that define this compression effect and extend to their reciprocal functions, $csc(e^x)\ and\ sec(e^x)$, as well as the tangent-based functions, $tan(e^x)\ and\ cot(e^x)$. These laws describe how the highest and lowest points of these functions behave, their limits, and how their waveforms transform under exponential growth.

The Natural Laws of Compressed Euler Wave Equations

First law:

"The sine or cosine wave of the function e^x *compresses tighter and tighter as it moves through the positive x-axis of the Cartesian coordinate system. The highest and lowest peak points of the equations state that any value of x at those specific points will always equal to 1, -1 and 0"*

- Unlike standard sine and cosine functions, which oscillate with a fixed periodicity, the functions $sin(e^x)$ and $cos(e^x)$ exhibit continuous compression as x increases. Despite this compression, the fundamental peak values remain unchanged. The local maxima and minima of these functions will always be constrained within the range [-1,1], and their zero crossings remain consistent with traditional sine and cosine behavior.

Second law:

"The local maximums and minimums of $csc(e^x)$ and $sec(e^x)$ *will only occur at* $y\ =\pm\ 1$*, but they will never reach 0."*

- Since Cosecant and secant are the reciprocals of sine and cosine, their maximum and minimum behavior depends on when sine and cosine reach their peak values. The compression effect of e^x forces their oscillations to become denser, but their local extrema remains at 1 and -1. However, these functions will never reach zero, as division by zero is undefined.

Third law:

"There is no local maximum or minimum in $tan(e^x)$ and $cot(e^x)$*, and any points their graph reaches within the positive x-axis always result in zero. Their graphs compress so much that they approach the form of a vertical line as the value of x increases."*

- The functions $tan(e^x)$ and $cot(e^x)$ exhibit unbounded behavior as they approach their asymptotes. Since their period compresses with increasing x, their oscillations become denser, creating an illusion of a near-vertical line in the graph. Unlike sine and cosine, which have fixed peaks, tangent and cotangent do not have local extrema because they do not oscillate within a bounded range.

The Mathematical Proof of the Natural Laws of Compressed Euler wave equations

First law:

First, take the derivatives of the functions $f(x) = sin(e^x)$ and $g(x) = cos(e^x)$ to determine their critical points (Use the Chain Rule):

$f'(x) = cos(e^x) \cdot \frac{d}{dx}e^x = e^x cos(e^x)$

$g'(x) =- sin(e^x) \cdot \frac{d}{dx}e^x =- e^x sin(e^x)$

Now we got the derivative of these two functions. We know that Critical points occur when $f'(x) = 0$ or $g'(x) = 0$.

$Setting\ f'(x) = 0$:

$e^x cos(e^x) = 0$

Since $e^x \neq 0$ for all real x, we must have:

$cos(e^x) = 0$

And the general solutions for $cos(y) = 0$ are:

$e^x = \frac{\pi}{2} + k\pi,\ k \in Z$

Take the natural logarithm to get the value of x:

$x = ln(\frac{\pi}{2} + k\pi)$

We will do the same procedure with g’(x)=0:

$- e^x sin(e^x) = 0$

$sin(e^x) = 0$

$sin(y) = 0 -> e^x = k\pi, k \in Z$

$x = ln(k\pi)$

The two values of x are the critical points. Where k is an element of the set of integers Z.

After that, we can now conduct a second Derivative test for the Maxima and Minima:

$Differentiating\ f'(x)\ again$:

$f''(x) = e^x cos(e^x) + e^x(- sin(e^x)e^x)$

$f''(x) = e^x cos(e^x) - e^{2x} sin(e^x)$

At $x = ln(\frac{\pi}{2} + k\pi)$, we know that $cos(e^x) = 0$, so:

$f''(x) =- e^{2x} sin(e^x)$

Since $sin(e^x)$ alternates between 1 and -1 at these points, we check:

- $If\ sin(e^x) = 1,\ then\ f''(x) =- e^{2x} < 0 -> Local\ Maximum$
- $If\ sin(e^x) =- 1,\ then\ f''(x) = e^{2x} > 0 -> Local\ Minimum$

Thus, the local extrema of $f(x) = sin(e^x)$ always occur at $y =\pm 1$.

Doing the same thing with $g'(x)$:

$g''(x) = - e^{x}sin(e^{x}) - e^{2x}cos(e^{x})$

$g''(x) = - e^{2x}cos(e^{x})$

- $If\ cos(e^{x}) = 1,\ then\ g''(x) = - e^{2x} < 0\ \ -> \ Local\ Maximum$
- $If\ cos(e^{x}) = - 1,\ then\ g''(x) = e^{2x} > 0\ \ -> \ Local\ Minimum$

$Thus,\ The\ Local\ extrema\ of\ g(x) = cos(e^{x})\ also\ occur\ at\ y = \pm\ 1\ only$

This also means that both functions cross zero at specific logarithmic intervals, Proving the first law.

Second law:

$$f(x) = csc(e^{x}) = \frac{1}{sin(e^{x})}$$

$$g(x) = sec(e^{x}) = \frac{1}{cos(e^{x})}$$

$$f'(x) = - \frac{cos(e^{x})}{sin^{2}(e^{x})}(e^{x})$$

$$f'(x) = - e^{x}\frac{cos(e^{x})}{sin^{2}(e^{x})}$$

$$g'(x) = e^{x}\frac{sin(e^{x})}{cos^{2}(e^{x})}$$

$$f'(x) = 0:\ \ - e^{x}\frac{cos(e^{x})}{sin^{2}(e^{x})} = 0$$

$$e^{x} \neq 0,\ cos(e^{x}) = 0$$

$$General\ solutions\ for\ cos(y) = 0:$$

$$e^{x} = \frac{\pi}{2} + k\pi,\ k \in Z$$

$$x = ln(\frac{\pi}{2} + k\pi)$$

$$g'(x) = 0:\ e^{x}\frac{sin(e^{x})}{cos^{2}(e^{x})} = 0,\ e^{x} \neq 0,\ sin(e^{x}) = 0.\ sin(y) = 0:$$

$$x = ln(k\pi)$$

- Yet again, the values of x are the critical points of the two functions $f(x)\ and\ g(x)$.

Conducting the second Derivative test for Maxima and Minima:

$$f''(x) = - e^{x}[\frac{-sin(e^{x})sin^{2}(e^{x}) - 2cos(e^{x})sin(e^{x})cos(e^{x})}{sin^{4}(e^{x})}] + f'(x)e^{x}$$

$Since\ cos(e^x)\ =\ 0\ at\ our\ critical\ points:$

$f''(x)\ =-\ e^x(\frac{-sin(e^x)sin^2(e^x)}{sin^4(e^x)})$

$f''(x)\ =\ e^x\frac{sin(e^x)}{sin^2(e^x)}$

$At\ x\ =\ ln(k\pi),\ we\ know\ that\ sin(e^x)\ =\pm\ 1:$

- $If\ sin(e^x)\ =\ 1,\ then\ f''(x)\ =\ e^x\ >\ 0\ \ ->\ \ Local\ Minimum$
- $If\ sin(e^x)\ =-\ 1,\ then\ f''(x)\ =-\ e^x\ <\ 0\ \ ->\ \ Local\ Maximum$

The Local extrema of f(x)=$csc(e^x)$ always occur at y=$\pm$1.
Same procedure with g'(x):

$$g''(x)\ =\ e^x(\frac{cos(e^x)cos^2(e^x)-2sin(e^x)cos(e^x)sin(e^x)}{cos^4(e^x)})\ +\ g'(x)e^x$$

$sin(e^x)\ =\ 0\ at\ the\ critical\ points:$

$$g''(x)\ =\ e^x\frac{cos(e^x)}{cos^2(e^x)}$$

$At\ x\ =\ ln(k\pi),\ cos(e^x)\ =\pm 1:$

- $If\ cos(e^x)\ =\ 1,\ then\ g''(x)\ =\ e^x\ >\ 0\ \ ->\ \ Local\ Minimum$
- $If\ cos(e^x)\ =-\ 1,\ then\ g''(x)\ =-\ e^x\ <\ 0\ \ ->\ \ Local\ Maximum$

The Local Extrema of g(x) always occur at y=$\pm$1.

Since $csc(y)\ =\frac{1}{sin\ (y)}$ and $sec(y)\ =\frac{1}{cos(y)}$, these functions are undefined at y=0 because division by zero is not possible. Therefore, neither $csc(e^x)\ nor\ sec(e^x)$ ever reach zero. Thus, proving the second law.

Third law:

$f'(x)\ =\ sec^2(e^x)(e^x)$

$g'(x)\ =-\ csc^2(e^x)(e^x)$

$f'(x)\ =\ 0,\ e^x\ \neq\ 0:$

$sec^2(e^x)\ =\ 0$

However, $sec^2(y)\ =\ 1\ +\ tan^2(y)$ is always positive and never zero. So this has no solution. Which also means the function f(x) has no local extrema.

$g'(x)\ =\ 0\ ->\ \ -\ csc^2(e^x)(e^x)\ =\ 0\ \ ->\ \ e^x\ \neq\ 0:\ csc^2(e^x)\ =\ 0$

But $csc^2(y)\ =\ 1\ +\ cot^2(y)$ is always positive and never zero. So it also has no solutions neither a local extrema.

Since $tan(y) = 0$ at y=kπ (where k is any integer), We solve:

$e^x = k\pi -> x = ln(k\pi)$

This shows that $tan(e^x) = 0$ at infinitely many discrete points on the x-axis. Similarly, since $cot(y) = 0\ at\ y = \frac{\pi}{2} = k\pi$, we solve:

$e^x = \frac{\pi}{2} + k\pi -> x = ln(\frac{\pi}{2} + k\pi)$

Thus, $cot(e^x) = 0$ also occurs at infinitely many discrete points.
So it is confirmed. The two functions reach zero at specific points but not continuously.
Finally, let's prove that the graphs actually compress into a vertical line.

As $x \to \infty$, the exponential term e^x grows extremely fast. The periodic functions $tan(y)$ and cot(y) oscillate between $-\ \infty$ and $+\ \infty$ with period π, but their inputs e^x are exponentially increasing. This means that within very small changes in x, the function cycles through entire periods of oscillation. The frequency of these oscillations increases exponentially, making the graph appear compressed into a vertical pattern. Mathematically, the horizontal change required for a full period shrinks as x approaches infinity:

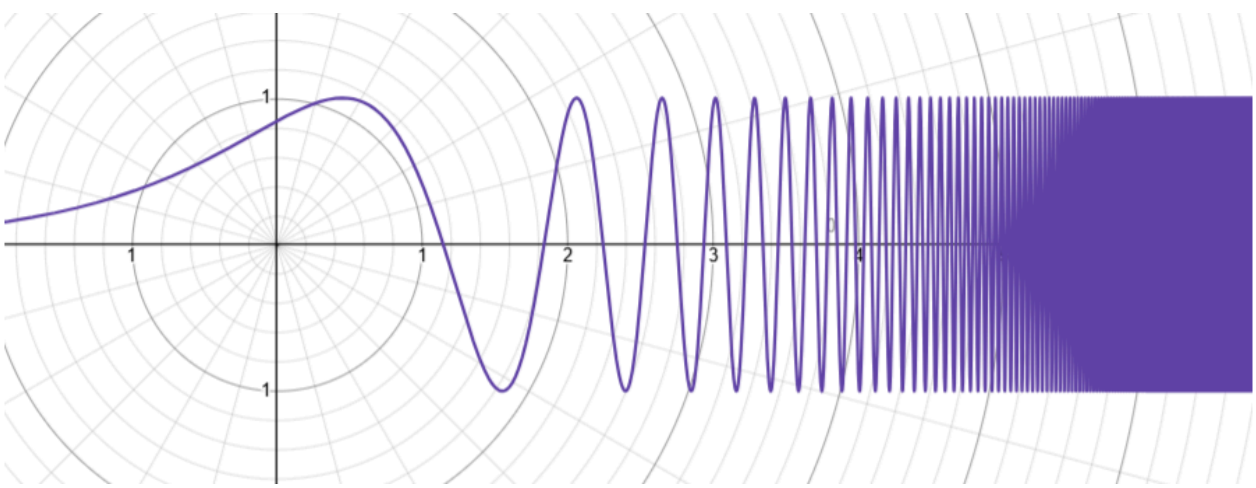
$\Delta x = ln((k + 1)\pi) - ln(k\pi) = ln(\frac{(k+1)\pi}{k\pi}) = ln(1 + \frac{1}{k})$

As $k \to \infty$:

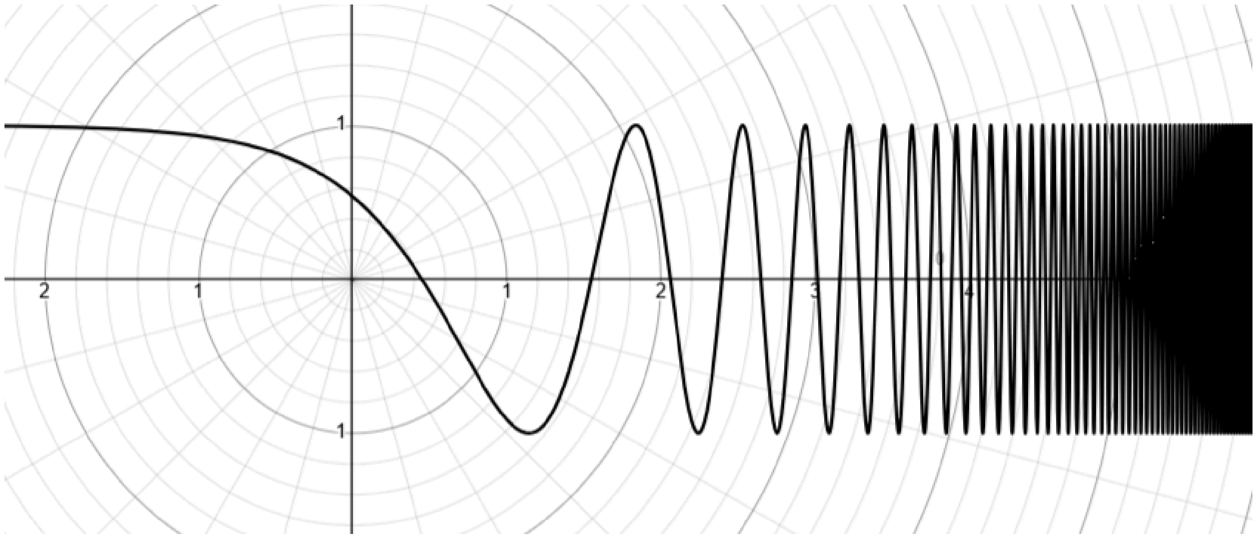
$$ln(1 + \frac{1}{k}) -> 0$$

Thus, the change in x required to complete one oscillation vanishes, making the function visually collapse into vertical lines. Proving the third law.

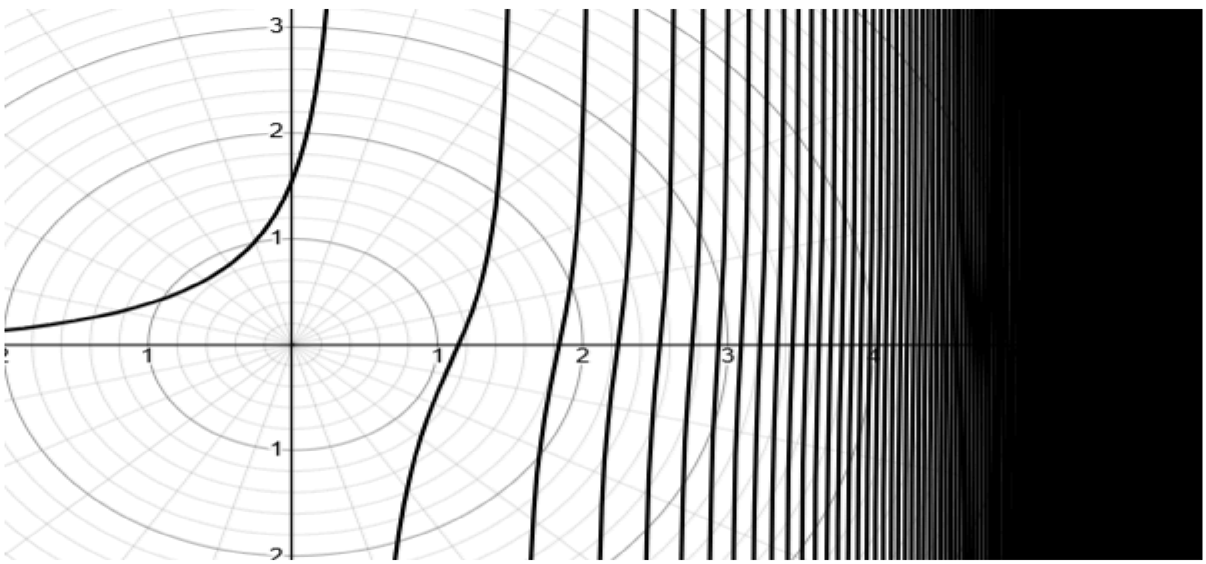
Graphical Visualization (Proof) of the Natural Laws of Compressed Euler Wave Equations



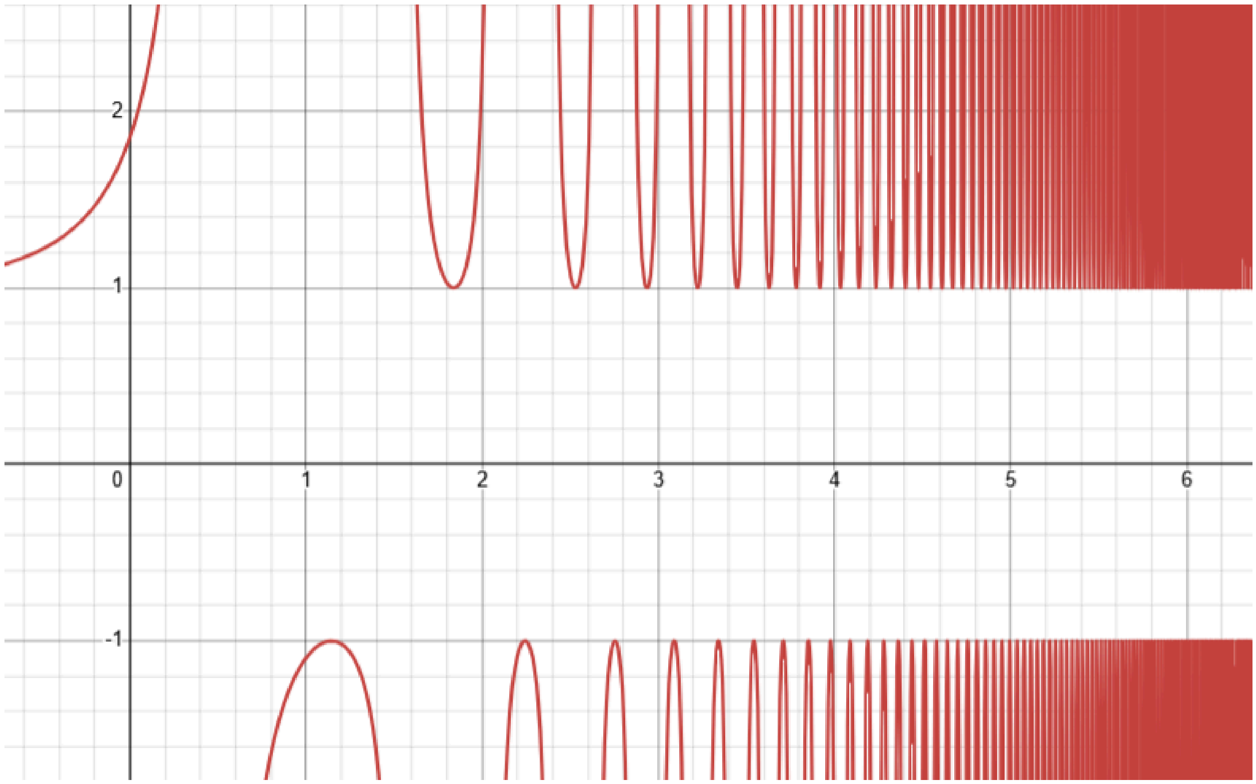
$sin(e^x)$



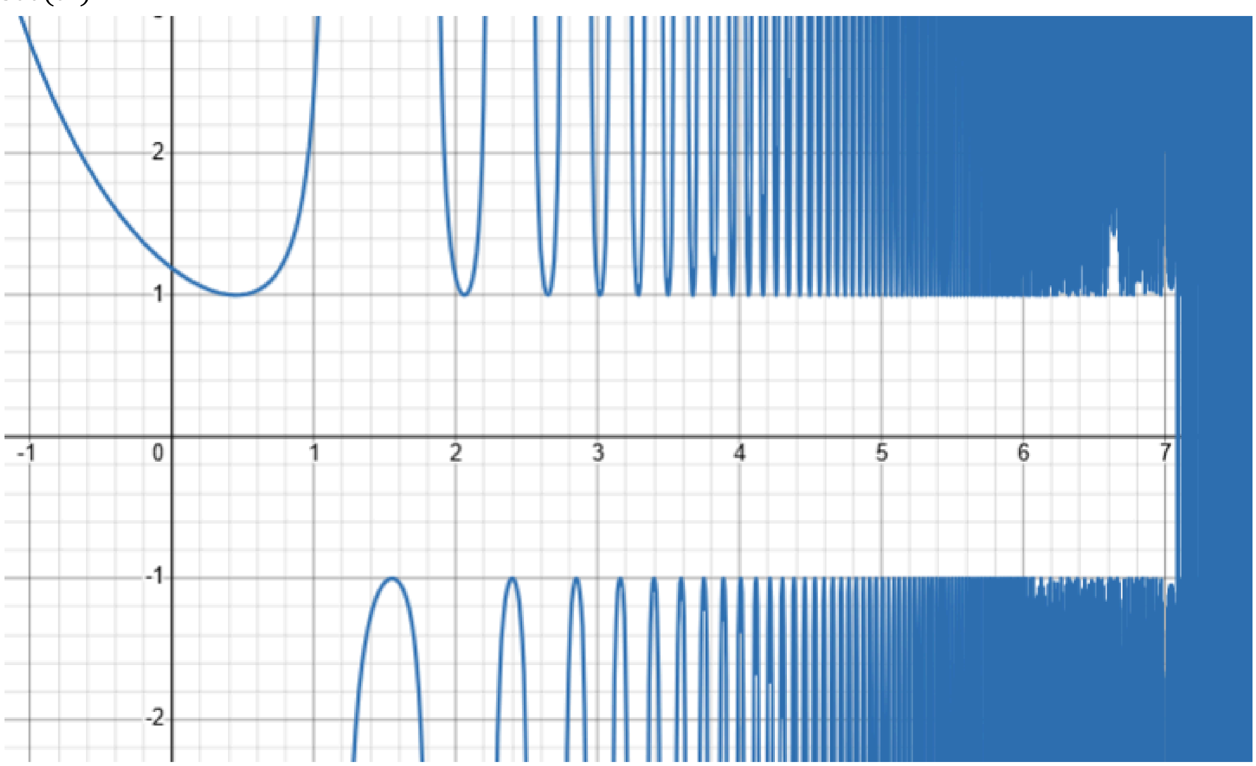
$cos(e^x)$



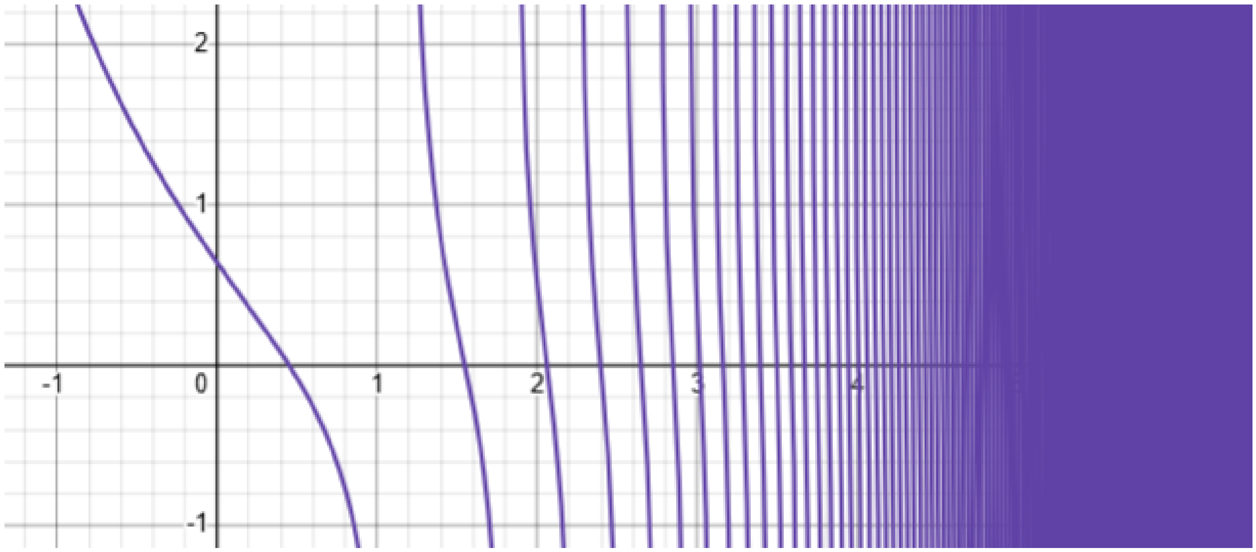
$Tan(e^x)$



$sec(e^x)$



$csc(e^x)$



$cot(e^x)$

Conclusion

The Study of Compressed Euler Wave Equations has revealed fundamental mathematical structures that govern the behavior of trigonometric functions when their inputs are transformed exponentially. Through rigorous derivation, Three natural Laws have been established, each describing a unique and intrinsic property of these compressed waves. Together, these natural laws form a comprehensive mathematical framework that explains how exponentiation influences trigonometric waveforms. The underlying structure of these equations showcases how standard wave behaviors—such as periodicity, extrema, and asymptotes—are modified when subject to an exponential input.

This research confirms that while the fundamental properties of trigonometric functions remain intact, their manifestation under exponential transformation follows a strict set of natural laws—laws that dictate how they contract, oscillate, and interact within the Cartesian coordinate system.