Tri-Quarter: Coordinates and Topological Zones with a Structured Orientation and Consistent Directional Separation Across the Complex Plane

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Abstract

In this paper, we introduce the Tri-Quarter Theorem, a new mathematical framework for defining a generalized complex-Cartesian-polar coordinate system on the complex plane $\mathbb{C}$. We discuss the coordinate system's construction and prove that the boundary zone T, identified as the unit circle, exhibits topological duality with the inner zone X_- ($||\vec{x}|| < 1$) and outer zone X_+ ($||\vec{x}|| > 1$), with a trichotomy-defined structured orientation that ensures consistent directional separation. This result unifies complex, Cartesian, and polar representations while offering new insights into topological separation and orientation. Its development was originally intended for quark confinement and black holes, but additional potential applications may include technologies that rely on complex numbers, boundary behaviors, or spatial partitioning in engineering, computing, and applied sciences. We discuss the coordinate system's construction and the theorem's proof. More work is needed to further develop and apply this framework via the methods of science and mathematics.

1 Introduction

The complex plane $\mathbb{C}$, viewed as a complex 1D Riemann surface, serves as a foundational structure in mathematics and its applications, bridging algebraic, geometric, and analytic perspectives. Traditional coordinate systems, such as Cartesian and polar forms, while powerful, often treat the unit circle as a mere boundary without fully exploiting its topological role. In this work, we address this gap by introducing the Tri-Quarter Theorem, which establishes a generalized complex-Cartesian-polar coordinate system with a new topological property.

Our main result demonstrates that the unit circle T is topologically dual to both the inner zone X_- ($||\vec{x}|| < 1$) and outer zone X_+ ($||\vec{x}|| > 1$), with a structured orientation derived from a trichotomy that ensures consistent directional separation across $\mathbb{C}$. This framework unifies three coordinate representations and elevates the unit circle's role from a passive boundary to an active separator with intrinsic properties. The significance lies in its potential to refine analyses and computations in fields requiring complex domain partitioning.

The original ideas behind this theorem were originally intended and developed for quark confinement and black holes, but additional potential applications may include sciences and technologies that rely on complex numbers and spatial partitioning where precise boundary handling and orientation can enhance modeling and design. In Section 2, we define the coordinate system, followed by the topological zones in Section 3, and the theorem's proof in Section 4. In Section 5, we conclude with a discussion of its origin, strengths, and potential future applications.

2 Generalized Complex-Cartesian-Polar Coordinates

Let $X = \mathbb{C}$ be the complex plane and complex 1D Riemann surface with the standard topology. We define the complex number-vector $\vec{x} = \vec{x}_{\mathbb{R}} + \vec{x}_{\mathbb{I}} = x_{\mathbb{R}} + x_{\mathbb{I}} \in X$ as a *position-point and position-vector* on X, such that

$$\vec{x}_{\mathbb{R}} \in \mathbb{R}^1 \times \{0\} = \{(x_{\mathbb{R}}, 0) \mid x_{\mathbb{R}} \in \mathbb{R}^1,\ 0 \in \{0\}\} \tag{1}$$

$$\vec{x}_{\mathbb{I}} \in \mathbb{I}^1 \times \{0\} = \{(0, x_{\mathbb{I}}) \mid 0 \in \{0\},\ x_{\mathbb{I}} \in \mathbb{I}^1\} \tag{2}$$

where $\mathbb{I}$ denotes imaginary (rather than irrational). It follows that $\vec{x} \in X$ is a Euclidean vector with *norm-amplitude* $||\vec{x}||$ and *phase* $\langle \vec{x} \rangle$, which are analogous to *magnitude* and *direction* in conventional notation. Furthermore, it follows that the orthogonal components of $\vec{x}$, namely $\vec{x}_{\mathbb{R}} \in \mathbb{R}^1 \times \{0\}$ and $\vec{x}_{\mathbb{I}} \in \{0\} \times \mathbb{I}^1$ as *axis-constrained* real and imaginary Euclidean vectors, respectively; they also have polar coordinates $(||\vec{x}_{\mathbb{R}}||, \langle \vec{x}_{\mathbb{R}} \rangle)$ and $(||\vec{x}_{\mathbb{I}}||, \langle \vec{x}_{\mathbb{I}} \rangle)$, respectively, such that $||\vec{x}_{\mathbb{R}}||, ||\vec{x}_{\mathbb{I}}|| \in [0, \infty)$, $\langle \vec{x}_{\mathbb{R}} \rangle \in \{0, \pi\}$, and $\langle \vec{x}_{\mathbb{I}} \rangle \in \{\frac{\pi}{2}, \frac{3\pi}{2}\}$ where the trichotomy axis-constraints are

$$\langle \vec{x} \rangle \in (0, \frac{\pi}{2}) \iff \langle \vec{x}_{\mathbb{R}} \rangle = 0 \text{ and } \langle \vec{x}_{\mathbb{I}} \rangle = \frac{\pi}{2} \text{ and } x_{\mathbb{R}} > 0 \text{ and } x_{\mathbb{I}} > 0 \tag{3}$$

$$\langle \vec{x} \rangle \in (\frac{\pi}{2}, \pi) \iff \langle \vec{x}_{\mathbb{R}} \rangle = \pi \text{ and } \langle \vec{x}_{\mathbb{I}} \rangle = \frac{\pi}{2} \text{ and } x_{\mathbb{R}} < 0 \text{ and } x_{\mathbb{I}} > 0 \tag{4}$$

$$\langle \vec{x} \rangle \in (\pi, \frac{3\pi}{2}) \iff \langle \vec{x}_{\mathbb{R}} \rangle = \pi \text{ and } \langle \vec{x}_{\mathbb{I}} \rangle = \frac{3\pi}{2} \text{ and } x_{\mathbb{R}} < 0 \text{ and } x_{\mathbb{I}} < 0 \tag{5}$$

$$\langle \vec{x} \rangle \in (\frac{3\pi}{2}, 2\pi) \iff \langle \vec{x}_{\mathbb{R}} \rangle = 0 \text{ and } \langle \vec{x}_{\mathbb{I}} \rangle = \frac{3\pi}{2} \text{ and } x_{\mathbb{R}} > 0 \text{ and } x_{\mathbb{I}} < 0 \tag{6}$$

with the boundary cases

$$\langle \vec{x} \rangle = 0 \iff \langle \vec{x}_{\mathbb{R}} \rangle = 0 \text{ and } \langle \vec{x}_{\mathbb{I}} \rangle = \nexists \text{ and } x_{\mathbb{R}} > 0 \text{ and } x_{\mathbb{I}} = 0 \tag{7}$$

$$\langle \vec{x} \rangle = \frac{\pi}{2} \iff \langle \vec{x}_{\mathbb{R}} \rangle = \nexists \text{ and } \langle \vec{x}_{\mathbb{I}} \rangle = \frac{\pi}{2} \text{ and } x_{\mathbb{R}} = 0 \text{ and } x_{\mathbb{I}} > 0 \tag{8}$$

$$\langle \vec{x} \rangle = \pi \iff \langle \vec{x}_{\mathbb{R}} \rangle = \pi \text{ and } \langle \vec{x}_{\mathbb{I}} \rangle = \nexists \text{ and } x_{\mathbb{R}} < 0 \text{ and } x_{\mathbb{I}} = 0 \tag{9}$$

$$\langle \vec{x} \rangle = \frac{3\pi}{2} \iff \langle \vec{x}_{\mathbb{R}} \rangle = \nexists \text{ and } \langle \vec{x}_{\mathbb{I}} \rangle = \frac{3\pi}{2} \text{ and } x_{\mathbb{R}} = 0 \text{ and } x_{\mathbb{I}} < 0 \tag{10}$$

where

$$x_{\mathbb{R}} = ||\vec{x}|| \cos\langle \vec{x} \rangle \text{ and } x_{\mathbb{I}} = ||\vec{x}|| \sin\langle \vec{x} \rangle \tag{11}$$

with Pythagorean form

$$||\vec{x}||^2 = ||\vec{x}_{\mathbb{R}}||^2 + ||\vec{x}_{\mathbb{I}}||^2 = x_{\mathbb{R}}^2 + x_{\mathbb{I}}^2, \quad \forall x \in X. \tag{12}$$

Thus, $\forall \vec{x} \in X \setminus \{0\}$ we define the 2D generalized complex-Cartesian-polar coordinate system as

$$(\vec{x}) = (\vec{x}_{\mathbb{R}} + \vec{x}_{\mathbb{I}}) = (\vec{x}_{\mathbb{R}}, \vec{x}_{\mathbb{I}}) = (x_{\mathbb{R}}, x_{\mathbb{I}}) = (||\vec{x}|| \cos\langle \vec{x} \rangle, ||\vec{x}|| \sin\langle \vec{x} \rangle) = (||\vec{x}||, \langle \vec{x} \rangle) \tag{13}$$

unifying complex, Cartesian, and polar representations. The trichotomy ensures a right-triangular structure via Equation (12). This system forms the foundation for the topological partitioning in Section 3.

3 Topological Zones

We use trichotomy to partition X into 3 distinct topological zones:

(1) the inner zone X_-,

(2) the boundary zone T, and

(3) the outer zone X_+.

$\forall \vec{x} \in X$ we know that precisely one of the following conditions must be satisfied:

$$||\vec{x}|| < 1 \iff \vec{x} \in X_- \tag{14}$$
$$||\vec{x}|| = 1 \iff \vec{x} \in T \tag{15}$$
$$||\vec{x}|| > 1 \iff \vec{x} \in X_+ \tag{16}$$

where clearly $X_- \cap T = T \cap X_+ = X_- \cap X_+ = \emptyset$ and $X_- \cup T \cup X_+ = X$, such that

$$X_- = \{\vec{x} \in X : ||\vec{x}|| < 1\} \tag{17}$$
$$T = \{\vec{x} \in X : ||\vec{x}|| = 1\} \tag{18}$$
$$X_+ = \{\vec{x} \in X : ||\vec{x}|| > 1\} \tag{19}$$

because the boundary zone T is a unit circle and the multiplicative group of all non-zero complex unit vectors separating X_- and X_+.

4 The Tri-Quarter Theorem

Here we prove the main result of this paper.

Theorem 4.1 (Tri-Quarter). In the generalized complex-Cartesian-polar coordinate system, the boundary zone T is topologically dual to both the inner zone X_- and outer zone X_+, with its trichotomy-defined structured orientation ensuring consistent directional separation across $X = \mathbb{C}$.

Proof. Let $X = \mathbb{C}$ be the complex 1D Riemann surface equipped with the 2D generalized complex-Cartesian-polar coordinate system given by Equation (13). Let $X_-, T, X_+ \subset X$ be given by Equations (14–17) so the boundary zone T separates X into the two disjoint open sets X_- and X_+ from the Jordan Curve Theorem, where

$$X \setminus T = X_- \cup X_+ \text{ and } X_- \cap T = T \cap X_+ = X_- \cap X_+ = \emptyset. \tag{20}$$

The coordinate system assigns phase values per Equations (3–7).

Next, we wish to show *structured orientation* on T. Now $\forall \vec{x} \in T$ we have $||\vec{x}|| = 1$ so

$$\vec{x} = (\vec{x}_{\mathbb{R}} + \vec{x}_{\mathbb{I}}) \tag{21}$$
$$= (\vec{x}_{\mathbb{R}}, \vec{x}_{\mathbb{I}}) \tag{22}$$
$$= (x_{\mathbb{R}}, x_{\mathbb{I}}) \tag{23}$$
$$= (||\vec{x}|| \cos\langle\vec{x}\rangle, ||\vec{x}|| \sin\langle\vec{x}\rangle) \tag{24}$$
$$= (1 \cdot \cos\langle\vec{x}\rangle, 1 \cdot \sin\langle\vec{x}\rangle) \tag{25}$$
$$= (\cos\langle\vec{x}\rangle, \sin\langle\vec{x}\rangle), \tag{26}$$

where the trichotomy of Equations (3–7) assigns the following structured orientation:

- $\langle\vec{x}\rangle \in (0, \frac{\pi}{2}) \Rightarrow (0, \frac{\pi}{2})$: all points in this arc of T have a positive real direction and a positive imaginary direction (e.g., $(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$).
- $\langle\vec{x}\rangle \in (\frac{\pi}{2}, \pi) \Rightarrow (\pi, \frac{\pi}{2})$: all points in this arc of T have a negative real direction and a positive imaginary direction (e.g., $(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$).

- $\langle\vec{x}\rangle \in (\pi, \frac{3\pi}{2}) \Rightarrow (\pi, \frac{3\pi}{2})$: all points in this arc of T have a negative real direction and a negative imaginary direction (e.g., $(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}})$).

- $\langle\vec{x}\rangle \in (\frac{3\pi}{2}, 2\pi) \Rightarrow (0, \frac{3\pi}{2})$: all points in this arc of T have a positive real direction and a negative imaginary direction (e.g., $(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}})$).

- Boundaries (e.g., $\langle\vec{x}\rangle = 0 \Rightarrow \vec{x} = (1, 0), (0, \nexists)$).

Thus, we can take any point on T to obtain a fixed phase pair; this defines a structured orientation that serves as an additional layer of encoded information.

Having established T's structured orientation, we now demonstrate its *simultaneous directional duality* with X_- and X_+. Since T separates X_- and X_+, we have $\partial X_- = T = \partial\overline{X}_+$, where ∂X_- is the boundary of the inner zone X_- and $\partial\overline{X}_+$ is the boundary of the closure of the outer zone X_+ with $\overline{X}_+ = \{\vec{x} \in X : ||\vec{x}|| \geq 1\}$. So we consider the following two cases:

- **Case 1: To X_-.** As the boundary of the inner zone (and open unit disk) X_-, T is approached radially by points in X_- along rays defined by $\langle\vec{x}\rangle$. So take a point $\vec{x} \in X_-$. Then $||\vec{x}|| < 1$. Now take $\langle\vec{x}\rangle \in (0, \frac{\pi}{2})$ as an example, with the trichotomy ensuring consistency across all $\langle\vec{x}\rangle \in [0, 2\pi)$. Then we have $\vec{x} = (||\vec{x}|| \cos\langle\vec{x}\rangle, ||\vec{x}|| \sin\langle\vec{x}\rangle)$ where $\vec{x}_{\mathbb{R}} > 0$ and $\vec{x}_{\mathbb{I}} > 0$. Then the system assigns $\langle\vec{x}_{\mathbb{R}}\rangle = 0$ and $\langle\vec{x}_{\mathbb{I}}\rangle = \frac{\pi}{2}$. So from within X_- as $||\vec{x}|| \to 1^-$, approaching T from inside, we have $\vec{x} \to (\cos\langle\vec{x}\rangle, \sin\langle\vec{x}\rangle) \in T$, so it follows that the phase assignment remains constant as $\langle\vec{x}_{\mathbb{R}}\rangle = 0$ and $\langle\vec{x}_{\mathbb{I}}\rangle = \frac{\pi}{2}$. This orientation (with a positive real direction and a positive imaginary direction) matches the inward approach from X_-.

- **Case 2: To X_+.** As the boundary of the closure of the outer zone X_+, T is approached radially by points in X_+ along rays defined by $\langle\vec{x}\rangle$. So take a point $\vec{x} \in X_+$. Then $||\vec{x}|| > 1$. Now take $\langle\vec{x}\rangle \in (0, \frac{\pi}{2})$ as an example, with the trichotomy ensuring consistency across all $\langle\vec{x}\rangle \in [0, 2\pi)$. Then we have $\vec{x} = (||\vec{x}|| \cos\langle\vec{x}\rangle, ||\vec{x}|| \sin\langle\vec{x}\rangle)$ where $\vec{x}_{\mathbb{R}} > 0$ and $\vec{x}_{\mathbb{I}} > 0$. Then (again) the system yields $\langle\vec{x}_{\mathbb{R}}\rangle = 0$ and $\langle\vec{x}_{\mathbb{I}}\rangle = \frac{\pi}{2}$. So from within X_+ as $||\vec{x}|| \to 1^+$, approaching T from outside, we have $\vec{x} \to (\cos\langle\vec{x}\rangle, \sin\langle\vec{x}\rangle) \in T$, so (again) it follows that the phase assignment remains constant as $\langle\vec{x}_{\mathbb{R}}\rangle = 0$ and $\langle\vec{x}_{\mathbb{I}}\rangle = \frac{\pi}{2}$. This orientation (with a positive real direction and a positive imaginary direction) matches the outward approach from X_+.

Therefore, $\forall\vec{x} \in T$, the assigned phase pair dualizes the approach: inward to X_- and outward to X_+; a consistent structured orientation dual to both X_- and X_+. This holds across all $\langle\vec{x}\rangle \in [0, 2\pi)$ where each point $\vec{x} \in T$ consistently identifies the transition from X_- to X_+.

Thus, T is topologically dual to X_- and X_+ with consistent directional separation across X. $\square$

5 Conclusion

I originally came up with the ideas for the Tri-Quarter Theorem back in 2012 when I was working on quark confinement [1]. In that work, the tri-quarter ideas are basically there but I never had the opportunity and realization to fully test, refine, and finalize them into a single consolidated theorem until now. Just recently, I started discussing these rusty old ideas with Grok v3 and it helped me "get my quadrant intervals squared away and dialed in" because apparently I was missing a couple things. Thanks Grok, you're the man.

This mathematical framework's strength resides in its unification of complex, Cartesian, and polar coordinates with a topological twist—duality and orientation across a boundary. This makes it versatile for:

- Problems involving separation of domains (inner vs. outer).
- Systems with circular symmetry or boundaries.
- Analyses requiring consistent directional or phase information in the complex plane.

My hypothesis is that this mathematical framework may be further developed and applied to help decipher great beasts like quark confinement [1] and black holes [2, 3, 4]. It could inspire or enhance technologies that rely on complex numbers, boundary behaviors, or spatial partitioning in engineering, computing, and applied sciences. Can it be applied to improve things like quantum computing hardware, cryptography, machine learning, digital signal processors, medical imaging devices, or wireless communication networks? Perhaps someday we will find out. Indeed, more work is needed to further develop and apply this framework via the methods of science and mathematics.

References

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