

The Higgs Boson May Be Made of Higher Dimensional Matter

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The Higgs boson may not be the heaviest particle created by man so far (it is exceeded by the top quark), but it may be the highest dimension particle created by man so far. If calculations are correct, it may be of dimension 20/21 (the top quark is dimension 9/10), that is, it may be composed of 20-dimensional matter (quarks) circulating in the surface of a 21-sphere. Recent Higgs boson mass measurements of high accuracy, and a mass factoring technique based on n-sphere surface volumes, which was derived from Planck's Energy-Frequency Relation, $E=hf$, were used to reach that conclusion. A power of two factoring, which is a possible indication of stability, was found within 3 MeV of the ATLAS group's recent 125,220 MeV improved accuracy measurement of the Higgs's mass. The S21h power of two factoring found, $(2^{16} - 2^{10})$ S21h, translates to a mass of 125,217.08 MeV, which is very close to the ATLAS group's result. Is the Higgs boson made of 20/21 dimensional matter?

Key to the Investigation of Hadron Masses

The key to the investigation of hadron masses with n-sphere surface volumes is the formula, $m=xSnh$, where m is the mass of the hadron in units of MeV/c^2 , x is a number, S_n is the value of the surface volume formula of a unit radius n-sphere, and h is Planck's constant's coefficient, but with different units. Used here, in the factoring formula, $m=xSnh$, it has units of MeV/c^2 , not J-s. (The factoring formula, $m=xSnh$, can be derived from Planck's Energy-Frequency Relation, $E=hf$, and how h gets its units changed to MeV/c^2 , and its factor of 10^{-34} removed is explained in the derivation of $m=xSnh$ on page 4.) When divided into experimental hadron masses (given in units of MeV/c^2) the result will be an integer, or an integer and a fraction, if the hadron's matter is of the same dimension as the factoring unit's dimension. It has been tested on hundreds of experimental hadron masses and has been found to factor many of them convincingly. See page 5 for examples.

Dimensional Analysis of the $W \times W = H$ Reaction

One way physicists believe the Higgs boson is created is by a reaction between two W bosons. The reaction can be written as $W \times W = H$. To find the dimension of the product matter (the Higgs boson) in this reaction, the dimensions of the matter in the reactant hadrons (W bosons) must be known. To find the dimension of the matter in a W boson, a convincing factoring of it must be found. The masses of the two W boson candidates most likely to be the W boson, according to experimentalists, are shown below, and both factor convincingly with **S11h**.

| <u>Hadron Names</u> | <u>W Boson</u> <u>ExpMass</u> | <u>Diff</u> <u>TM-EM</u> | <u>W Boson</u> <u>ThrMass</u> | <u>HSSV</u> <u>Factoring</u> |
|----------------------|----------------------------------|-----------------------------|----------------------------------|---------------------------------|
| W boson candidate #1 | 80354 | 1.47 | 80355.47 = | 4096/7 S11h |
| W boson candidate #2 | 80433.5 | 0.44 | 80433.94 = | 4100/7 S11h |

The factorings of the W boson candidates with S11 (and h) means that the W boson is likely composed of 10 dimensional matter circulating in the surface of an 11-sphere. That's what the factorings tell us. According to conclusions drawn from factoring hundreds of hadrons with n-sphere surface volumes (times h), it appears that when two hadrons collide and form a different hadron, *higher dimensional matter is created*. That is, the matter (quarks) in the *product hadron* is of a higher dimension than the matter (quarks) in the *reactant hadrons*, and the amazing thing is, the dimension of the *product hadron's matter* can be calculated if the dimensions of the

reactant hadrons are known. Just multiply together all the surface volume formulae associated with each of the reactant hadrons (quarks) in the reaction. Examine the resulting formula. Do the powers of ' π ' and ' r ' in the resulting formula match the powers of ' π ' and ' r ' in a valid surface volume formula? If so, the resultant hadron is composed of matter (quarks) of the dimension of the matching surface volume formula. To find out what dimension a Higgs boson might be, multiply **S11** by **S11**.

$$(\mathbf{S11})(\mathbf{S11}) = (64 / 945 \pi^5 r^{10}) (64 / 945 \pi^5 r^{10})$$

$$(\mathbf{S11})(\mathbf{S11}) = 4096 / 893025 \pi^{10} r^{20}$$

The resulting formula above has the same powers of ' π ' and ' r ' as the formula for the surface volume of a 21-sphere, shown below.

$$\mathbf{S21} = 2048 / 654729075 \pi^{10} r^{20}$$

The conclusion to be drawn from this is that the Higgs boson - since it can be created by the collision of two W bosons - likely has a dimension of 20/21, that is, it is likely composed of 20-dimensional matter, circulating in the surface of a 21-sphere.

To confirm that the matter in the Higgs boson is of this dimension, (20/21), divide the Higgs's experimental masses by **S21h** and see if any convincing factorings can be found (such as an integer result, or even better - a power of two result, or a sum of powers of two result.). After checking all available Higgs boson mass measurements, one was found roughly in the middle of the range of measurements, with a highly significant, that is, a convincing factoring. It is $(2^{16} - 2^{10}) \mathbf{S21h}$, which was found when the experimental mass (a recent estimate of improved accuracy made by the ATLAS group) 125,220 MeV, was divided by **S21h**. See the table below. Masses are in units of MeV/c².

| <u>TECH</u> | <u>Higgs</u> <u>ExpMass</u> | <u>ExpErr</u> | <u>Higgs</u> <u>ThrMass</u> | <u>HSSV</u> <u>Factoring</u> |
|-------------|--------------------------------|---------------|--------------------------------|-------------------------------------|
| ATLAS | 125,220 | 110 | 125,217.08 | = $(2^{16} - 2^{10}) \mathbf{S21h}$ |

It looks like $(2^{16} - 2^{10}) \mathbf{S21h}$ is the mass found by the Atlas group. Is it the Higgs boson? If the Higgs is supposed to be an elementary particle - one not composed of other parts - then no, it isn't the Higgs. How can this particle, $(2^{16} - 2^{10}) \mathbf{S21h}$, be called elementary when physicists know it decays four or five different ways to less massive particles? So, if the Higgs has to be an elementary particle then, no, this isn't the Higgs.

Why was this factoring, $(2^{16} - 2^{10})\mathbf{S21h}$, and not $(2^{16})\mathbf{S21h}$, which translates to a mass of 127,204.65 MeV, found for the Higgs boson's mass? Is the mass with the factoring of $(2^{16} - 2^{10})\mathbf{S21h}$ more stable, or formed more readily, than the mass with the factoring $(2^{16})\mathbf{S21h}$? Which factorings of **S21h** are most stable and why? Is what we currently know about physics enough to find answers to questions about higher dimensional matter, and specifically, about the structure and dynamics of hadrons?

On the next page (page 3) is a table of some Higgs boson mass measurements between plus and minus 1000 MeV of $(2^{16} - 2^{10}) \mathbf{S21h}$, approximately, matched to their factorings with **S21h**. Notice the factorings are all the result of additions or subtractions of multiples of smaller powers of two (or sums of smaller powers of two) to $(2^{16} - 2^{10})$ times **S21h**.

Some Higgs Boson Mass Measurements
Matched with
Hypersphere Surface Volume Factorings of Them
(Mass in units of MeV/c²)

| <u>TECN</u> | <u>Higgs</u> <u>ExpMass</u> | <u>+/-</u> | <u>Higgs</u> <u>ThrMass</u> | <u>HSS Volume</u> <u>Factoring</u> | <u>ExpM-ThrM</u> <u>MassDiff</u> | <u>Range</u> |
|-------------|--------------------------------|------------|--------------------------------|--|-------------------------------------|--------------|
| | | | 124,223.29 = | (2 ¹⁶ -2 ¹⁰ -512) S21h | | -1000 MeV |
| | | | 124,409.63 = | (2 ¹⁶ -2 ¹⁰ -416) S21h | | |
| | | | 124,471.74 = | (2 ¹⁶ -2 ¹⁰ -384) S21h | | |
| | | | 124,627.02 = | (2 ¹⁶ -2 ¹⁰ -304) S21h | | |
| | | | 124,595.96 = | (2 ¹⁶ -2 ¹⁰ -320) S21h | | |
| CMS | 124,700 | 310 | 124,720.19 = | (2 ¹⁶ -2 ¹⁰ -256) S21h | 20.19 | |
| ATLS | 124,860 | 270 | 124,844.41 = | (2 ¹⁶ -2 ¹⁰ -192) S21h | 15.59 | |
| ATLS | 124,970 | 240 | 124,968.63 = | (2 ¹⁶ -2 ¹⁰ -128) S21h | 1.37 | |
| LHC | 125,090 | 210 | 125,092.86 = | (2 ¹⁶ -2 ¹⁰ - 64) S21h | 2.86 | |
| LHC | 125,150 | 370 | 125,154.97 = | (2 ¹⁶ -2 ¹⁰ - 32) S21h | 4.97 | |
| ATLS | 125,170 | 110 | 125,170.50 = | (2 ¹⁶ -2 ¹⁰ - 24) S21h | 0.50 | |
| | | | 125,186.02 = | (2 ¹⁶ -2 ¹⁰ - 16) S21h | | |
| PDG AVG | 125,200 | 110 | 125,201.55 = | (2 ¹⁶ -2 ¹⁰ - 8) S21h | 1.55 | |
| ATLS | 125,220 | 110 | 125,217.08 = | (2¹⁶-2¹⁰) S21h | | 0 MeV |
| | | | 125,232.61 = | (2 ¹⁶ -2 ¹⁰ + 8) S21h | | |
| | | | 125,248.14 = | (2 ¹⁶ -2 ¹⁰ + 16) S21h | | |
| | | | 125,263.66 = | (2 ¹⁶ -2 ¹⁰ + 24) S21h | | |
| | | | 125,279.19 = | (2 ¹⁶ -2 ¹⁰ + 32) S21h | | |
| ATLS | 125,360 | 370 | 125,341.30 = | (2 ¹⁶ -2 ¹⁰ + 64) S21h | 18.70 | |
| CMS | 125,460 | 160 | 125,465.53 = | (2 ¹⁶ -2 ¹⁰ +128) S21h | 5.53 | |
| CMS | 125,590 | 420 | 125,589.75 = | (2 ¹⁶ -2 ¹⁰ +192) S21h | 0.25 | |
| | | | 125,713.97 = | (2 ¹⁶ -2 ¹⁰ +256) S21h | | |
| CMS | 125,800 | 400 | 125,807.14 = | (2 ¹⁶ -2 ¹⁰ +304) S21h | 7.14 | |
| | | | 125,838.20 = | (2 ¹⁶ -2 ¹⁰ +320) S21h | | |
| | | | 125,962.42 = | (2 ¹⁶ -2 ¹⁰ +384) S21h | | |
| ATLS | 126,000 | 400 | 126,024.53 = | (2 ¹⁶ -2 ¹⁰ +416) S21h | 24.53 | |
| CMS | 126,200 | 600 | 126,210.87 = | (2 ¹⁶ -2 ¹⁰ +512) S21h | 10.87 | +1000 MeV |

Some of the Higgs boson's experimental mass measurements factor as smaller powers of two added or subtracted from (2¹⁶ - 2¹⁰) S21h. As can be seen in the table, some of the matches between experimental and theoretical masses are quite close. 64 S21h is equal to 124 MeV approximately. The source of the Higgs boson experimental mass data in the table comes from Particle Data Group's 2024 report.

Source of Mass Data: S. Navaset al.(Particle Data Group), Phys. Rev. D110, 030001 (2024)

Derivation of the *Hypersphere Surface Volume* Factoring Formula

$$\mathbf{m}_{\text{MeV}} = \mathbf{h}_{\text{MeV}}(\mathbf{xSn})$$

The HSSV factoring formula, $\mathbf{m} = \mathbf{h}(\mathbf{xSn})$, which is used to discover hadron dimensions and exact masses, can be derived from Planck's Energy-Frequency Relation: $\mathbf{E} = \mathbf{hf}$. The key to the derivation is associating a frequency with a unit of hypervolume. A main benefit of the derivation is that it explains how the (10^{-34}) factor was removed from \mathbf{h} , and its units changed from J-s to MeV/c².

If $\mathbf{m} = \mathbf{h}(\mathbf{xSn})$ is correct, (and the factorings of hundreds of hadrons says it is) then a frequency of, $1.602176634 \times 10^{21}$ Hz, is associated with each unit of hypervolume (each unit of \mathbf{Sn}) of a hadron, no matter the dimension. In the example with \mathbf{Ds} (See next page), \mathbf{Ds} 's hypervolume is **10.000 S9**, which equals $1967.053/\mathbf{h} = 296.8657$ hypervolume units. Multiplying 296.8657 by ($1.602176634 \times 10^{21}$ Hz/vol) - the frequency per unit hypervolume constant - will give you a frequency of $4.75631288 \times 10^{23}$ Hz as the frequency associated with the entire particle, which is correct. (Putting that frequency in Planck's energy-frequency law ($\mathbf{E}=\mathbf{hf}$) will give you the particle's mass in Joules.) So in terms of particle *hypervolume*, Planck's energy-frequency law can be rewritten as:

$$\mathbf{E}_J = \mathbf{h}_{\text{J-s}}(\mathbf{xSn}) (1.602176634 \times 10^{21} \text{ Hz/vol}) \quad (\text{here } \mathbf{h} = 6.62607015 \times 10^{-34} \text{ J-s})$$

Which says a frequency (and therefore energy) is associated with a volume. To convert \mathbf{h} to units of MeV/c² divide the right hand side by $1.602176634 \times 10^{-13}$ Joules/MeV/c² (the Joules to MeV/c² conversion factor). The result is \mathbf{h} in units of MeV/c² and a factor of (1×10^{34}) times $\mathbf{h}(\mathbf{xSn})$ on the right. (\mathbf{E} on the left hand side of the equation then has units of MeV/c² by default.) When that factor, (1×10^{34}), is multiplied by Planck's constant, ($6.62607015 \times 10^{-34}$ MeV/c²), you are left with just Planck's constant's coefficient (6.62607015 MeV/c²) for \mathbf{h} . The result is:

$$\mathbf{m}_{\text{MeV}} = \mathbf{h}_{\text{MeV}}(\mathbf{xSn}) \quad (\text{So, here } \mathbf{h} = 6.62607015 \text{ MeV/c}^2, \text{ not } 6.62607015 \times 10^{-34} \text{ J-s.})$$

Where \mathbf{m} is in units of MeV/c², $\mathbf{h} = 6.62607015$ MeV/c², and \mathbf{Sn} is the hypervolume calculated from the surface volume formula for an n-sphere using a radius of one (a unit radius). (\mathbf{Snh} values are given in an appendix for all \mathbf{n} from dimensions 2 to 21.) That formula seems to work on any dimension of hadron, *which implies that the mass density of the hypervolume of hadrons remains the same over all dimensions*. What is the density of the hypervolume of any hadron? It is 6.62607015 MeV/c² per unit hypervolume. That's what the formula says if it is rearranged.

$$\mathbf{h}_{\text{MeV}} = \mathbf{m}_{\text{MeV}} / (\mathbf{xSn})$$

So, if $\mathbf{m}=\mathbf{h}(\mathbf{xSn})$ is valid, it means that if a correct factoring can be found for a hadron then, a dimension and a precise mass can be assigned to it.

Evidence That Hadrons Are Made of Higher Dimensional Matter

Theoretical and Experimental Masses Compared

(Masses in units of MeV/c²)

| <u>HSS Volume</u> <u>Factoring</u> | <u>Hadron's</u> <u>ThrMass</u> | <u>TM-EM</u> | <u>Hadron's</u> <u>ExpMass</u> | <u>ExpErr</u> | <u>Hadron's</u> <u>Name</u> | |
|---------------------------------------|-----------------------------------|--------------|-----------------------------------|---------------|--------------------------------|-----|
| 4.4444 | S5h = 775.071 | 0.051 | 775.02 | .35 | ρ (775) | |
| 6.0000 | S6h = 1232.698 | 0.202 | 1232.9 | 1.2 | Δ (1232) | |
| 2.5000 | S7h = 547.866 | 0.001 | 547.865 | 0.031 | η | |
| 25/7 | S7h = 782.665 | 0.015 | 782.65 | 0.12 | ω | |
| 6.00000 | S7h = 1314.878 | 0.018 | 1314.86 | 0.20 | Xi° | |
| 6.03125 | S7h = 1321.726 | 0.016 | 1321.71 | 0.07 | Xi⁻ | |
| 26.6666 | S8h = 5737.239 | 0.039 | 5737.2 | 0.7 | B1 (5747) | |
| 10.0000 | S9h = 1967.053 | 0.053 | 1967.0 | 1.0 | Ds | |
| 15.0000 | S10h = 2534.634 | 0.034 | 2534.6 | 0.3 | Ds1 (2536) | |
| 16.0000 | S11h = 2197.219 | 0.181 | 2197.4 | 4.4 | Xc0 (1P) | |
| 29.0000 | S11h = 3982.461 | 0.039 | 3982.5 | 1.8 | Zcs (3982) | |
| 4096/7 | S11h = 80355.47 | 1.473 | 80354 | 23 | W Boson | [2] |
| 4100/7 | S11h = 80433.94 | 0.445 | 80433.5 | 9.4 | W boson | [2] |
| 26.0000 | S12h = 2760.433 | 0.333 | 2760.1 | 1.1 | D3* (2750) | |
| 27.0000 | S12h = 2866.605 | 0.005 | 2866.6 | AVG | Ds3 (2860)⁺ | |
| 28.0000 | S12h = 2972.775 | 0.975 | 2971.8 | 8.7 | D (3000)⁰ | |
| 50.0000 | S13h = 3922.028 | 0.013 | 3922.15 | 1.2 | X (3930) | |
| 61.4400 | S14h = 3415.496 | 0.004 | 3415.5 | 0.4 | Xc0 (1P) | |
| 64.0000 | S14h = 3557.808 | 0.008 | 3557.8 | 1.2 | Xc2 (1P) | |
| 93.0000 | S15h = 3525.820 | 0.020 | 3525.8 | 0.2 | h1 (1P) | |
| 2 ¹⁷ /900 | S16h = 3633.472 | 0.128 | 3633.6 | 1.7 | nc (2s) | |
| 2 ¹⁷ +128 /900 | S16h = 3637.020 | 0.020 | 3637.0 | 5.7 | nc (2s) | |
| 2 ¹⁷ +256 /900 | S16h = 3640.569 | 0.069 | 3640.5 | 3.2 | nc (2s) | |
| 17160/70 | S17h = 3893.006 | 0.006 | 3893.0 | 2.3 | Zc (3900) | |
| 18304/70 | S17h = 4152.540 | 0.040 | 4152.5 | 1.7 | Xc1 (4140) | |
| 20736/70 | S17h = 4704.049 | | 4704 | 10 | Xc0 (4700) | |
| 222.0000 | S17h = 3525.484 | 0.084 | 3525.40 | 0.13 | hc (1P) | |
| 384.0000 | S17h = 6098.135 | 0.135 | 6098.0 | 1.7 | Σb (6097) | |
| 100.5000 | S18h = 984.646 | 0.054 | 984.7 | 0.4 | fo (980) | |
| 280.0000 | S20h = 957.590 | 0.090 | 957.5 | 0.2 | η' (958) | |
| (2 ¹⁶ -2 ¹⁰) | S21h = 125217.08 | 2.920 | 125220 | 110 | Higgs Boson | [2] |

Note: **17160** = 16384 + 512 + 256 + 8
18304 = 16384 + 1024 + 512 + 256 + 128
20736 = 16384 + 4096 + 2048 + 256

Source of Mass Data: P.A. Zyla et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2020, 083C01 (2020) and 2021 update

APPENDIX A

n-Sphere Surface Volume Formulae

(Dimension 2 - Dimension 21)

| <u>Sphere Dimension</u> | <u>S_n</u> | <u>Surface Volume Formula</u> | <u>(π, r) Powers</u> |
|-----------------------------|----------------------|------------------------------------|---|
| 2 | S2 = | 2 $\pi^1 r^1$ | (1, 1) |
| 3 | S3 = | 4 $\pi^1 r^2$ | (1, 2) |
| 4 | S4 = | 2 $\pi^2 r^3$ | (2, 3) |
| 5 | S5 = | 8/3 $\pi^2 r^4$ | (2, 4) |
| 6 | S6 = | $\pi^3 r^5$ | (3, 5) |
| 7 | S7 = | 16/15 $\pi^3 r^6$ | (3, 6) |
| 8 | S8 = | 1/3 $\pi^4 r^7$ | (4, 7) |
| 9 | S9 = | 32/105 $\pi^4 r^8$ | (4, 8) |
| 10 | S10 = | 1/12 $\pi^5 r^9$ | (5, 9) |
| 11 | S11 = | 64 / 945 $\pi^5 r^{10}$ | (5, 10) |
| 12 | S12 = | 1 / 60 $\pi^6 r^{11}$ | (6, 11) |
| 13 | S13 = | 128 / 10395 $\pi^6 r^{12}$ | (6, 12) |
| 14 | S14 = | 1 / 360 $\pi^7 r^{13}$ | (7, 13) |
| 15 | S15 = | 256 / 135135 $\pi^7 r^{14}$ | (7, 14) |
| 16 | S16 = | 1 / 2520 $\pi^8 r^{15}$ | (8, 15) |
| 17 | S17 = | 512 / 2027025 $\pi^8 r^{16}$ | (8, 16) |
| 18 | S18 = | 1 / 20160 $\pi^9 r^{17}$ | (9, 17) |
| 19 | S19 = | 1024 / 34459425 $\pi^9 r^{18}$ | (9, 18) |
| 20 | S20 = | 1 / 181440 $\pi^{10} r^{19}$ | (10, 19) |
| 21 | S21 = | 2048 / 654729075 $\pi^{10} r^{20}$ | (10, 20) |

APPENDIX B

Values of n-Sphere Surface Volume
Units of Factorization

(Below $h = 6.62607015 \text{ MeV}/c^2$, not $6.62607015 \times 10^{-34} \text{ J-s}$)

(Dimension 2 - Dimension 21)

| <u>Sphere Dimension</u> | <u>Unit of Factorization</u> | <u>Formula</u> | <u>Value (MeV/c²)</u> |
|-------------------------|------------------------------|--|----------------------------------|
| 2 | S2h = | $2 \pi^1 r^1 h =$ | 41.63282661 |
| 3 | S3h = | $4 \pi^1 r^2 h =$ | 83.26565322 |
| 4 | S4h = | $2 \pi^2 r^3 h =$ | 130.7933822 |
| 5 | S5h = | $8/3 \pi^2 r^4 h =$ | 174.3911763 |
| 6 | S6h = | $\pi^3 r^5 h =$ | 205.4497644 |
| 7 | S7h = | $16/15 \pi^3 r^6 h =$ | 219.1464153 |
| 8 | S8h = | $1/3 \pi^4 r^7 h =$ | 215.1464901 |
| 9 | S9h = | $32/105 \pi^4 r^8 h =$ | 196.7053624 |
| 10 | S10h = | $1/12 \pi^5 r^9 h =$ | 168.9756582 |
| 11 | S11h = | $64 / 945 \pi^5 r^{10} h =$ | 137.3262492 |
| 12 | S12h = | $1 / 60 \pi^6 r^{11} h =$ | 106.1705373 |
| 13 | S13h = | $128 / 10395 \pi^6 r^{12} h =$ | 78.44057013 |
| 14 | S14h = | $1 / 360 \pi^7 r^{13} h =$ | 55.59076334 |
| 15 | S15h = | $256 / 135135 \pi^7 r^{14} h =$ | 37.91204905 |
| 16 | S16h = | $1 / 2520 \pi^8 r^{15} h =$ | 24.94907624 |
| 17 | S17h = | $512 / 2027025 \pi^8 r^{16} h =$ | 15.88056197 |
| 18 | S18h = | $1 / 20160 \pi^9 r^{17} h =$ | 9.797479330 |
| 19 | S19h = | $1024 / 34459425 \pi^9 r^{18} h =$ | 5.869441980 |
| 20 | S20h = | $1 / 181440 \pi^{10} r^{19} h =$ | 3.419965454 |
| 21 | S21h = | $2048 / 654729075 \pi^{10} r^{20} h =$ | 1.940989032 |

Quark Assignments to n-Sphere Surface Volume Formulae

| Sphere Dimension | Quark Names | | | Corresponding n-Sphere Surface Formula |
|------------------|-------------|-----|---|---|
| | Old | New | | |
| 2 | u | q1 | = | $2 \pi^1 r^1$ |
| 3 | d | q2 | = | $4 \pi^1 r^2$ |
| 4 | s | q3 | = | $2 \pi^2 r^3$ |
| 5 | c | q4 | = | $8/3 \pi^2 r^4$ |
| 6 | b | q5 | = | $\pi^3 r^5$ |
| 7 | t | q6 | = | $16/15 \pi^3 r^6$ |
| 8 | ----- | q7 | = | $1/3 \pi^4 r^7$ |
| 9 | ----- | q8 | = | $32/105 \pi^4 r^8$ |
| 10 | ----- | q9 | = | $1/12 \pi^5 r^9$ |
| 11 | ----- | q10 | = | $64 / 945 \pi^5 r^{10}$ |
| 12 | ----- | q11 | = | $1 / 60 \pi^6 r^{11}$ |
| 13 | ----- | q12 | = | $128 / 10395 \pi^6 r^{12}$ |
| 14 | ----- | q13 | = | $1 / 360 \pi^7 r^{13}$ |
| 15 | ----- | q14 | = | $256 / 135135 \pi^7 r^{14}$ |
| 16 | ----- | q15 | = | $1 / 2520 \pi^8 r^{15}$ |
| 17 | ----- | q16 | = | $512 / 2027025 \pi^8 r^{16}$ |
| 18 | ----- | q17 | = | $1 / 20160 \pi^9 r^{17}$ |
| 19 | ----- | q18 | = | $1024 / 34459425 \pi^9 r^{18}$ |
| 20 | ----- | q19 | = | $1 / 181440 \pi^{10} r^{19}$ |
| 21 | ----- | q20 | = | $2048 / 654729075 \pi^{10} r^{20}$ |

Smallest Formation Quarks per n-Sphere

(Dimension 2 - Dimension 21)

| Sphere Dimension | S _n | Surface Volume Formula | (π, r) Powers | Formation Quarks |
|---------------------|----------------|--|------------------|---|
| 2 | S2 = | 2 π ¹ r ¹ | (1, 1) | u |
| 3 | S3 = | 4 π ¹ r ² | (1, 2) | d |
| 4 | S4 = | 2 π ² r ³ | (2, 3) | du = 8 π ² r ³ = 4 S4 |
| 5 | S5 = | 8/3 π ² r ⁴ | (2, 4) | dd = 64 π ² r ⁴ = 24 S5 |
| 6 | S6 = | π ³ r ⁵ | (3, 5) | ddu = 32 π ³ r ⁵ = 32 S6 |
| 7 | S7 = | 16/15 π ³ r ⁶ | (3, 6) | ddd = 256 π ³ r ⁶ = 273.. S7 |
| 8 | S8 = | 1/3 π ⁴ r ⁷ | (4, 7) | dddd = 128 π ⁴ r ⁷ = 384 S8 |
| 9 | S9 = | 32/105 π ⁴ r ⁸ | (4, 8) | dddd = 1024 π ⁴ r ⁸ = 312.. S9 |
| 10 | S10 = | 1/12 π ⁵ r ⁹ | (5, 9) | ddddu |
| 11 | S11 = | 64 / 945 π ⁵ r ¹⁰ | (5, 10) | ddddd |
| 12 | S12 = | 1 / 60 π ⁶ r ¹¹ | (6, 11) | ddddd |
| 13 | S13 = | 128 / 10395 π ⁶ r ¹² | (6, 12) | ddddd |
| 14 | S14 = | 1 / 360 π ⁷ r ¹³ | (7, 13) | ddddd |
| 15 | S15 = | 256 / 135135 π ⁷ r ¹⁴ | (7, 14) | ddddd |
| 16 | S16 = | 1 / 2520 π ⁸ r ¹⁵ | (8, 15) | ddddd |
| 17 | S17 = | 512 / 2027025 π ⁸ r ¹⁶ | (8, 16) | ddddd |
| 18 | S18 = | 1 / 20160 π ⁹ r ¹⁷ | (9, 17) | ddddd |
| 19 | S19 = | 1024 / 34459425 π ⁹ r ¹⁸ | (9, 18) | ddddd |
| 20 | S20 = | 1 / 181440 π ¹⁰ r ¹⁹ | (10, 19) | ddddd |
| 21 | S21 = | 2048 / 654729075 π ¹⁰ r ²⁰ | (10, 20) | ddddd |

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