Fraction Algebra[1]

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This paper will explore fractions and introduce brand new concepts related to fractions. We will also introduce related ideas such as Iverson transformation, inverse Iverson transformation. We will also introduce theorems related to flooring and ceiling functions.

Prologue

Hello everyone, thank you for your kind and generous readership //:-D This is rather a serious research paper, but I will keep it as entertaining as possible. Please enjoy-

1. Fraction and Complement

Let's say there is a real number 2.3. Then, we can express that number using integers and fractions:

$$2.3 = 2 + 0.3 = 3 - 0.7$$

$$2.3 = \lfloor 2.3 \rfloor + \{2.3\} = \lceil 2.3 \rceil - /2.3/$$

In order, we used flooring function,[3] fraction function, ceiling function, and complement function above. Flooring of 2.3 is 2, so the flooring function extracts the integer part of 2.3. Faction of 2.3 is 0.3, so the fraction function extracts the fraction part of 2.3.

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[1] This paper is dedicated to the People in the world who support this author's 2028 US Presidential campaign: his social media and internet Friends (in DailyMotion, YouTube, Facebook, Instagram, TikTok, Clapper, SSRN, and VIXRA and other websites), his past and current in-person Friends, and his Family in Korea. Started being written on 1/26/2025. He's a secular-religious, politically independent, and a private academic. The author is running for the US President in 2028 as an independent thinker.

[2] A lawyer by trade, a scientist by hobby, a humanologist by mission, a U.S. Army veteran by record, a former computer programmer, a prior PhD candidate in computational biology (withdrawn after 2 years without a degree), a former actor/writer/director/indie-filmmaker/background-music-composer. Born in the USA, 1978. Grew up in Seoul, South Korea as a child and a teenager. Returned to America as a college student. Still growing up in Alaska, America as a person //!-)

[3] See https://en.wikipedia.org/wiki/Floor_and_ceiling_functions .

Ceiling of 2.3 is 3, so it makes 2.3 to the next bigger integer. Complement of 2.3 is 0.7 and it is the fraction that is needed to make 2.3 to 3. In summary,

$$\{x\} \equiv x - \lfloor x \rfloor$$

$$/x/ \equiv \lceil x \rceil - x$$

Fraction of x, we call it fraction x or curly x. Complement of x, we pronounce it as complement x or slash x. In our example above,

$$\{2.3\} = 2.3 - \lfloor 2.3 \rfloor = 2.3 - 2 = 0.3$$

$$/2.3/ = \lceil 2.3 \rceil - 2.3 = 3 - 2.3 = 0.7$$

Now, let's examine negative numbers inside the fraction function.

$$\{-2.3\} = -2.3 - \lfloor -2.3 \rfloor = -2.3 - (-3) = 0.7 = /2.3/$$

In general,

$$\{-x\} = /x/$$

And here is the proof:

$$\{-x\} = -x - \lfloor -x \rfloor = -x + \lceil x \rceil = /x/$$

We used the following property of the floor-ceiling conversion. To convert a floor to a ceiling, multiply -1 to both inside and outside of the floor function and then flip the floor to a ceiling:

$$\lfloor x \rfloor = -\lceil -x \rceil$$

Likewise,

$$\lceil x \rceil = -\lfloor -x \rfloor$$

Next, let's look at a negative number inside the complement function:

$$/-2.3/ = \lceil -2.3 \rceil - (-2.3) = -2 + 2.3 = 0.3 = \{2.3\}$$

In general,

$$/-x/ = \{x\}$$

And here is the proof:

$$/-x/ = \lceil -x \rceil - (-x) = -\lfloor x \rfloor + x = \{x\}$$

The interesting thing is thus this: fraction function behaves like multiplication by +1, while complement function behaves like multiplication by -1. The following properties reinforces that analogical notion:

$$//x//=/\lceil x\rceil - x/= \lceil \lceil x\rceil - x\rceil - (\lceil x\rceil - x) = \lceil x\rceil + \lceil -x\rceil - \lceil x\rceil + x = -\lfloor x\rfloor + x = \{x\}$$

That is, slash slash x is equal to curly x. It's like -1 time -1 equals +1. In general, applying complement function even times result in fraction function and applying complement function odd times result in complement function:

$$/x/^{2n}= \{x\}$$

$$/x/^{2n+1}=/x/$$

And that is analogous to:

$$(-1)^{2n} = +1$$

$$(-1)^{2n+1} = -1$$

Likewise,

$$\{x\}^n = \{x\}$$

And that is analogous to:

$$(+1)^n = +1$$

Next, let's examine the interactions between fraction function and complement function:

$$/\{x\}/=/x - \lfloor x\rfloor/= \lceil x - \lfloor x\rfloor\rceil - (x - \lfloor x\rfloor) = -\lfloor x\rfloor + \lceil x\rceil - x + \lfloor x\rfloor = \lceil x\rceil - x =/x/$$

And this is analogous to:

$$-1 * (+1) = -1$$

Likewise,

$$\{/x/\} = \{\lceil x\rceil - x\} = \lceil x\rceil - x - \lfloor \lceil x\rceil - x\rfloor = \lceil x\rceil - x - (\lceil x\rceil + \lfloor -x\rfloor) = -x + \lceil x\rceil =/x/$$

And this is analogous to:

$$+1 * (-1) = -1$$

In the two previous proofs above, we used the following well known properties of ceiling and flooring functions. An integer, n, inside of the ceiling and flooring functions can crawl out of the ceiling and flooring functions (we will call this "integer crawl out theorem"):

$$\lceil x + n\rceil = \lceil x\rceil + n$$

$$\lfloor x + n\rfloor = \lfloor x\rfloor + n$$

Flooring and ceiling functions are "integer functions", meaning that their outputs are integers. Non-integers, like 2.3, we will call them fractional numbers, or fractionals.

Analogously, we have "integer disappearance theorem". In both fraction function and complement function, an integer inside of the two functions disappears:

$$\{x+n\}=\{x\}$$

$$/x+n/=/x/$$

2. Distributive Properties

We will look at the distributive properties of fraction function, complement function, floor function, and ceiling function upon summation, subtraction, multiplication, and division. So we will deal with 16 situations.

(1) Floor Function and Summation

We want to find a function 'f' such that:

$$\lfloor x+y\rfloor = f(\lfloor x\rfloor, \lfloor y\rfloor)$$

Potentially,

$$\lfloor x+y\rfloor = \lfloor x\rfloor + \lfloor y\rfloor + e(x,y)$$

Where e(x,y) is an error correction term. Let's use the definition of fraction function:

$$\lfloor x+y\rfloor = \lfloor(\lfloor x\rfloor+\{x\})+(\lfloor y\rfloor+\{y\})\rfloor = \lfloor x\rfloor+\lfloor y\rfloor+\lfloor\{x\}+\{y\}\rfloor$$

(2) Floor Function and Subtraction

We will proceed likewise:

$$\lfloor x-y\rfloor = \lfloor(\lfloor x\rfloor+\{x\})-(\lfloor y\rfloor+\{y\})\rfloor = \lfloor x\rfloor-\lfloor y\rfloor+\lfloor\{x\}-\{y\}\rfloor$$

(3) Ceiling Function and Summation

We will proceed likewise. We will use floor/ceiling conversion formula:

$$\lceil x+y\rceil = \lceil(\lceil x\rceil - /x/)+(\lceil y\rceil - /y/)\rceil = \lceil x\rceil+\lceil y\rceil+\lceil -/x/-/y/\rceil = \lceil x\rceil+\lceil y\rceil-\lfloor /x/+/y/\rfloor$$

(4) Ceiling Function and Subtraction

We will proceed likewise:

$$\lceil x-y\rceil = \lceil(\lceil x\rceil - /x/) - (\lceil y\rceil - /y/)\rceil = \lceil x\rceil - \lceil y\rceil + \lceil -/x/+/y/\rceil = \lceil x\rceil + \lceil y\rceil - \lfloor /x/-/y/\rfloor$$

(5) Fraction Function and Summation

We want to find a function ‘f’ such that:

$$\{x+y\} = f(\{x\},\{y\})$$

Potentially,

$$\{x+y\} = \{x\} + \{y\} + e(x,y)$$

Where e(x,y) is an error correction term, which is a function of x and y. Let’s use the definition of fraction function, the distributive formula for flooring function and summation, and integer-crawl-out-property of the flooring function:

$$\{x+y\} = (x+y) - \lfloor x+y\rfloor = \big((\lfloor x\rfloor + \{x\}) + (\lfloor y\rfloor + \{y\})\big) - \lfloor(\lfloor x\rfloor + \{x\}) + (\lfloor y\rfloor + \{y\})\rfloor$$

$$= \big((\lfloor x\rfloor + \{x\}) + (\lfloor x\rfloor + \{y\})\big) - (\lfloor x\rfloor + \lfloor y\rfloor + \lfloor\{x\} + \{y\}\rfloor) = \{x\} + \{y\} - \lfloor\{x\} + \{y\}\rfloor$$

(6) Fraction Function and Subtraction

We will proceed likewise:

$$\{x-y\} = (x-y) - \lfloor x-y\rfloor = \big((\lfloor x\rfloor + \{x\}) - (\lfloor y\rfloor + \{y\})\big) - (\lfloor x\rfloor - \lfloor y\rfloor + \lfloor\{x\} - \{y\}\rfloor)$$

$$= \{x\} - \{y\} - \lfloor\{x\} - \{y\}\rfloor$$

(7) Complement Function and Summation

We will proceed likewise:

$$/x+y/ = \lceil x+y\rceil - (x+y) = \lceil(\lceil x\rceil - /x/) + (\lceil y\rceil - /y/)\rceil - ((\lceil x\rceil - /x/) + (\lceil y\rceil - /y/))$$

$$= (\lceil x\rceil + \lceil y\rceil + \lceil -/x/-/y/\rceil) - ((\lceil x\rceil - /x/) + (\lceil y\rceil - /y/))$$

$$= /x/+/y/ - \lfloor /x/+/y/\rfloor$$

(8) Complement Function and Subtraction

We will proceed likewise:

$$/x-y/=\lceil x-y\rceil-(x-y)=\lceil(\lceil x\rceil-/x/)-(\lceil y\rceil-/y/)\rceil-((\lceil x\rceil-/x/)-(\lceil y\rceil-/y/))$$

$$=(\lceil x\rceil-\lceil y\rceil+\lceil-/x/+/y/\rceil)-((\lceil x\rceil-/x/)-(\lceil y\rceil-/y/))$$

$$=/x/-/y/-\lfloor/x/-/y/\rfloor$$

(9) Floor Function and Multiplication

We will proceed likewise:

$$\lfloor x*y\rfloor=\lfloor(\lfloor x\rfloor+\{x\})*(\lfloor y\rfloor+\{y\})\rfloor=\left\lfloor\lfloor x\rfloor\lfloor y\rfloor+\lfloor x\rfloor\{y\}+\lfloor y\rfloor\{x\}+\{x\}\{y\}\right\rfloor$$

$$=\lfloor x\rfloor\lfloor y\rfloor+\left\lfloor\lfloor x\rfloor\{y\}+\lfloor y\rfloor\{x\}+\{x\}\{y\}\right\rfloor$$

(10) Ceiling Function and Multiplication

We will proceed likewise:

$$\lceil x*y\rceil=\lceil(\lceil x\rceil-/x/)*(\lceil y\rceil-/y/)\rceil=\left\lceil\lceil x\rceil\lceil y\rceil-\lceil x\rceil/y/-\lceil y\rceil/x/+/x//y/\right\rceil$$

$$=\lceil x\rceil\lceil y\rceil+\lceil-\lceil x\rceil/y/-\lceil y\rceil/x/+/x//y/\rceil=\lceil x\rceil\lceil y\rceil-\lfloor\lceil x\rceil/y/+\lceil y\rceil/x/-/x//y/\rfloor$$

(11) Fraction Function and Multiplication

We will proceed likewise. We will use integer disappearance theorem and curly addition distribution theorem from section (5):

$$\{x*y\}=\{(\lfloor x\rfloor+\{x\})*(\lfloor y\rfloor+\{y\})\}=\{\lfloor x\rfloor\lfloor y\rfloor+\lfloor x\rfloor\{y\}+\lfloor y\rfloor\{x\}+\{x\}\{y\}\}$$

$$=\{\lfloor x\rfloor\lfloor y\rfloor+\lfloor x\rfloor\{y\}+\lfloor y\rfloor\{x\}+\{x\}\{y\}\}=\{\lfloor x\rfloor\{y\}+\lfloor y\rfloor\{x\}+\{x\}\{y\}\}$$

$$=\{(\{x\}\{y\})+(\lfloor x\rfloor\{y\}+\lfloor y\rfloor\{x\})\}$$

$$=\left\{\{x\}\{y\}\right\}+\left\{\lfloor x\rfloor\{y\}+\lfloor y\rfloor\{x\}\right\}-\left\lfloor\left\{\{x\}\{y\}\right\}+\left\{\lfloor x\rfloor\{y\}+\lfloor y\rfloor\{x\}\right\}\right\rfloor$$

$$=\{x\}\{y\}+\left\{\lfloor x\rfloor\{y\}+\lfloor y\rfloor\{x\}\right\}-\left\lfloor\{x\}\{y\}+\left\{\lfloor x\rfloor\{y\}+\lfloor y\rfloor\{x\}\right\}\right\rfloor$$

Above, used the following property of the fraction function:

$$\left\{\{x\}\{y\}\right\}=\{x\}\{y\}$$

The reason for that is, when two fractions are multiplied, the resulting number is also a fraction.

In this paper, a fraction denotes a number that is equal to or bigger than 0, and also is less than 1.

(12) Complement Function and Multiplication

We will proceed likewise. We will use slash subtraction distribution theorem of section (8):

$/x*y/=/(\lceil x\rceil -/x/)*(\lceil y\rceil -/y/)/=/\lceil x\rceil\lceil y\rceil - \lceil x\rceil/y/-\lceil y\rceil/x/+/x//y//$

$=/(/x//y/)-(\lceil x\rceil/y/+\lceil y\rceil/x/)/$

$=/(/x//y/)/-/(\lceil x\rceil/y/+\lceil y\rceil/x/)/-\lfloor/(/x//y/)/-/(\lceil x\rceil/y/+\lceil y\rceil/x/)/\rfloor$

Now, let us look at:

$$/(/x//y/)/$$

In chapter 1, we have slash-curly conversion formula. Basically, in order to convert a slash to a curly, or vice versa, we need multiply -1 to the inside of that function:

$$/x/=\{-x\}$$

Then,

$/(/x//y/)/=\{-(/x//y/)\}$

For example,

$/(/0.2//0.3/)/=/0.8*0.7/=/0.56/=0.44$

Also, by the definition of the slash function,

$/(/x//y/)/=\lceil/x//y/\rceil -/x//y/$

Then, combining all these and using floor-ceiling conversion theorem, we get:

$/x*y/=/(/x//y/)/-/(\lceil x\rceil/y/+\lceil y\rceil/x/)/-\lfloor/(/x//y/)/-/(\lceil x\rceil/y/+\lceil y\rceil/x/)/\rfloor$

$=\lceil/x//y/\rceil -/x//y/-/(\lceil x\rceil/y/+\lceil y\rceil/x/)/+\lceil/(\lceil x\rceil/y/+\lceil y\rceil/x/)/-/(/x//y/)/\rceil$

This is quite a bulky formula and it does not has the form of:

$$/x * y/=/x/*/y/+e(/x/,/y/)$$

But still, we did indeed find a function that expresses /x*y/ in terms of /x/*/y/ :

$$/x * y/= f(/x/*/y/)$$

Then a more practical question would be, is the lengthy formula we found above has some practical applications or values? Well, that is for the future scientists to find out. Here, in humanology school, we proceed in mathematics with intellectual curiosity and the spirit of scientific discovery and academic exploration and the enjoyment of creativity. The process of discovering fraction algebra is chronicled in humanology episodes filmed and published in late 2024 and early 2025.[4]

(13) Floor Function and Division

For divisions, we will use an algebraic trick. It does look tautological, but we will achieve our goal of expressing a floor function in a division distribution form:

$$\left\lfloor \frac{x}{y} \right\rfloor = \left\lfloor \frac{x + y * \frac{\lfloor x \rfloor}{\lfloor y \rfloor} - y * \frac{\lfloor x \rfloor}{\lfloor y \rfloor}}{y} \right\rfloor = \left\lfloor \frac{\lfloor x \rfloor}{\lfloor y \rfloor} + \frac{x}{y} - \frac{\lfloor x \rfloor}{\lfloor y \rfloor} \right\rfloor = \left\lfloor \left\lfloor \frac{\lfloor x \rfloor}{\lfloor y \rfloor} \right\rfloor + \left\{ \frac{\lfloor x \rfloor}{\lfloor y \rfloor} \right\} + \frac{x}{y} - \frac{\lfloor x \rfloor}{\lfloor y \rfloor} \right\rfloor$$

$$= \left\lfloor \frac{\lfloor x \rfloor}{\lfloor y \rfloor} \right\rfloor + \left\lfloor \left\{ \frac{\lfloor x \rfloor}{\lfloor y \rfloor} \right\} + \frac{x}{y} - \frac{\lfloor x \rfloor}{\lfloor y \rfloor} \right\rfloor = f\left(\frac{\lfloor x \rfloor}{\lfloor y \rfloor} \right)$$

(14) Ceiling Function and Division

We will proceed likewise:

$$\left\lceil \frac{x}{y} \right\rceil = \left\lceil \frac{\lceil x \rceil}{\lceil y \rceil} \right\rceil + \left\lceil \left\{ \frac{\lceil x \rceil}{\lceil y \rceil} \right\} + \frac{x}{y} - \frac{\lceil x \rceil}{\lceil y \rceil} \right\rceil$$

[4] For example, see https://www.dailymotion.com/video/x9ax090 .

(15) Fraction Function and Division

We will proceed likewise:

$$\left\{\frac{x}{y}\right\} = \left\{\frac{x + y * \frac{\{x\}}{\{y\}} - y * \frac{\{x\}}{\{y\}}}{y}\right\} = \left\{\frac{\{x\}}{\{y\}} + \frac{x}{y} - \frac{\{x\}}{\{y\}}\right\} = \left\{\left\lfloor\frac{\{x\}}{\{y\}}\right\rfloor + \left\{\frac{\{x\}}{\{y\}}\right\} + \frac{x}{y} - \frac{\{x\}}{\{y\}}\right\}$$

$$= \left\{\left\{\frac{\{x\}}{\{y\}}\right\} + \frac{x}{y} - \frac{\{x\}}{\{y\}}\right\} = f\left(\frac{\{x\}}{\{y\}}\right)$$

(16) Fraction Function and Division

We will proceed likewise:

$$/\frac{x}{y}/=/\frac{/x/}{/y/} + \frac{x}{y} - \frac{/x/}{/y/}/=/\left\lfloor\frac{/x/}{/y/}\right\rfloor + \left\{\frac{/x/}{/y/}\right\} + \frac{x}{y} - \frac{/x/}{/y/}/=/\left\{\frac{/x/}{/y/}\right\} + \frac{x}{y} - \frac{/x/}{/y/}/= f(\frac{/x/}{/y/})$$

Alternatively,

$$/\frac{x}{y}/=/\frac{/x/}{/y/} + \frac{x}{y} - \frac{/x/}{/y/}/=/\left\lceil\frac{/x/}{/y/}\right\rceil - /\frac{/x/}{/y/}/ + \frac{x}{y} - \frac{/x/}{/y/}/=/-/\frac{/x/}{/y/}/ + \frac{x}{y} - \frac{/x/}{/y/}/$$

$$= \{/\frac{/x/}{/y/}/ - \frac{x}{y} + \frac{/x/}{/y/}\} = g(\frac{/x/}{/y/})$$

Above, we used slash-curly conversion formula to convert slash to curly, by multiplying -1 to the terms inside the slash function.

3. Iverson Transformation and Inverse Iverson Transformation

(1) Analogy with Fourier Transformation[5] and Iverson Bracket

Fourier Transformation expresses any arbitrary continuous function[6] with infinite summation of sines and cosines. Inverse Fourier Transformation retrieves the original function from the sine/cosine infinite summation form, i.e., the Fourier form of the original function.

Similarly and surprisingly, Iverson transformation expresses any arbitrary discontinuous (a.k.a. discrete) function with finite summation of Iverson brackets.[7] Inverse Iverson

[5] See https://en.wikipedia.org/wiki/Fourier_transform .

[6] Of course, discontinuous functions like saw tooth function can be approximated by Fourier transform as well. See https://en.wikipedia.org/wiki/Sawtooth_wave .

[7] See https://en.wikipedia.org/wiki/Iverson_bracket .

transformation converts the function in Iverson form into a more regular algebraic form, using a finite summation of ceiling functions and floor functions.

Iverson bracket is a Boolean function whose input is a propositional logical[8] function p(x) and whose output is binary, i.e., 0 or 1. For instance,

$$p(x) = [p(x)] = [x > 0] = \left\{ \frac{1, if\ x > 0}{0, if\ x \leq 0} \right\}$$

Above, the curly bracket does not denote a fraction function but it just means the outputs bifurcates depending on the value of x. The central line above is merely a delimiter and it does not denote a fraction.

It is a known fact that:

$$[p(x) \wedge q(x)] = p(x) * q(x)$$

$$[p(x) \vee q(x)] = p(x) + q(x) - p(x) * q(x)$$

$$[\sim p(x)] = 1 - p(x)$$

We will call them Iverson conjunction formula, Iverson disjunction formula, and Iverson negation formula, respectively.

(2) Iversonization of a Function

We will say "let's iversonize this function" when we conduct an Iverson transform on a function. Let's design an arbitrary discontinuous function:

$$f(x) = \left\{ \begin{array}{c} 2^x, if\ x < 1 \\ \hline 3, if\ x = 1 \\ \hline 5, if\ 1 < x < 2 \\ \hline 3, if\ x = 2 \\ \hline x^2, if\ x > 2 \end{array} \right\}$$

The iversonized function would be:

$$f(x) = 2^x * [x < 1] + 3 * [x = 1] + 5 * [1 < x < 2] + 3 * [x = 2] + x^2 * [x > 2]$$

Note that the function does not need to be one-to-one (i.e. injective) function in order to be iversonized. The only requirement for a function to be iversonizable is that it is a traditional function, that is, a function where an x value maps to a one and only one f(x) value.

[8] See https://en.wikipedia.org/wiki/Propositional_calculus .

(3) De-iversonization of a Function

We will say "let's de-iversonize this function" when we conduct an inverse Iverson transform on a function. To de-iversonize a function is to convert the iversonized function (i.e., a function that contains Iverson brackets) to a de-iversonized function (i.e., a function that does not contain Iverson brackets). De-iversonized functions will contain floor functions and/or ceiling functions, just like Fourier-formed functions contain sines and cosines.

Let us start with an easy example:

$$[x = 3]$$

Our goal is to express the above Iversonized function into a function that does not contain Iverson brackets. Basically, this is a mapping problem and we may call it, mappology. Also, we may call this approach as mathematical engineering, as we are approaching mathematics like an engineer. We are designing an algebraic function that is equivalent to the Iversonized function and there can be many equivalent algebraic functions.

We want to find an algebraic function that maps 3 to 1, and that maps all other numbers to 0. Can our kind and knowledgeable reader think of one such function? The author wants this discourse to be more interactive, so he gives the reader 5 minutes to think about it.

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One way to do it is to use exponential functions and flooring functions:

$$[x = 3] = \lfloor 2^{x-3} \rfloor * \lfloor 2^{3-x} \rfloor$$

An exponential function is always positive and it becomes 1 only when x is 0. So we're taking advantage of that property and also the floor function as well. Of course, the base of the exponential function needs not be 2, but it can be any positive number.

Now, can the reader think of an alternative algebraic function that does not involve an exponential function, as it is an expensive function, computationally speaking. The author gives the reader, 5 minutes, as before.

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An alternative way to do it is to use absolute value function and a floor function:

$$[x = 3] = \left\lfloor \frac{1}{|x-3| + 1} \right\rfloor$$

Of course, the 1 in the denominator and the 1 in the numerator, they don't have to be two 1's. They could be any same positive number that appears both upstairs and downstairs of the fractional number inside the flooring function.

(4) De-iversonized Kronecker Delta[9]

First, let's review the traditional, bifurcated definition of Kronecker delta function:

$$\delta_i^j = [i = j] = \left\{ \frac{1, if\ i = j}{0, if\ i \neq j} \right\}$$

Like before, there are two ways to do this. First, exponential form:

$$\delta_i^j = \lfloor a^{i-j} \rfloor * \lfloor a^{j-i} \rfloor \ , where\ a > 0\ and\ a \neq 1$$

This works because an exponential function, $y=a^x$, is more than 1 on one side and less than 1 on the other side of the y-axis and they're always positive. An exponential function is 1 only when x is 0.

Now, let's give it a formal proof.

(proof begins)

First, assume i=j.

$$\delta_i^i = \lfloor a^{i-i} \rfloor * \lfloor a^{i-i} \rfloor = \lfloor a^{i-i} \rfloor * \lfloor a^{i-i} \rfloor = \lfloor a^0 \rfloor * \lfloor a^0 \rfloor = \lfloor 1 \rfloor * \lfloor 1 \rfloor = 1$$

Next, assume i>j and 0<a<1. Let b=1/a. Then b>1.

$$\delta_i^j = \lfloor a^{i-j} \rfloor * \lfloor a^{j-i} \rfloor = \lfloor b^{j-i} \rfloor * \lfloor b^{i-j} \rfloor = \left\lfloor \frac{1}{b^{i-j}} \right\rfloor * \lfloor b^{i-j} \rfloor = 0 * \lfloor b^{i-j} \rfloor = 0$$

Next, assume i>j and a>1.

$$\delta_i^j = \lfloor a^{i-j} \rfloor * \lfloor a^{j-i} \rfloor = \lfloor a^{i-j} \rfloor * \left\lfloor \frac{1}{a^{i-j}} \right\rfloor = \lfloor a^{i-j} \rfloor * 0 = 0$$

Next, assume i<j and 0<a<1. Let b=1/a. Then b>1.

$$\delta_i^j = \lfloor a^{i-j} \rfloor * \lfloor a^{j-i} \rfloor = \lfloor b^{j-i} \rfloor * \lfloor b^{i-j} \rfloor = \lfloor b^{j-i} \rfloor * \left\lfloor \frac{1}{b^{j-i}} \right\rfloor = \lfloor b^{j-i} \rfloor * 0 = 0$$

Next, assume i<j and a>1.

$$\delta_i^j = \lfloor a^{i-j} \rfloor * \lfloor a^{j-i} \rfloor = \left\lfloor \frac{1}{a^{j-i}} \right\rfloor * \lfloor a^{j-i} \rfloor = 0 * \lfloor a^{j-i} \rfloor = 0$$

(proof ends)

[9] See https://en.wikipedia.org/wiki/Kronecker_delta .

Next, non-exponential form:

$$\delta_i^j = \left\lfloor \frac{a}{|i-j|+a} \right\rfloor \text{ , } where\ a > 0$$

This works because when i=j, a/a is 1. When i is not equal to j, |i-j| is a positive number and it makes the denominator bigger than the numerator in the fractional inside of the flooring function, making it zero.

Now, let's give it a formal proof:

(proof begins)

First, assume i=j.

$$\delta_i^i = \left\lfloor \frac{a}{|i-i|+a} \right\rfloor = \left\lfloor \frac{a}{a} \right\rfloor = \lfloor 1 \rfloor = 1$$

Next, assume i is not equal to j. Then, |i-j|=b, where b>0.

$$\delta_i^j = \left\lfloor \frac{a}{|i-j|+a} \right\rfloor = \left\lfloor \frac{a}{b+a} \right\rfloor = 0$$

(proof ends)

The formulas above are most likely to be brand new discoveries as of today, 2/1/2025. A lesson here is thus this: you don't have to be a very advanced mathematician in order to discover a new mathematical formula, equation, or a theorem. The mathematics involved here is more or less a high school level. The author is an amateur mathematician who neither majored nor minored in mathematics. His math level is about intermediate.

(5) De-iversonization of Sign Function[10]

Let us review the traditional trifurcated definition of the sign function (a.k.a., signum):

$$sgn(x) = \left\{ \begin{array}{c} 1, if\ x > 0 \\ \hline 0, if\ x = 0 \\ \hline -1, if\ x < 0 \end{array} \right\}$$

In Wikipedia, as of 2/1/2025, they have a very nice Iverson expression of the sign function:

$$sgn(x) = \left\lfloor \frac{x}{|x|+1} \right\rfloor - \left\lfloor \frac{-x}{|x|+1} \right\rfloor$$

[10] See https://en.wikipedia.org/wiki/Sign_function .

Of course, we can improve upon that, with the new knowledge of fraction algebra. We will use floor-ceiling conversion formula and also generalize the formula further:

$$sgn(x) = \left\lfloor \frac{x}{|x| + a} \right\rfloor + \left\lceil \frac{x}{|x| + a} \right\rceil \ , where\ a > 0$$

Now, let's prove it formally:

(proof begins)

First, assume x=0.

$$sgn(0) = \left\lfloor \frac{0}{|0| + a} \right\rfloor + \left\lceil \frac{0}{|0| + a} \right\rceil = \lfloor 0 \rfloor + \lceil 0 \rceil = 0 + 0 = 0$$

Next, assume x>0.

$$sgn(x) = \left\lfloor \frac{x}{|x| + a} \right\rfloor + \left\lceil \frac{x}{|x| + a} \right\rceil = \left\lfloor \frac{|x|}{|x| + a} \right\rfloor + \left\lceil \frac{|x|}{|x| + a} \right\rceil = 0 + 1 = 1$$

Next, assume x<0.

$$sgn(x) = \left\lfloor \frac{x}{|x| + a} \right\rfloor + \left\lceil \frac{x}{|x| + a} \right\rceil = \left\lfloor \frac{-|x|}{|x| + a} \right\rfloor + \left\lceil \frac{-|x|}{|x| + a} \right\rceil = -1 + 0 = -1$$

(proof ends)

(6) Generalization of Sign Function

In mathematics, there are many ways to generalize a given formula. In Wikipedia, as of 2/1/2025, they generalized the sign function using complex numbers or matrix in linear algebra. Here in the humanology school, we will generalize the sign function in real number world, as follows:

$$sgn_a^b(x) = \left\{ \begin{array}{c} 1, if\ x > b \\ \hline 0, if\ a \le x \le b \\ \hline -1, if\ x < a \end{array} \right\}$$

Basically, we are enlarging the middle part of the sign function, increasing the bandwidth of a communication channel, so to speak.

Well, first, let's iversonize the expanded sign function above:

$$sgn_a^b(x) = 1 * [x > b] + 0 * [a \le x \le b] + (-1) * [x < a]$$

Then, we need to learn how to de-iversonize [x>b] and [x<a]. We'll do that in the next chapter. For now, let's make it backward compatible:

$$sgn_0^0(x) = \left\{ \begin{array}{c} \frac{\frac{1, if\ x > 0}{0, if\ 0 \le x \le 0}}{-1, if\ x < 0} \end{array} \right\}$$

$$sgn(x) = sgn_0^0(x) = \left\lfloor \frac{x}{|x| + a} \right\rfloor + \left\lceil \frac{x}{|x| + a} \right\rceil \ , where\ a > 0$$

The horizontal shifting of the function above by ‘c’ would be:

$$sgn_c^c(x) = \left\{ \begin{array}{c} \frac{\frac{1, if\ x > c}{0, if\ c \le x \le c}}{-1, if\ x < c} \end{array} \right\}$$

$$sgn_c^c(x) = \left\lfloor \frac{x - c}{|x - c| + a} \right\rfloor + \left\lceil \frac{x - c}{|x - c| + a} \right\rceil \ , where\ a > 0$$

Here, ‘c’ can be any real number.

4. De-iversonization of Inequality Operators

(1) De-iversonization of x>a, in Exponential Form

This is not an easy problem to solve. So let’s simplify it. We will de-iversonize x>0, then do the horizontal shifting.

Basically, we want to find a function ‘f’ that maps -1 and 0 to 0, and that maps +1 to +1. First, we will find an exponential form. Next, a non-exponential form. Let’s go.

$$[x > 0] = \left\lfloor \frac{\left\lfloor 2^{\left(1 - \left\lfloor \frac{1}{2^x} \right\rfloor\right)} \right\rfloor}{2} \right\rfloor = f(x)$$

Let’s plug in 0 to the function:

$$f(0) = \left\lfloor \frac{\left\lfloor 2^{\left(1 - \left\lfloor \frac{1}{2^0} \right\rfloor\right)} \right\rfloor}{2} \right\rfloor = \left\lfloor \frac{\left\lfloor 2^{(1 - \lfloor 1 \rfloor)} \right\rfloor}{2} \right\rfloor = \left\lfloor \frac{\lfloor 2^0 \rfloor}{2} \right\rfloor = \left\lfloor \frac{1}{2} \right\rfloor = 0$$

Let’s plug in -1 to the function:

$$f(-1) = \left\lfloor \frac{\left\lfloor 2^{\left(1 - \left\lfloor \frac{1}{2^{-1}} \right\rfloor\right)} \right\rfloor}{2} \right\rfloor = \left\lfloor \frac{\left\lfloor 2^{(1 - \lfloor 2 \rfloor)} \right\rfloor}{2} \right\rfloor = \left\lfloor \frac{\lfloor 2^{-1} \rfloor}{2} \right\rfloor = \left\lfloor \frac{0}{2} \right\rfloor = 0$$

Let's plug in +1 to the function:

$$f(1) = \left\lfloor \frac{\left\lfloor 2^{\left(1 - \left\lfloor \frac{1}{2^1} \right\rfloor\right)} \right\rfloor}{2} \right\rfloor = \left\lfloor \frac{\lfloor 2^{(1-0)} \rfloor}{2} \right\rfloor = \left\lfloor \frac{\lfloor 2^1 \rfloor}{2} \right\rfloor = \left\lfloor \frac{2}{2} \right\rfloor = 1$$

Let's plug in -0.5 to the function:

$$f(-0.5) = \left\lfloor \frac{\left\lfloor 2^{\left(1 - \left\lfloor \frac{1}{2^{-0.5}} \right\rfloor\right)} \right\rfloor}{2} \right\rfloor = \left\lfloor \frac{\left\lfloor 2^{(1-\lfloor 2^{0.5} \rfloor)} \right\rfloor}{2} \right\rfloor = \left\lfloor \frac{\left\lfloor 2^{(1-\lfloor \sqrt{2} \rfloor)} \right\rfloor}{2} \right\rfloor = \left\lfloor \frac{2^{(1-1)}}{2} \right\rfloor = \left\lfloor \frac{2^0}{2} \right\rfloor = \left\lfloor \frac{1}{2} \right\rfloor = 0$$

Let's plug in +0.5 to the function:

$$f(0.5) = \left\lfloor \frac{\left\lfloor 2^{\left(1 - \left\lfloor \frac{1}{2^{0.5}} \right\rfloor\right)} \right\rfloor}{2} \right\rfloor = \left\lfloor \frac{\left\lfloor 2^{\left(1 - \left\lfloor \frac{1}{1.414\ldots} \right\rfloor\right)} \right\rfloor}{2} \right\rfloor = \left\lfloor \frac{\lfloor 2^{(1-0)} \rfloor}{2} \right\rfloor = \left\lfloor \frac{2^1}{2} \right\rfloor = \left\lfloor \frac{2}{2} \right\rfloor = \lfloor 1 \rfloor = 1$$

So, it works. A formal proof? Well. We will let our current or future mathematicians do it. Why? Look. We don't have to do everything. Let's call it a division of labor, so to speak. Let's move on. We contribute to mathematics somewhat. Let others fill in the gap. We got a lot left to do in this paper. We're busy. So how the author came up with the formula above? Again, it's chronicled in "humanology series" and "Friday night live with huhnkie lee" series in daily motion dot come, filmed and published in late 2024 and early 2025. It took him several months.

Anyway, in general:

$$[x > 0] = \left\lfloor \frac{\left\lfloor b^{\left(1 - \left\lfloor \frac{1}{d^x} \right\rfloor\right)} \right\rfloor}{b} \right\rfloor = f(x, 0) \quad , \quad where\ b > 1, \qquad d > 1$$

Next, all we need to do is to conduct a horizontal shifting by 'a' on the function 'f':

$$[x > a] = \left\lfloor \frac{\left\lfloor b^{\left(1 - \left\lfloor \frac{1}{d^{x-a}} \right\rfloor\right)} \right\rfloor}{b} \right\rfloor = f(x, a) \quad , \quad where\ b > 1, \qquad d > 1$$

Well. On a second thought, let's practice discipline and diligence and give it a formal proof. I learned better than that //xD

(proof begins)

First, assume x=a.

$$f(a,a) = [a > a] = \left\lfloor \frac{\left\lfloor b^{\left(1-\left\lfloor \frac{1}{d^{a-a}} \right\rfloor\right)} \right\rfloor}{b} \right\rfloor = \left\lfloor \frac{\left\lfloor b^{\left(1-\left\lfloor \frac{1}{d^{0}} \right\rfloor\right)} \right\rfloor}{b} \right\rfloor = \left\lfloor \frac{\left\lfloor b^{\left(1-\left\lfloor \frac{1}{1} \right\rfloor\right)} \right\rfloor}{b} \right\rfloor = \left\lfloor \frac{\left\lfloor b^{(1-\lfloor 1 \rfloor)} \right\rfloor}{b} \right\rfloor$$

$$= \left\lfloor \frac{\left\lfloor b^{(1-1)} \right\rfloor}{b} \right\rfloor = \left\lfloor \frac{\lfloor b^{0} \rfloor}{b} \right\rfloor = \left\lfloor \frac{\lfloor 1 \rfloor}{b} \right\rfloor = \left\lfloor \frac{1}{b} \right\rfloor = 0$$

Next, assume x>a. Let x-a=c. Then, c>0.

$$f(x,a) = [x > a] = \left\lfloor \frac{\left\lfloor b^{\left(1-\left\lfloor \frac{1}{d^{x-a}} \right\rfloor\right)} \right\rfloor}{b} \right\rfloor = \left\lfloor \frac{\left\lfloor b^{\left(1-\left\lfloor \frac{1}{d^{c}} \right\rfloor\right)} \right\rfloor}{b} \right\rfloor = \left\lfloor \frac{\left\lfloor b^{(1-0)} \right\rfloor}{b} \right\rfloor = \left\lfloor \frac{\lfloor b^{1} \rfloor}{b} \right\rfloor = \left\lfloor \frac{b}{b} \right\rfloor = 1$$

Next, assume x<a. Let a-x=c. Then, c>0. Then, $d^c > 1$. Let e = (1 - d^c) Then, e < 0.

$$f(x,a) = [x > a] = \left\lfloor \frac{\left\lfloor b^{\left(1-\left\lfloor \frac{1}{d^{x-a}} \right\rfloor\right)} \right\rfloor}{b} \right\rfloor = \left\lfloor \frac{\left\lfloor b^{\left(1-\left\lfloor \frac{1}{d^{-c}} \right\rfloor\right)} \right\rfloor}{b} \right\rfloor = \left\lfloor \frac{\left\lfloor b^{(1-\lfloor d^{c} \rfloor)} \right\rfloor}{b} \right\rfloor = \left\lfloor \frac{\lfloor b^{e} \rfloor}{b} \right\rfloor = \left\lfloor \frac{0}{b} \right\rfloor = 0$$

(proof ends)

(2) De-iversonization of x>a, in Non-exponential Form

Previously, we established that:

$$sgn(x) = \left\lfloor \frac{x}{|x| + a} \right\rfloor + \left\lceil \frac{x}{|x| + a} \right\rceil \quad , where\ a > 0$$

To simplify, let's use 1 as 'a':

$$sgn(x) = \left\lfloor \frac{x}{|x| + 1} \right\rfloor + \left\lceil \frac{x}{|x| + 1} \right\rceil$$

Then,

$$[x > 0] = \left\lceil \frac{\left\lfloor \frac{x}{|x| + 1} \right\rfloor + \left\lceil \frac{x}{|x| + 1} \right\rceil}{2} \right\rceil = \left\lceil \frac{sgn(x)}{2} \right\rceil$$

Basically, sign function maps negative numbers to -1, 0 to 0, positive numbers to +1. If we divide -1, 0, +1 by 2, they become -1/2, 0, +1/2. If we ceiling them, they become 0, 0, 1. This is multi-step mapping problem. Welcome to the mappology in humanology school. It's a study of mapping methodologies.

In general:

$$[x > 0] = \left\lceil \frac{\left\lfloor \frac{x}{|x| + c} \right\rfloor + \left\lceil \frac{x}{|x| + c} \right\rceil}{b} \right\rceil = f(x, 0) \ \ , where\ b > 1, c > 0$$

Next, let us conduct a horizontal shift by ‘a’ on function ‘f’:

$$[x > a] = \left\lceil \frac{\left\lfloor \frac{x - a}{|x - a| + c} \right\rfloor + \left\lceil \frac{x - a}{|x - a| + c} \right\rceil}{b} \right\rceil = \left\lceil \frac{sgn_a^a(x)}{b} \right\rceil = f(x, a) \ \ , where\ b > 1, c > 0$$

Now, let’s prove it:

(proof begins)

First, assume x>a. Let x-a=d. Then, d>0.

$$f(x, a) = [x > a] = \left\lceil \frac{\left\lfloor \frac{x - a}{|x - a| + c} \right\rfloor + \left\lceil \frac{x - a}{|x - a| + c} \right\rceil}{b} \right\rceil = \left\lceil \frac{\left\lfloor \frac{d}{|d| + c} \right\rfloor + \left\lceil \frac{d}{|d| + c} \right\rceil}{b} \right\rceil$$

$$= \left\lceil \frac{\left\lfloor \frac{|d|}{|d| + c} \right\rfloor + \left\lceil \frac{|d|}{|d| + c} \right\rceil}{b} \right\rceil = \left\lceil \frac{0 + 1}{b} \right\rceil = \left\lceil \frac{1}{b} \right\rceil = 1$$

Next, assume x=a.

$$f(a, a) = [a > a] = \left\lceil \frac{\left\lfloor \frac{a - a}{|a - a| + c} \right\rfloor + \left\lceil \frac{a - a}{|a - a| + c} \right\rceil}{b} \right\rceil = \left\lceil \frac{\left\lfloor \frac{0}{|0| + c} \right\rfloor + \left\lceil \frac{0}{|0| + c} \right\rceil}{b} \right\rceil = 0$$

Next, assume x<a. Let a-x=d. Then, d>0.

$$f(x, a) = [x > a] = \left\lceil \frac{\left\lfloor \frac{x - a}{|x - a| + c} \right\rfloor + \left\lceil \frac{x - a}{|x - a| + c} \right\rceil}{b} \right\rceil = \left\lceil \frac{\left\lfloor \frac{-d}{|-d| + c} \right\rfloor + \left\lceil \frac{-d}{|-d| + c} \right\rceil}{b} \right\rceil$$

$$= \left\lceil \frac{\left\lfloor \frac{-|d|}{|d| + c} \right\rfloor + \left\lceil \frac{-|d|}{|d| + c} \right\rceil}{b} \right\rceil = \left\lceil \frac{-1 + 0}{b} \right\rceil = \left\lceil \frac{-1}{b} \right\rceil = 0$$

(proof ends)

(3) De-iversonization of x<a, in Non-exponential Form

As for de-iversonization of x<a in exponential form, we'll let other mathematicians figure it out. Again, we contribute to mathematics somewhat, actually, we contribute to mathematics significantly, but we don't have to do everything. We are busy. We also do martial arts, push-ups, sit-ups, pull-ups, running and dancing and singing too. The author is 46 years old. He sounds like a grumpy old man already, lol. My apologies //:-) Jokes aside, ok, let's go:

$$[x < a] = -\left\lfloor \frac{\left\lfloor \frac{x-a}{|x-a|+c} \right\rfloor + \left\lceil \frac{x-a}{|x-a|+c} \right\rceil}{b} \right\rfloor = -\left\lfloor \frac{sgn_a^a(x)}{b} \right\rfloor = f(x,a)\,, where\ b > 1, c > 0$$

If we use the floor-ceiling conversion formula,

$$[x < a] = -\left\lfloor \frac{\left\lfloor \frac{x-a}{|x-a|+c} \right\rfloor + \left\lceil \frac{x-a}{|x-a|+c} \right\rceil}{b} \right\rfloor = \left\lceil \frac{-\left\lfloor \frac{x-a}{|x-a|+c} \right\rfloor - \left\lceil \frac{x-a}{|x-a|+c} \right\rceil}{b} \right\rceil$$

$$= \left\lceil \frac{\left\lceil \frac{a-x}{|x-a|+c} \right\rceil + \left\lfloor \frac{a-x}{|x-a|+c} \right\rfloor}{b} \right\rceil$$

Now, let's prove it:

(proof begins)

First, assume x<a. Let a-x=d. Then d>0.

$$f(x,a) = [x < a] = \left\lceil \frac{\left\lceil \frac{a-x}{|x-a|+c} \right\rceil + \left\lfloor \frac{a-x}{|x-a|+c} \right\rfloor}{b} \right\rceil = \left\lceil \frac{\left\lceil \frac{d}{|d|+c} \right\rceil + \left\lfloor \frac{d}{|d|+c} \right\rfloor}{b} \right\rceil$$

$$= \left\lceil \frac{\left\lceil \frac{|d|}{|d|+c} \right\rceil + \left\lfloor \frac{|d|}{|d|+c} \right\rfloor}{b} \right\rceil = \left\lceil \frac{1+0}{b} \right\rceil = \left\lceil \frac{1}{b} \right\rceil = 1$$

Next, assume x=a.

$$f(a,a) = [a < a] = \left\lceil \frac{\left\lceil \frac{a-a}{|a-a|+c} \right\rceil + \left\lfloor \frac{a-a}{|x-a|+c} \right\rfloor}{b} \right\rceil = \left\lceil \frac{\left\lceil \frac{0}{|0|+c} \right\rceil + \left\lfloor \frac{0}{|0|+c} \right\rfloor}{b} \right\rceil = 0$$

First, assume x>a. Let x-a=d. Then d>0.

$$f(x,a) = [x < a] = \left\lceil \frac{\left\lceil \frac{a-x}{|x-a|+c} \right\rceil + \left\lfloor \frac{a-x}{|x-a|+c} \right\rfloor}{b} \right\rceil = \left\lceil \frac{\left\lceil \frac{-d}{|-d|+c} \right\rceil + \left\lfloor \frac{-d}{|-d|+c} \right\rfloor}{b} \right\rceil$$

$$= \left\lceil \frac{\left\lceil \frac{-|d|}{|d|+c} \right\rceil + \left\lfloor \frac{-|d|}{|d|+c} \right\rfloor}{b} \right\rceil = \left\lceil \frac{0-1}{b} \right\rceil = \left\lceil \frac{-1}{b} \right\rceil = 0$$

(proof ends)

(4) De-iversonization of x<=a, in Non-exponential Form, First Attempt

(this section contains errors and attempts to correct the errors. If the reader wants to see only the correct result, see the next section. We will leave this section as it is, as it contains valid and valuable methodologies and it also has a good educational value of demonstrating how a scientific theory is developed via trials and errors. However, to understand the next section, probably it is necessary to look at this section before looking at the next section.)

An easy way to do this is to use Iverson negation formula:

$$[x \le a] = [\sim(x > a)] = 1 - [x > a] = 1 + \left\lfloor \frac{\left\lfloor \frac{x-a}{|x-a|+1} \right\rfloor + \left\lceil \frac{x-a}{|x-a|+1} \right\rceil}{2} \right\rfloor$$

$$= \left\lfloor \frac{\left\lfloor \frac{x-a}{|x-a|+1} \right\rfloor + \left\lceil \frac{x-a}{|x-a|+1} \right\rceil}{2} + 1 \right\rfloor = \left\lfloor \frac{\left\lfloor \frac{x-a}{|x-a|+1} \right\rfloor + \left\lceil \frac{x-a}{|x-a|+1} \right\rceil + 2}{2} \right\rfloor$$

Now, let's find an alternative way to this. Let's think of an easier case and make a hypothesis:

$$f(x,0) = [x \le 0] = \left\lceil \frac{sgn(x)}{-2} + \frac{1}{4} \right\rceil$$

Let's plug in some numbers:

$$f(-1,0) = [-1 \le 0] = \left\lceil \frac{sgn(-1)}{-2} + \frac{1}{4} \right\rceil = \left\lceil \frac{-1}{-2} + \frac{1}{4} \right\rceil = \left\lceil \frac{3}{4} \right\rceil = 1$$

$$f(0,0) = [0 \le 0] = \left\lceil \frac{sgn(0)}{-2} + \frac{1}{4} \right\rceil = \left\lceil \frac{0}{-2} + \frac{1}{4} \right\rceil = \left\lceil \frac{1}{4} \right\rceil = 1$$

$$f(1,0) = [1 \le 0] = \left\lceil \frac{sgn(1)}{-2} + \frac{1}{4} \right\rceil = \left\lceil \frac{1}{-2} + \frac{1}{4} \right\rceil = \left\lceil \frac{-1}{4} \right\rceil = 0$$

So, it works. Our next step is to generalize the denominator numbers above:

$$f(x,0) = [x \le 0] = \left\lceil \frac{sgn(x)}{-a} + \frac{1}{a*b} \right\rceil \;, \qquad where\ a > 1, b > 1$$

Then, we further generalize above by conducting a horizontal shifting on 'f' by 'c':

$$f(x,c) = [x \le c] = \left\lceil \frac{sgn_c^c(x)}{-a} + \frac{1}{a*b} \right\rceil \;, \qquad where\ a > 1, b > 1$$

Previously, we have (after replacing recurring letter 'a' with 'd'):

$$sgn_c^c(x) = \left\lfloor \frac{x-c}{|x-c|+d} \right\rfloor + \left\lceil \frac{x-c}{|x-c|+d} \right\rceil \;, where\ d > 0$$

Let's plug it in and prove the formula:

(proof begins)

$$f(x,c) = [x \le c] = \left\lceil \frac{\left\lfloor \frac{x-c}{|x-c|+d} \right\rfloor + \left\lceil \frac{x-c}{|x-c|+d} \right\rceil}{-a} + \frac{1}{a*b} \right\rceil \;, where\ a > 1, b > 1, d > 0$$

First, assume x=c:

$$f(c,c) = [c \le c] = \left\lceil \frac{\left\lfloor \frac{c-c}{|c-c|+d} \right\rfloor + \left\lceil \frac{c-c}{|c-c|+d} \right\rceil}{-a} + \frac{1}{a*b} \right\rceil = \left\lceil \frac{0}{-a} + \frac{1}{a*b} \right\rceil = \left\lceil \frac{1}{a*b} \right\rceil = 1$$

Next, assume x<c. Let c – x = e. Then, e > 0.

$$f(x,c) = [x \le c] = \left\lceil \frac{\left\lfloor \frac{x-c}{|x-c|+d} \right\rfloor + \left\lceil \frac{x-c}{|x-c|+d} \right\rceil}{-a} + \frac{1}{a*b} \right\rceil$$

$$= \left\lceil \frac{\left\lfloor \frac{-e}{|-e|+d} \right\rfloor + \left\lceil \frac{-e}{|-e|+d} \right\rceil}{-a} + \frac{1}{a*b} \right\rceil = \left\lceil \frac{\left\lfloor -\left(\frac{|e|}{|e|+d}\right) \right\rfloor + \left\lceil -\left(\frac{|e|}{|e|+d}\right) \right\rceil}{-a} + \frac{1}{a*b} \right\rceil$$

$$= \left\lceil \frac{-1+0}{-a} + \frac{1}{a*b} \right\rceil = \left\lceil \frac{1}{a} + \frac{1}{a*b} \right\rceil = \left\lceil \frac{b+1}{a*b} \right\rceil$$

Whoops, it seems our hypothesis contains some error. The final term above is not always 1. To make it 1, we need to further limit the ranges of 'a' and 'b':

$$b + 1 \le a * b, \quad 1 \le b(a-1), \quad b \ge \frac{1}{a-1}$$

For instance, if a=1.1, then:

$$b \ge \frac{1}{a-1} = \frac{1}{1.1-1} = \frac{1}{0.1} = 10$$

For another instance, if a=2, then:

$$b \ge \frac{1}{a-1} = \frac{1}{2-1} = 1$$

For another instance, if a=11, then:

$$b \ge \frac{1}{11-1} = 0.1$$

Anyhow, we disproved our original hypothesis and then came up with a new hypothesis that works and here it is:

$$f(x,c) = [x \le c] = \left\lceil \frac{\left\lfloor \frac{x-c}{|x-c|+d} \right\rfloor + \left\lceil \frac{x-c}{|x-c|+d} \right\rceil}{-a} + \frac{1}{a*b} \right\rceil \text{, } where\ a > 1, b \ge \frac{1}{a-1}, d > 0$$

But, perhaps there is a simpler way to do this. Let's let 1/ab be 1/f instead and see what happens. For the equation to be true, what we needs is:

$$a + f \leq a * f$$

Then the question is, when a > 1, what would be the range of 'f', in order to satisfy the inequality above?

$$\frac{a + f}{a * f} \leq 1$$

$$\frac{a}{a * f} + \frac{f}{a * f} \leq 1$$

$$\frac{1}{f} + \frac{1}{a} \leq 1$$

$$\frac{1}{f} \leq 1 - \frac{1}{a}$$

$$\frac{1}{f} \leq \frac{a - 1}{a}$$

$$f \geq \frac{a}{a - 1}$$

$$f \geq \frac{a - 1 + 1}{a - 1}$$

$$f \geq 1 + \frac{1}{a - 1}$$

This means that f > 1. But, 'f' is dependent on 'a'. 'f' cannot be any number larger than or equal to 1. For example, if a = 1.1,

$$f \geq 1 + \frac{1}{1.1 - 1} = 11$$

That is, the formula doesn't work in situations like when a=f=1.1. Alright. Then we came up with another formula that works:

$$f(x, c) = [x \leq c] = \left\lceil \frac{\left\lfloor \frac{x - c}{|x - c| + d} \right\rfloor + \left\lceil \frac{x - c}{|x - c| + d} \right\rceil}{-a} + \frac{1}{f} \right\rceil \text{ ,} where\ a > 1, f \geq 1 + \frac{1}{a - 1}, d > 0$$

Now that we modified our hypothesis, we need to redo the case when x=c:

$$f(c,c) = [c \leq c] = \left\lceil \frac{\left\lfloor \frac{c-c}{|c-c|+d} \right\rfloor + \left\lceil \frac{c-c}{|c-c|+d} \right\rceil}{-a} + \frac{1}{f} \right\rceil = \left\lceil \frac{1}{f} \right\rceil$$

So, it works.

Next, assume x>c. Let x – c = e. Then, e > 0.

$$f(x,c) = [x \leq c] = \left\lceil \frac{\left\lfloor \frac{x-c}{|x-c|+d} \right\rfloor + \left\lceil \frac{x-c}{|x-c|+d} \right\rceil}{-a} + \frac{1}{f} \right\rceil$$

$$= \left\lceil \frac{\left\lfloor \frac{x-c}{|x-c|+d} \right\rfloor + \left\lceil \frac{x-c}{|x-c|+d} \right\rceil}{-a} + \frac{1}{f} \right\rceil = \left\lceil \frac{\left\lfloor \frac{e}{|e|+d} \right\rfloor + \left\lceil \frac{e}{|e|+d} \right\rceil}{-a} + \frac{1}{f} \right\rceil$$

$$= \left\lceil \frac{0+1}{-a} + \frac{1}{f} \right\rceil = \left\lceil \frac{1}{-a} + \frac{1}{f} \right\rceil = \left\lceil \frac{1}{f} - \frac{1}{a} \right\rceil$$

Previously, we have:

$$\frac{1}{f} \leq 1 - \frac{1}{a}$$

$$\frac{1}{f} - \frac{1}{a} \leq 1 - \frac{2}{a}$$

We also have:

$$a > 1$$

$$-a < -1$$

$$-\frac{a}{2} < -\frac{1}{2}$$

$$-\frac{2}{a} > -2$$

$$1 - \frac{2}{a} > -1$$

Also, if we send ‘a’ to positive infinity, we have:

$$-1 < 1 - \frac{2}{a} < 1$$

Well. It's not quite giving us the result we want. Let's go back and examine the range of 'f', as 'a' ranges from 1 to positive infinity:

$$f \geq 1 + \frac{1}{a-1}$$

'a' is more than 1. So, as 'a' approached 1^+, which is 1 plus positive infinitesimality, i.e., 0^+, 1/(a-1) approaches positive infinity. So, 'f' has upper boundary of positive infinity. In that case,

$$\lim_{a \to 1^+} \left\lceil \frac{1}{f} - \frac{1}{a} \right\rceil = \left\lceil \frac{1}{+\infty} - \frac{1}{1^+} \right\rceil = \lceil 0^+ - 1^- \rceil = 0$$

It's basically like,

$$\lceil 0.01 - 0.99 \rceil = \lceil -0.98 \rceil = 0$$

Next, as 'a' approached positive infinity, the lower boundary of 'f' becomes 1^+, which means that 'f' is larger than 1.

$$\lim_{a \to +\infty} \left\lceil \frac{1}{f} - \frac{1}{a} \right\rceil = \left\lceil \frac{1}{1^+} - \frac{1}{+\infty} \right\rceil = \lceil 1^- - \ 0^+ \rceil = 1$$

It's basically like,

$$\lceil 0.99 - 0.01 \rceil = \lceil 0.98 \rceil = 1$$

Well, this is not the result we want, but it was good introduction and exercise in the algebra of infinitesimality, which may be new in mathematics. As a side, let's explore infinitesimality algebra a little bit more:

$$\frac{1}{2^+} = 0.5^- = 2^{-1^+} = (2^{-1})^-$$

This is definitely interesting discovery.[11] Now, let's get back to the main problem.

[11] Actually, the equations above contains some errors and we will correct them later in this paper when we deal with infinitesimal algebra. Actually, we will correct the errors there in the next paper, "Infinitesimal Algebra and Infinity Algebra", which will most likely be published ssrn dot com. This "Fraction Algebra" paper will most likely be published in vixra dot org.

We basically need to come up with an alternative hypothesis. Let's take a five minute break.

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Let's get back to our latest hypothesis:

$$f(x,c) = [x \leq c] = \left\lceil \frac{\left\lfloor \frac{x-c}{|x-c|+d} \right\rfloor + \left\lceil \frac{x-c}{|x-c|+d} \right\rceil}{-a} + \frac{1}{f} \right\rceil \text{ , where } a > 1, f > 1, \quad d > 0$$

Our goal is to find the range of 'f', such that:

$$\left\lceil \frac{1}{a} + \frac{1}{f} \right\rceil = 1$$

And,

$$\left\lceil -\frac{1}{a} + \frac{1}{f} \right\rceil = 0$$

Let's come up with inequalities that satisfy the first requirement above:

$$\left\lceil \frac{1}{a} + \frac{1}{f} \right\rceil = 1$$

$$0 < \frac{1}{a} + \frac{1}{f} \leq 1$$

Let's come up with inequalities that satisfy the second requirement above:

$$\left\lceil -\frac{1}{a} + \frac{1}{f} \right\rceil = 0$$

$$-1 < -\frac{1}{a} + \frac{1}{f} \leq 0$$

Let’s add the two sets of inequalities:

$$0 < \frac{1}{a} + \frac{1}{f} \leq 1$$

$$-1 < -\frac{1}{a} + \frac{1}{f} \leq 0$$

Then we get:

$$-1 < \frac{2}{f} \leq 1$$

$$-\frac{1}{2} < \frac{1}{f} \leq \frac{1}{2}$$

We know that f is more than 1. So,

$$0 < \frac{1}{f} \leq \frac{1}{2}$$

Then,

$$2 \leq f < +\infty$$

Well, that does not look right because:

$$\left\lceil \frac{1}{1.1} + \frac{1}{2} \right\rceil = 2 \neq 1$$

Now, let’s go back to our earlier hypothesis where we express ‘f’ as ‘ab’:

$$f(x,c) = [x \leq c] = \left\lceil \frac{\left\lfloor \frac{x-c}{|x-c|+d} \right\rfloor + \left\lceil \frac{x-c}{|x-c|+d} \right\rceil}{-a} + \frac{1}{a * b} \right\rceil \text{ , where } a > 1, b > 1, d > 0$$

Our goal is to find the range of ‘b’, such that:

$$\left\lceil \frac{1}{a} + \frac{1}{ab} \right\rceil = 1$$

And,

$$\left\lceil -\frac{1}{a} + \frac{1}{ab} \right\rceil = 0$$

Let's come up with inequalities that satisfy the first requirement above:

$$\left\lceil \frac{1}{a} + \frac{1}{ab} \right\rceil = 1$$

$$0 < \frac{1}{a} + \frac{1}{ab} \leq 1$$

Let's come up with inequalities that satisfy the second requirement above:

$$\left\lceil -\frac{1}{a} + \frac{1}{ab} \right\rceil = 0$$

$$-1 < -\frac{1}{a} + \frac{1}{ab} \leq 0$$

Let's add the two sets of inequalities:

$$0 < \frac{1}{a} + \frac{1}{ab} \leq 1$$

$$-1 < -\frac{1}{a} + \frac{1}{ab} \leq 0$$

Then we get:

$$-1 < \frac{2}{ab} \leq 1$$

$$-\frac{1}{2} < \frac{1}{ab} \leq \frac{1}{2}$$

We know ab>1. Then,

$$0 < \frac{1}{ab} \leq \frac{1}{2}$$

$$2 \leq ab < +\infty$$

Then,

$$b > \frac{1}{a}$$

This does not look right either, because:

$$\left\lceil \frac{1}{2} + \frac{1}{1} \right\rceil = 2 \neq 1$$

(proof ends)

(5) De-iversonization of x<=a, in Non-exponential Form, Second Attempt

The author went back to the white board in humanology series in daily motion dot com and then came up with yet another hypothesis and this time, it works. Let's go.

$$f(x,c) = [x \le c] = \left\lceil \frac{\left\lfloor \frac{x-c}{|x-c|+d} \right\rfloor + \left\lceil \frac{x-c}{|x-c|+d} \right\rceil}{-a} + \frac{1}{f} \right\rceil \text{ , } where\ a > 1, f > 0, d > 0$$

It turns out that the range of 'f' bifurcates depending on whether 'a' is less than 2 or more than 2. When 'a' is 2, either range of 'f' works equally. Above, we have 'f' being larger than 2, but we need to narrow the range down further to make the formula work correctly.

Basically, we are dealing with the following situation. We want to find the range of 'f' such that, when a > 1 and f > 0, 'f' must satisfy the two equations:

$$\left\lceil \frac{1}{a} + \frac{1}{f} \right\rceil = 1$$

$$\left\lceil \frac{1}{-a} + \frac{1}{f} \right\rceil = 0$$

First, let's look at the case where 'a' is between 1 and 2, right-inclusive:

$$1 < a \le 2$$

Then,

$$0.5 \le \frac{1}{a} < 1$$

Let's say, 1/a is 0.6. Then, to make the first equation true,

$$\left\lceil \frac{1}{a} + \frac{1}{f} \right\rceil = 1$$

1/f should be a number less than the complement of 1/a. Complement of 0.6 is 0.4. Let's say 1/f is 0.3:

$$\lceil 0.6 + 0.3 \rceil = 1$$

Next, let's look at the second equation:

$$\left\lceil \frac{1}{-a} + \frac{1}{f} \right\rceil = 0$$

$$\lceil -0.6 + 0.3 \rceil = 0$$

So, it works. Now, let's find the range of 'f' and express it in terms of 'a':

$$0 \le \frac{1}{f} < \quad /\frac{1}{a}/ \;=\; 1 - \frac{1}{a} = \frac{a-1}{a}$$

Therefore,

$$f > \frac{a}{a-1}$$

The above is the correct range. The following is true but it is too broad a range and the following should not be used in our final formula because it sometimes leads to erroneous result. But it is an interesting fact, so we will observe it:

$$1 < a \le 2$$

$$0 < a - 1 \le 1$$

$$1 \le \frac{1}{a-1} < +\infty$$

$$2 \le 1 + \frac{1}{a-1} < +\infty$$

And we have:

$$f > \frac{a}{a-1} = \frac{a-1+1}{a-1} = 1 + \frac{1}{a-1} \ge 2$$

Therefore,

$$f > 2$$

Now, let's bring the two equations and plug in some numbers:

$$\left\lceil \frac{1}{a} + \frac{1}{f} \right\rceil = 1$$

$$\left\lceil \frac{1}{-a} + \frac{1}{f} \right\rceil = 0$$

Let's say, 'a' is a number slightly more than 1, like 1.1. Then 1/a will be a number slightly less than 1, like 0.99. And let's say 'f' is some number bigger than 2, like 4. Then,

$$\left\lceil \frac{1}{a} + \frac{1}{f} \right\rceil = \left\lceil \frac{1}{1.1} + \frac{1}{4} \right\rceil = \lceil 0.91 + 0.25 \rceil = \lceil 1.16 \rceil = 2 \neq 1$$

Next, let's use our correct, narrow range formula for 'f':

$$f > \frac{a}{a-1} = \frac{1.1}{1.1-1} = \frac{1.1}{0.1} = 11$$

Let's see if it works. Let's use the example of 'f' being 12:

$$\left\lceil \frac{1}{a} + \frac{1}{f} \right\rceil = \left\lceil \frac{1}{1.1} + \frac{1}{12} \right\rceil = \lceil 0.91 + 0.083 \rceil = \lceil 0.993 \rceil = 1$$

So, 'f' is more than 2, yes. But, 'f' being more than 2 is not good enough. 'f' must be bigger than 11, in this example.

Next, let's see if the second equation works:

$$\left\lceil \frac{1}{-a} + \frac{1}{f} \right\rceil = \left\lceil \frac{1}{-1.1} + \frac{1}{12} \right\rceil = \lceil -0.91 + 0.083 \rceil = \lceil -0.827 \rceil = 0$$

So, it works.

So far, we took care of the case when:

$$1 < a \leq 2$$

Next, let's take a look at the case when:

$$a > 2$$

Again, we have two equations for the range of 'f' to satisfy:

$$\left\lceil \frac{1}{a} + \frac{1}{f} \right\rceil = 1$$

$$\left\lceil \frac{1}{-a} + \frac{1}{f} \right\rceil = 0$$

We have:

$$a > 2$$

$$0 < \frac{1}{a} < 0.5$$

Let's look at the first equation above. Let's say 1/a is 0.3. Then, 1/f should be between 0 and 0.7, right inclusive. Let's say 1/f is 0.6:

$$\left\lceil \frac{1}{a} + \frac{1}{f} \right\rceil = \lceil 0.3 + 0.6 \rceil = 1$$

So, we have:

$$0 < \frac{1}{f} \leq / \frac{1}{a} / = 1 - \frac{1}{a}$$

Now, let's look at the second equation:

$$\left\lceil \frac{1}{-a} + \frac{1}{f} \right\rceil = 0$$

We have:

$$\left\lceil \frac{1}{-a} + \frac{1}{f} \right\rceil = \lceil -0.3 + 0.6 \rceil = 1 \neq 0$$

So, in this case, we need to further narrow down the range of 1/f. Here, what we need is, we need 1/f to be between 0 and 1/a, right inclusive:

$$0 < \frac{1}{f} \leq \frac{1}{a}$$

Let's say 1/f is 0.2:

$$\left\lceil \frac{1}{-a} + \frac{1}{f} \right\rceil = \lceil -0.3 + 0.2 \rceil = 0$$

So, it works. Now, let's see if it works for the first equation as well:

$$\left\lceil \frac{1}{a} + \frac{1}{f} \right\rceil = \lceil 0.3 + 0.2 \rceil = 1$$

Yep, it works. So, we have:

$$0 < \frac{1}{f} \leq \frac{1}{a}$$

$$+\infty > f > a$$

Or, simply:

$$f > a$$

Now, we are ready to formally prove our latest hypothesis. Let's go.

(proof begins)

$$f(x,c) = [x \le c] = \left\lceil \frac{\left\lfloor \frac{x-c}{|x-c|+d} \right\rfloor + \left\lceil \frac{x-c}{|x-c|+d} \right\rceil}{-a} + \frac{1}{f} \right\rceil,$$

$$where \left\{ \frac{1 < a \le 2, f > \frac{a}{a-1}, d > 0, or}{2 < a < f, d > 0,} \right\}$$

It took the author several days to come up with a right hypothesis. Proving it would be a lot easier. It's Friday, 2/7/25. Let's do the proof tomorrow. Let's not work too hard //:-)

Ok, we're back. Let's start with the easiest case when x is c.

$$f(c,c) = [c \le c] = \left\lceil \frac{\left\lfloor \frac{c-c}{|c-c|+d} \right\rfloor + \left\lceil \frac{c-c}{|c-c|+d} \right\rceil}{-a} + \frac{1}{f} \right\rceil = \left\lceil \frac{1}{f} \right\rceil$$

If 1<a<=2,

$$f > \frac{a}{a-1}$$

$$0 < \frac{1}{f} < \frac{a-1}{a} = 1 - \frac{1}{a} < 1$$

Therefore,

$$\left\lceil \frac{1}{f} \right\rceil = 0$$

It works.

$$if\ 2 < a < f,$$

$$1 > \frac{1}{2} > \frac{1}{f} > 0$$

Therefore,

$$\left\lceil \frac{1}{f} \right\rceil = 0$$

It works there too.

Next, let's see what happens when x < c. Let c-x=e. Then, e>0:

$$f(x,c) = [x \leq c] = \left\lceil \frac{\left\lfloor \frac{x-c}{|x-c|+d} \right\rfloor + \left\lceil \frac{x-c}{|x-c|+d} \right\rceil}{-a} + \frac{1}{f} \right\rceil$$

$$= \left\lceil \frac{\left\lfloor \frac{-e}{|-e|+d} \right\rfloor + \left\lceil \frac{-e}{|-e|+d} \right\rceil}{-a} + \frac{1}{f} \right\rceil = \left\lceil \frac{\left\lfloor \frac{-|e|}{|e|+d} \right\rfloor + \left\lceil \frac{-|e|}{|e|+d} \right\rceil}{-a} + \frac{1}{f} \right\rceil$$

$$= \left\lceil \frac{-1+0}{-a} + \frac{1}{f} \right\rceil = \left\lceil \frac{1}{a} + \frac{1}{f} \right\rceil$$

Ok. Now let's go to the first range of 'a' and 'f':

$$if\ 1 < a \leq 2, f > \frac{a}{a-1},$$

$$f > \frac{a}{a-1}$$

$$0 < \frac{1}{f} < \frac{a-1}{a} = 1 - \frac{1}{a} < 1$$

$$\frac{1}{a} < \frac{1}{a} + \frac{1}{f} < \frac{1}{a} + 1 - \frac{1}{a} = 1$$

So, we have:

$$0 < \frac{1}{a} + \frac{1}{f} < 1$$

Therefore,

$$\left\lceil \frac{1}{a} + \frac{1}{f} \right\rceil = 1$$

Next, let's go to the second range of 'a' and 'f':

$$if\ 2 < a < f,$$

$$\frac{1}{2} > \frac{1}{a} > \frac{1}{f}$$

So, we have:

$$\frac{1}{2} > \frac{1}{a}$$

$$\frac{1}{2} > \frac{1}{f}$$

If we add the two inequalities, we have:

$$1 > \frac{1}{a} + \frac{1}{f} > 0$$

Therefore,

$$\left\lceil \frac{1}{a} + \frac{1}{f} \right\rceil = 1$$

Alright, it worked. Now, let's handle the third and last possibility where x > 0. Let x-c be e. Then, e > 0:

$$f(x,c) = [x \leq c] = \left\lceil \frac{\left\lfloor \frac{x-c}{|x-c|+d} \right\rfloor + \left\lceil \frac{x-c}{|x-c|+d} \right\rceil}{-a} + \frac{1}{f} \right\rceil$$

$$= \left\lceil \frac{\left\lfloor \frac{e}{|e|+d} \right\rfloor + \left\lceil \frac{e}{|e|+d} \right\rceil}{-a} + \frac{1}{f} \right\rceil = \left\lceil \frac{\left\lfloor \frac{e}{e+d} \right\rfloor + \left\lceil \frac{e}{e+d} \right\rceil}{-a} + \frac{1}{f} \right\rceil = \left\lceil \frac{0+1}{-a} + \frac{1}{f} \right\rceil = \left\lceil \frac{1}{-a} + \frac{1}{f} \right\rceil$$

Let's take care of the first ranges of 'a' and 'f':

$$if\ 1 < a \leq 2, f > \frac{a}{a-1},$$

$$f > \frac{a}{a-1}$$

$$0 < \frac{1}{f} < \frac{a-1}{a} = 1 - \frac{1}{a} < 1$$

So, we have:

$$0 < \frac{1}{f} < 1 - \frac{1}{a}$$

And, we have:

$1 < a \leq 2$

$\frac{1}{2} \leq \frac{1}{a} < 1$

$-1 < -\frac{1}{a} \leq -\frac{1}{2}$

If we add the two inequalities about 1/f and -1/a, we have:

$0 < \frac{1}{f} < 1 - \frac{1}{a}$

$-1 < -\frac{1}{a} \leq -\frac{1}{2}$

$-1 < \frac{1}{f} - \frac{1}{a} < \frac{1}{2} - \frac{1}{a}$

Now, let's look at the upper boundary:

$\frac{1}{2} - \frac{1}{a}$

Let's manipulate the range of 'a':

$1 < a \leq 2$

$\frac{1}{2} \leq \frac{1}{a} < 1$

$-1 < -\frac{1}{a} \leq -\frac{1}{2}$

$-\frac{1}{2} < \frac{1}{2} - \frac{1}{a} \leq 0$

Ok. Let us combine above inequality with the previous one:

$-1 < \frac{1}{f} - \frac{1}{a} < \frac{1}{2} - \frac{1}{a}$

Then we have:

$-1 < \frac{1}{f} - \frac{1}{a} < \frac{1}{2} - \frac{1}{a} \leq 0$

Thus,

$-1 < \frac{1}{f} - \frac{1}{a} < 0$

Therefore,

$$\left\lfloor\frac{1}{-a}+\frac{1}{f}\right\rfloor = \left\lfloor\frac{1}{f}-\frac{1}{a}\right\rfloor = 0$$

So, it works. Next, let's take care of the second range of 'a' and 'f':

$if\ 2 < a < f,$

$$\frac{1}{2} > \frac{1}{a} > \frac{1}{f} > 0$$

So, we have:

$$\frac{1}{2} > \frac{1}{a}$$

$$\frac{1}{2} > \frac{1}{f}$$

$$\frac{1}{a} > \frac{1}{f}$$

And we have:

$$-\frac{1}{a} > -\frac{1}{2}$$

From the following and from the above inequalities:

$$\frac{1}{a} > \frac{1}{f} > 0$$

We have:

$$0 > -\frac{1}{a}+\frac{1}{f} > -\frac{1}{a} > -\frac{1}{2}$$

Thus,

$$0 > -\frac{1}{a}+\frac{1}{f} > -\frac{1}{2}$$

$$\left\lfloor\frac{1}{-a}+\frac{1}{f}\right\rfloor = \left\lfloor\frac{1}{f}-\frac{1}{a}\right\rfloor = 0$$

This concludes the proof.

(proof ends)

(6) De-iversonization of x>=a, in Non-exponential Form, Second Attempt

Let's design a hypothetical formula for [x>=a] based on pattern recognitions. We have three proven formulas:

$$[x > a] = \left\lceil \frac{\left\lfloor \frac{x-a}{|x-a|+c} \right\rfloor + \left\lceil \frac{x-a}{|x-a|+c} \right\rceil}{b} \right\rceil$$

$$[x < a] = \left\lceil \frac{\left\lfloor \frac{a-x}{|x-a|+c} \right\rfloor + \left\lceil \frac{a-x}{|x-a|+c} \right\rceil}{b} \right\rceil$$

$$[x \le c] = \left\lceil \frac{\left\lfloor \frac{x-c}{|x-c|+d} \right\rfloor + \left\lceil \frac{x-c}{|x-c|+d} \right\rceil}{-a} + \frac{1}{f} \right\rceil, where \left\{ \frac{1 < a \le 2, f > \frac{a}{a-1}, d > 0, or}{2 < a < f, d > 0,} \right\}$$

The author apologizes for the inconsistency of the letters but it's been just too busy, so.

First, an easy way to do this is to use Iverson negation formula and floor-ceiling conversion formula and floor/ceiling-crawl-in/out formula:

$$[x \ge a][\sim(x < a)] = 1 - \left\lceil \frac{\left\lfloor \frac{a-x}{|x-a|+c} \right\rfloor + \left\lceil \frac{a-x}{|x-a|+c} \right\rceil}{b} \right\rceil$$

$$= 1 + \left\lfloor \frac{-\left\lfloor \frac{a-x}{|x-a|+c} \right\rfloor - \left\lceil \frac{a-x}{|x-a|+c} \right\rceil}{b} \right\rfloor = 1 + \left\lfloor \frac{\left\lceil \frac{x-a}{|x-a|+c} \right\rceil + \left\lfloor \frac{x-a}{|x-a|+c} \right\rfloor}{b} \right\rfloor$$

$$= 1 + \left\lfloor \frac{\left\lfloor \frac{x-a}{|x-a|+c} \right\rfloor + \left\lceil \frac{x-a}{|x-a|+c} \right\rceil}{b} \right\rfloor = \left\lfloor 1 + \frac{\left\lfloor \frac{x-a}{|x-a|+c} \right\rfloor + \left\lceil \frac{x-a}{|x-a|+c} \right\rceil}{b} \right\rfloor$$

Next, let's do it the other, the hard way. First, let's observe the following two inequalities:

$$[x > a] = \left\lceil \frac{\left\lfloor \frac{x-a}{|x-a|+c} \right\rfloor + \left\lceil \frac{x-a}{|x-a|+c} \right\rceil}{b} \right\rceil$$

$$[x < a] = \left\lceil \frac{\left\lfloor \frac{a-x}{|x-a|+c} \right\rfloor + \left\lceil \frac{a-x}{|x-a|+c} \right\rceil}{b} \right\rceil$$

The only difference between the two is in the numerator: x-a becomes a-x. Now let's look at:

$$[x \le c] = \left\lfloor \frac{\left\lfloor \frac{x-c}{|x-c|+d} \right\rfloor + \left\lceil \frac{x-c}{|x-c|+d} \right\rceil}{-a} + \frac{1}{f} \right\rfloor, where \left\{ \frac{1 < a \le 2, f > \frac{a}{a-1}, d > 0, or}{2 < a < f, d > 0,} \right\}$$

Using floor/ceiling conversion formula like before, we get:

$$[x \le c] = \left\lfloor \frac{\left\lfloor \frac{c-x}{|x-c|+d} \right\rfloor + \left\lceil \frac{c-x}{|x-c|+d} \right\rceil}{a} + \frac{1}{f} \right\rfloor, where \left\{ \frac{1 < a \le 2, f > \frac{a}{a-1}, d > 0, or}{2 < a < f, d > 0,} \right\}$$

Then, a reasonable hypothesis would be:

$$[x \ge c] = \left\lfloor \frac{\left\lfloor \frac{x-c}{|x-c|+d} \right\rfloor + \left\lceil \frac{x-c}{|x-c|+d} \right\rceil}{a} + \frac{1}{f} \right\rfloor, where \left\{ \frac{1 < a \le 2, f > \frac{a}{a-1}, d > 0, or}{2 < a < f, d > 0,} \right\}$$

Before proving attempt, let's make some examples to see if the formula hold the water or not. Let 'c' be zero, for instance.

$$[x \ge 0] = \left\lfloor \frac{\left\lfloor \frac{x}{|x|+d} \right\rfloor + \left\lceil \frac{x}{|x|+d} \right\rceil}{a} + \frac{1}{f} \right\rfloor, where \left\{ \frac{1 < a \le 2, f > \frac{a}{a-1}, d > 0, or}{2 < a < f, d > 0,} \right\}$$

Now, let 'x' be zero.

$$[0 \geq 0] = \left\lceil \frac{\left\lfloor \frac{0}{|0| + d} \right\rfloor + \left\lceil \frac{0}{|0| + d} \right\rceil}{a} + \frac{1}{f} \right\rceil, where \left\{ \frac{1 < a \leq 2, f > \frac{a}{a-1}, d > 0, or}{2 < a < f, d > 0,} \right\}$$

Ok.

$$f > \frac{a}{a-1} = \frac{a-1+1}{a-1} = 1 + \frac{1}{a-1}$$

And we have:

$$1 < a \leq 2$$

$$0 < a - 1 \leq 1$$

$$1 \leq \frac{1}{a-1} < \infty$$

$$2 \leq 1 + \frac{1}{a-1} < f$$

Thus,

$$2 < f$$

Therefore,

$$[0 \geq 0] = \left\lceil \frac{\left\lfloor \frac{0}{|0| + d} \right\rfloor + \left\lceil \frac{0}{|0| + d} \right\rceil}{a} + \frac{1}{f} \right\rceil = \left\lceil \frac{1}{f} \right\rceil = 1$$

Next, let 'x' be 1:

$$[x \geq 0] = \left\lceil \frac{\left\lfloor \frac{x}{|x|+d} \right\rfloor + \left\lceil \frac{x}{|x|+d} \right\rceil}{a} + \frac{1}{f} \right\rceil, where \left\{ \frac{1 < a \leq 2, f > \frac{a}{a-1}, d > 0, or}{2 < a < f, d > 0,} \right\}$$

$$[1 \geq 0] = \left\lceil \frac{\left\lfloor \frac{1}{|1|+d} \right\rfloor + \left\lceil \frac{1}{|1|+d} \right\rceil}{a} + \frac{1}{f} \right\rceil, where \left\{ \frac{1 < a \leq 2, f > \frac{a}{a-1}, d > 0, or}{2 < a < f, d > 0,} \right\}$$

Then,

$$\left\lceil \frac{\left\lfloor \frac{1}{|1|+d} \right\rfloor + \left\lceil \frac{1}{|1|+d} \right\rceil}{a} + \frac{1}{f} \right\rceil = \left\lceil \frac{0+1}{a} + \frac{1}{f} \right\rceil = \left\lceil \frac{1}{a} + \frac{1}{f} \right\rceil$$

If 1<a<=2,

$$f > \frac{a}{a-1}$$

$$\frac{1}{f} < \frac{a-1}{a}$$

$$\frac{1}{a} + \frac{1}{f} < \frac{1}{a} + \frac{a-1}{a} = 1$$

Thus,

$$0 < \frac{1}{a} + \frac{1}{f} < 1$$

Therefore,

$$[1 \geq 0] = \left\lceil \frac{1}{a} + \frac{1}{f} \right\rceil = 1$$

Next, If 2<a<f,

$$2 < a < f$$

$$\frac{1}{f} < \frac{1}{a} < \frac{1}{2}$$

Then,

$$\frac{1}{f} < \frac{1}{2}$$

$$\frac{1}{a} < \frac{1}{2}$$

Thus,

$$0 < \frac{1}{a} + \frac{1}{f} < 1$$

Therefore,

$$[1 \geq 0] = \left\lceil \frac{1}{a} + \frac{1}{f} \right\rceil = 1$$

Next, let's say 'x' is -1:

$$[x \geq 0] = \left\lceil \frac{\left\lfloor \frac{x}{|x| + d} \right\rfloor + \left\lceil \frac{x}{|x| + d} \right\rceil}{a} + \frac{1}{f} \right\rceil, where \left\{ \frac{1 < a \leq 2, f > \frac{a}{a-1}, d > 0, or}{2 < a < f, d > 0,} \right\}$$

$$[-1 \geq 0] = \left\lceil \frac{\left\lfloor \frac{-1}{|-1| + d} \right\rfloor + \left\lceil \frac{-1}{|-1| + d} \right\rceil}{a} + \frac{1}{f} \right\rceil, where \left\{ \frac{1 < a \leq 2, f > \frac{a}{a-1}, d > 0, or}{2 < a < f, d > 0,} \right\}$$

$$\left\lceil \frac{\left\lfloor \frac{-1}{|-1| + d} \right\rfloor + \left\lceil \frac{-1}{|-1| + d} \right\rceil}{a} + \frac{1}{f} \right\rceil = \left\lceil \frac{\left\lfloor \frac{-1}{1 + d} \right\rfloor + \left\lceil \frac{-1}{1 + d} \right\rceil}{a} + \frac{1}{f} \right\rceil = \left\lceil \frac{-1 + 0}{a} + \frac{1}{f} \right\rceil = \left\lceil \frac{-1}{a} + \frac{1}{f} \right\rceil$$

Like before, if 1<a<=2,

$$f > \frac{a}{a-1}$$

$$\frac{1}{f} < \frac{a-1}{a}$$

$$0 < \frac{1}{f} < 1 - \frac{1}{a}$$

We have:

$1 < a \leq 2$

$\frac{1}{2} \leq \frac{1}{a} < 1$

$-1 < -\frac{1}{a} \leq -\frac{1}{2}$

Now, let's add the following two sets of inequalities:

$0 < \frac{1}{f} < 1 - \frac{1}{a}$

$-1 < -\frac{1}{a} \leq -\frac{1}{2}$

Then, we get:

$-1 < -\frac{1}{a} + \frac{1}{f} < \frac{1}{2} - \frac{1}{a}$

Like before, let's look at the upper boundary:

$\frac{1}{2} - \frac{1}{a}$

Let's manipulate the range of 'a':

$1 < a \leq 2$

$\frac{1}{2} \leq \frac{1}{a} < 1$

$-1 < -\frac{1}{a} \leq -\frac{1}{2}$

$-\frac{1}{2} < \frac{1}{2} - \frac{1}{a} \leq 0$

Ok. Let us combine above inequality with the previous one:

$-1 < \frac{1}{f} - \frac{1}{a} < \frac{1}{2} - \frac{1}{a}$

Then we have:

$-1 < \frac{1}{f} - \frac{1}{a} < \frac{1}{2} - \frac{1}{a} \leq 0$

Thus,

$$-1 < \frac{1}{f} - \frac{1}{a} < 0$$

Therefore,

$$[-1 \geq 0] = \left\lceil \frac{1}{-a} + \frac{1}{f} \right\rceil = \left\lceil \frac{1}{f} - \frac{1}{a} \right\rceil = 0$$

So, we got what we wanted. Our next step is to formally prove our hypothesis. Today is 2/24/25. Let's do that tomorrow //:-)

Ok, we are back on 3/3/25. I needed some break from all that math. Let's go.

(proof begins)

$$[x \geq c] = \left\lceil \frac{\left\lfloor \frac{x-c}{|x-c|+d} \right\rfloor + \left\lceil \frac{x-c}{|x-c|+d} \right\rceil}{a} + \frac{1}{f} \right\rceil, where \left\{ \frac{1 < a \leq 2, f > \frac{a}{a-1}, d > 0, or}{2 < a < f, d > 0,} \right\}$$

The first case is when x=c.

$$[c \geq c] = \left\lceil \frac{\left\lfloor \frac{c-c}{|c-c|+d} \right\rfloor + \left\lceil \frac{c-c}{|c-c|+d} \right\rceil}{a} + \frac{1}{f} \right\rceil, where \left\{ \frac{1 < a \leq 2, f > \frac{a}{a-1}, d > 0, or}{2 < a < f, d > 0,} \right\}$$

$$[c \geq c] = \left\lceil \frac{\left\lfloor \frac{0}{|0|+d} \right\rfloor + \left\lceil \frac{0}{|0|+d} \right\rceil}{a} + \frac{1}{f} \right\rceil = \left\lceil \frac{1}{f} \right\rceil$$

First, when 1<a<=2,

$$1 < a \leq 2$$

$$0 < a - 1 \leq 1$$

$$1 \leq \frac{1}{a-1} < \infty$$

$$2 \leq 1 + \frac{1}{a-1} < f$$

Thus,

$$2 < f$$

Therefore,

$$[c \geq c] = \left\lceil \frac{1}{f} \right\rceil = 1$$

Second, when 2<a<f:

$$2 < f$$

Therefore,

$$[c \geq c] = \left\lceil \frac{1}{f} \right\rceil = 1$$

So, it works.

The second case is when x>c. Like before, let 'e' be x-c. Then, e>0.

$$[x \geq c] = \left\lceil \frac{\left\lfloor \frac{x-c}{|x-c|+d} \right\rfloor + \left\lceil \frac{x-c}{|x-c|+d} \right\rceil}{a} + \frac{1}{f} \right\rceil, where \left\{ \frac{1 < a \leq 2, f > \frac{a}{a-1}, d > 0, or}{2 < a < f, d > 0,} \right\}$$

$$[x \geq c] = \left\lceil \frac{\left\lfloor \frac{e}{|e|+d} \right\rfloor + \left\lceil \frac{e}{|e|+d} \right\rceil}{a} + \frac{1}{f} \right\rceil = \left\lceil \frac{0+1}{a} + \frac{1}{f} \right\rceil = \left\lceil \frac{1}{a} + \frac{1}{f} \right\rceil$$

If 1<a<=2,

$$f > \frac{a}{a-1}$$

$$\frac{1}{f} < \frac{a-1}{a}$$

$$\frac{1}{a} + \frac{1}{f} < \frac{1}{a} + \frac{a-1}{a} = 1$$

Thus,

$$0 < \frac{1}{a} + \frac{1}{f} < 1$$

Therefore,

$$[x \geq c] = \left\lceil \frac{1}{a} + \frac{1}{f} \right\rceil = 1$$

Next, If 2<a<f,

$$2 < a < f$$

$$\frac{1}{f} < \frac{1}{a} < \frac{1}{2}$$

Then,

$$\frac{1}{f} < \frac{1}{2}$$

$$\frac{1}{a} < \frac{1}{2}$$

Thus,

$$0 < \frac{1}{a} + \frac{1}{f} < 1$$

Therefore,

$$[x \geq c] = \left\lceil \frac{1}{a} + \frac{1}{f} \right\rceil = 1$$

So, it works. Our strategy here is, first, make some concrete examples using numbers instead of letters first, and then prove the hypothesis. Then, use the same method when we prove the hypothesis using letters.

The third case is when x<c. Let ‘e’ be c-x. Then, e > 0.

$$[x \geq c] = \left\lceil \frac{\left\lfloor \frac{x-c}{|x-c|+d} \right\rfloor + \left\lceil \frac{x-c}{|x-c|+d} \right\rceil}{a} + \frac{1}{f} \right\rceil, where \left\{ \frac{1 < a \leq 2, f > \frac{a}{a-1}, d > 0, or}{2 < a < f, d > 0,} \right\}$$

$$[x \geq c] = \left\lceil \frac{\left\lfloor \frac{-e}{|-e|+d} \right\rfloor + \left\lceil \frac{-e}{|-e|+d} \right\rceil}{a} + \frac{1}{f} \right\rceil = \left\lceil \frac{\left\lfloor \frac{-e}{e+d} \right\rfloor + \left\lceil \frac{-e}{e+d} \right\rceil}{a} + \frac{1}{f} \right\rceil$$

$$= \left\lceil \frac{-1+0}{a} + \frac{1}{f} \right\rceil = \left\lceil \frac{-1}{a} + \frac{1}{f} \right\rceil$$

Like before, if 1<a<=2,

$$f > \frac{a}{a-1}$$

$$\frac{1}{f} < \frac{a-1}{a}$$

$$0 < \frac{1}{f} < 1 - \frac{1}{a}$$

We have:

$$1 < a \leq 2$$

$$\frac{1}{2} \leq \frac{1}{a} < 1$$

$$-1 < -\frac{1}{a} \leq -\frac{1}{2}$$

Now, let's add the following two sets of inequalities:

$$0 < \frac{1}{f} < 1 - \frac{1}{a}$$

$$-1 < -\frac{1}{a} \leq -\frac{1}{2}$$

Then, we get:

$$-1 < -\frac{1}{a} + \frac{1}{f} < \frac{1}{2} - \frac{1}{a}$$

Like before, let's look at the upper boundary:

$$\frac{1}{2} - \frac{1}{a}$$

Let's manipulate the range of 'a':

$$1 < a \leq 2$$

$$\frac{1}{2} \leq \frac{1}{a} < 1$$

$$-1 < -\frac{1}{a} \leq -\frac{1}{2}$$

$$-\frac{1}{2} < \frac{1}{2} - \frac{1}{a} \leq 0$$

Ok. Let us combine above inequality with the previous one:

$$-1 < \frac{1}{f} - \frac{1}{a} < \frac{1}{2} - \frac{1}{a}$$

Then we have:

$$-1 < \frac{1}{f} - \frac{1}{a} < \frac{1}{2} - \frac{1}{a} \leq 0$$

Thus,

$$-1 < \frac{1}{f} - \frac{1}{a} < 0$$

Therefore,

$$[x \geq c] = \left\lceil \frac{1}{-a} + \frac{1}{f} \right\rceil = \left\lceil \frac{1}{f} - \frac{1}{a} \right\rceil = 0$$

So, it works.

Next, let’s see what happens when 2<a<f:

$$2 < a < f$$

$$0 < \frac{1}{f} < \frac{1}{a} < \frac{1}{2}$$

$$-\frac{1}{a} < \frac{1}{f} - \frac{1}{a} < 0$$

Ok. Now, let’s find out the lower boundary:

$$2 < a$$

$$\frac{1}{a} < \frac{1}{2}$$

$$\frac{-1}{2} < \frac{-1}{a}$$

Thus,

$$-\frac{1}{2} < \frac{1}{f} - \frac{1}{a} < 0$$

Therefore,

$$[x \geq c] = \left\lceil \frac{1}{-a} + \frac{1}{f} \right\rceil = \left\lceil \frac{1}{f} - \frac{1}{a} \right\rceil = 0$$

This completes the proof. Awesome //:-)

(proof ends)

(7) Inverse Iverson Transformation Revisited: Putting Them Altogether

Alright. Let's express a random discontinuous function with Iverson brackets.

$$f(x) = \begin{cases} f_0 \, , if \; x < k_1 \\ f_1 \, , if \; x = k_1 \\ f_{12} \, , if \; k_1 < x < k_2 \\ f_2 \, , if \; x = k_2 \\ f_3 \, , if \; x > k_2 \end{cases}$$

Note that the five sub-functions above, they can be all different. Some or all of them can be the same as well. Now, let's iversonize the above function.

$$f(x) = f_0 * [x < k_1] + f_1 * [x = k_1] + f_{12} * [k_1 < x < k_2] + f_2 * [x = k_2] + f_3 * [x > k_2]$$

We will use Iverson conjunction formula, which can be verified using the truth table:

$$[p(x) \wedge q(x)] = p(x) * q(x)$$

Then,

$$[k_1 < x < k_2] = [k_1 < x] * [x < k_2]$$

Alright. Let's plug it in.

$$f(x) = f_0 * [x < k_1] + f_1 * [x = k_1] + f_{12} * [k_1 < x] * [x < k_2] + f_2 * [x = k_2] + f_3 * [x > k_2]$$

Now, let's enumerate the formulas that we found before:

$$\delta_x^k = [x = k] = \left\lfloor \frac{a}{|x-k| + a} \right\rfloor \; , where \; a > 0$$

$$[x > k] = \left\lceil \frac{\left\lfloor \frac{x-k}{|x-k| + c} \right\rfloor + \left\lceil \frac{x-k}{|x-k| + c} \right\rceil}{b} \right\rceil , where \; b > 1, c > 0$$

$$[x < k] = \left\lceil \frac{\left\lceil \frac{k-x}{|k-x| + c} \right\rceil + \left\lfloor \frac{k-x}{|k-x| + c} \right\rfloor}{b} \right\rceil , where \; b > 1, c > 0$$

Actually, in the two formulas above, the ‘c’ appears twice. Let’s generalize them even further:

$$[x > k] = \left\lceil \frac{\left\lfloor \frac{x-k}{|x-k|+c} \right\rfloor + \left\lceil \frac{x-k}{|x-k|+d} \right\rceil}{b} \right\rceil, where\ b > 1, c > 0, d > 0$$

$$[x < k] = \left\lceil \frac{\left\lceil \frac{k-x}{|k-x|+c} \right\rceil + \left\lfloor \frac{k-x}{|k-x|+d} \right\rfloor}{b} \right\rceil, where\ b > 1, c > 0, d > 0$$

In other words, ‘c’ and ‘d’ may be the same but they do not need to.

Now, let’s bring them altogether.

$$f(x) = f_0 * [x < k_1] + f_1 * [x = k_1] + f_{12} * [k_1 < x] * [x < k_2] + f_2 * [x = k_2] + f_3 * [x > k_2]$$

$$f(x) = f_0 * \left\lceil \frac{\left\lceil \frac{k_1-x}{|k_1-x|+c_1} \right\rceil + \left\lfloor \frac{k_1-x}{|k_1-x|+d_1} \right\rfloor}{b_1} \right\rceil$$

$$+f_1 * \left\lfloor \frac{a_1}{|x-k_1|+a_1} \right\rfloor$$

$$+f_{12} * \left\lceil \frac{\left\lfloor \frac{x-k_1}{|x-k_1|+c_2} \right\rfloor + \left\lceil \frac{x-k_1}{|x-k_1|+d_2} \right\rceil}{b_2} \right\rceil * \left\lceil \frac{\left\lceil \frac{k_2-x}{|k_2-x|+c_3} \right\rceil + \left\lfloor \frac{k_2-x}{|k_2-x|+d_3} \right\rfloor}{b_3} \right\rceil$$

$$+f_2 * \left\lfloor \frac{a_2}{|x-k_2|+a_2} \right\rfloor$$

$$+f_3 * \left\lceil \frac{\left\lfloor \frac{x-k_2}{|x-k_2|+c_4} \right\rfloor + \left\lceil \frac{x-k_2}{|x-k_2|+d_4} \right\rceil}{b_4} \right\rceil$$

$$, where\ a_1, a_2, c_1, c_2, c_3, c_4, d_1, d_2, d_3, d_4 > 0, and\ b_1, b_2, b_3, b_4 > 1$$

Well. The formula above is correct. But. Is it useful? //xD Well, we’re pure mathematics here, so usefulness is irrelevant. But, it is possible that some of the formulas in this paper may find applications in electronics and signal processing in the future. Either way, practical applications or not, at least it’s a good education and entertainment for mathematics enthusiasts like this author //:-)

5. Other Formulas Discovered Along The Way

(1) Integer Membership Function

We will first formulate integer non-membership function, which outputs 1 if its input is a non-integer, and outputs 0 if its input is an integer. 'I' denotes the set of all integers.

$$f(x) = [x \in I] = [\{x\} = 0] = \left\lfloor \frac{a}{|\{x\}| + a} \right\rfloor = \left\lfloor \frac{a}{\{x\} + a} \right\rfloor, where\ a > 0$$

We will next formulate integer non-membership function, which outputs 1 if its input is a non-integer, and outputs 0 if its input is an integer.

$$g(x) = [x \notin I] = 1 - f(x) = 1 - [x \in I] = 1 - \left\lfloor \frac{a}{\{x\} + a} \right\rfloor, where\ a > 0$$

Now, let's use the floor-ceiling negation formula and integer-crawl-in formula:

$$g(x) = [x \notin I] = 1 - \left\lfloor \frac{a}{\{x\} + a} \right\rfloor = 1 + \left\lceil \frac{-a}{\{x\} + a} \right\rceil = \left\lceil 1 + \frac{-a}{\{x\} + a} \right\rceil = \left\lceil \frac{\{x\} + a - a}{\{x\} + a} \right\rceil$$
$$= \left\lceil \frac{\{x\}}{\{x\} + a} \right\rceil, where\ a > 0$$

A good application of the above formula is as follows:

$$\{x\} + /x/ = [x \notin I] = \left\lceil \frac{\{x\}}{\{x\} + a} \right\rceil, where\ a > 0$$

That is, when x is a non-integer, fraction x plus complement x equals one. When x is an integer, both fraction x and complement x are zero, making their sum zero.

(2) Another Formula

This is a formula that was discovered along the way. Let's go ahead and prove it. We will use lefthand side (L) and righthand side (R) method:

$$L = \{x\} - \{y\} * \frac{\lfloor x \rfloor}{\lfloor y \rfloor} =^? x - y * \frac{\lfloor x \rfloor}{\lfloor y \rfloor} = R$$

(proof begins)

$$R = x - y * \frac{\lfloor x \rfloor}{\lfloor y \rfloor} = \lfloor x \rfloor + \{x\} - (\lfloor y \rfloor + \{y\}) * \frac{\lfloor x \rfloor}{\lfloor y \rfloor} = \lfloor x \rfloor + \{x\} - \lfloor x \rfloor - \{y\} * \frac{\lfloor x \rfloor}{\lfloor y \rfloor}$$
$$= \{x\} - \{y\} * \frac{\lfloor x \rfloor}{\lfloor y \rfloor} = R$$

(proof ends)

6. Appendix: A Fundamental Unsolved Problem in Complex Analysis

In this chapter, we will discuss an important topic unrelated to the main topic of fraction algebra.

The author published two papers related to complex analysis.[12] Those two papers may contain some errors but they contain valuable and valid methodologies. The author left the project of Riemann hypothesis proof, early in the year 2024 after he discovered an even more fundamental and important unsolved problem in complex analysis.

As of today, 3/9/25, a Wikipedia article[13] acknowledges that complex exponentiation is not well defined. The author tried to find a consistent definition of complex exponentiation, but was unable to do so. The author's opinion is that, without a logically consistent definition of complex exponentiation, it is nonsensical to talk about Riemann hypothesis,[14] because Riemann hypothesis involves complex exponentiation. Only after we have a consistent definition of what a complex exponentiation is, then we can talk about Riemann hypothesis and proceed with its proof or disproof.

Let's make an example of complex exponentiation and understand why it's not well defined, at this time.

Well, let's step back and deal with basics first. Let 'k' be any integer. Let's find a complex square root of 1:

$$z^2 = 1 = r * e^{i(\theta+2k\pi)} = 1 * e^{i(0+2k\pi)} = 1 * e^{i(2k\pi)} = e^{i(2k\ \ )}$$

We will apply fractional exponentiation of both sides:

$$(z^2)^{\frac{1}{2}} = z = \left(e^{i(2k\pi)}\right)^{\frac{1}{2}} = e^{\frac{i(2k\pi)}{2}} = e^{i(k\pi)} = \{-1,1\}$$

Now, let's go to the famous Euler's identity:

$$e^{i\theta} = \cos\theta + i * \sin\theta$$

Let theta be 1:

$$e^{i} = \cos 1 + i * \sin 1$$

Above, e^i is one complex number, not a set of infinitely many complex numbers.

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[12] See https://vixra.org/abs/2308.0064 ; https://papers.ssrn.com/sol3/papers.cfm?abstract_id=4643304 .
[13] See https://en.wikipedia.org/wiki/Exponentiation .
[14] See https://en.wikipedia.org/wiki/Riemann_hypothesis .

Now, we understand that complex exponentiation is not yet defined. But, for now, let's see what happens if we apply the multiplication rule to complex exponentiation. Here, let -k be m. Since k is an integer, m is also an integer.

$$(e^1)^i = \left(e^{i(-i)}\right)^i = \left(e^{i(-i+2k\ \ )}\right)^i \overset{?}{=} e^{i*i(-i+2k\pi)} = e^{-1*(-i+2k\pi)} = e^{i-2k\pi} = e^{i+2m\pi}$$
$$= e^i * e^{2m\pi} = (\cos 1 + i * \sin 1) * \{\dots, e^{-4\pi}, e^{-2\pi}, 1, e^{2\pi}, e^{4\pi}, \dots\}$$

Above, we have an infinite set of complex numbers.

So, basically, we cannot apply the multiplication rule when it comes to a situation where an exponent is a complex number. In other words, we do not have a consistent definition of complex exponentiation. The author tried to find a good definition of complex exponentiation, but was not able to do so. In this author's opinion, without a good definition of complex exponentiation, it is nonsensical to proceed with the project of proving or disproving Riemann hypothesis. That's why the author left that project, early in the year of 2024, though he may get back to it in the future, after he or others find a good definition of complex exponentiation. In the meantime, there are tons of other undiscovered areas in the mathematics universe, so we will continue to explore different areas of mathematics //:-)

Epilogue[15]

Hello everyone, thank you for your kind and generous readership //:-D We hope you enjoyed the show. Our next article to write and publish will be "Infinitesimal Algebra and Infinity Algebra". Our next next article to write and publish will be titled, "The Secret Forest of Forey Sequence". There, we'll introduce some interesting concepts in the world of symmetric distribution of coprimes.[16]

Thank you for your time and see you later, kind and generous ladies and gentlemen //:-)

[15] This paper was started being written on 1/26/2025. It was finished being written on 3/9/2025 //:-)
[16] See https://en.wikipedia.org/wiki/The_Road_Not_Taken .