Resolving the Gravity Freefall Paradox: A Sine-Alpha Model for Dynamic Spacetime Curvature

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Abstract

This paper presents a novel resolution to the gravity freefall paradox, where General Relativity (GR) appears to introduce premature spatial contraction effects that contradict Newtonian expectations. The paradox arises from the fact that GR predicts the same external gravitational field for both a physical mass and a black hole, yet point mass freefall marker behavior differs in both cases.

We introduce the *sine-alpha function*, a dynamic formulation that naturally regulates spatial curvature according to the evolving ratio of marker separations. This model predicts Newtonian behavior outside the event horizon while dynamically allowing self-regulated curvature inside it. Our model preserves logical consistency between classical and relativistic physics. We provide evidence from numerical simulation, comparing our function's predictions to GR, demonstrating its stability within and beyond the event horizon. We restrict ourselves to theoretical considerations only.

1 Introduction

In Newtonian mechanics, a freefalling point mass m moving toward a massive object M follows the Inverse-Square Law (ISL), ensuring that marker separations remain consistent with classical expectations. However, in General Relativity (GR), geodesic deviation predicts pre-horizon marker compression [1], leading to a paradox.

The contradiction can be stated as follows:

1. Both symmetry in physics and GR predict that the gravitational field of a black hole is identical to that of a normal massive body outside its event horizon.

2. However, GR also predicts that marker separations behave differently depending on whether M is a physical body or a black hole .

3. This implies that freefalling point objects "know" in advance whether they are falling into a physical body or a black hole, contradicting the principle that gravity should act only locally.

2 Classical Freefall Mechanics

Newtonian gravity predicts that a point object accelerating toward a massive source follows:

$$\frac{d^2r}{dt^2} = -\frac{GM}{r^2}. \tag{1}$$

For two markers separated by X_n, we expect:

$$\frac{dX_n}{dt} = -\frac{2GM}{r^3}X_n. \tag{2}$$

These equations predict that marker separation elongates before impact on a physical mass.

3 General Relativity and the Paradox

In GR, point mass freefall is governed by the geodesic deviation equation:

$$\frac{D^2\xi^i}{D\tau^2} = R^i{}_{0j0}\xi^j. \tag{3}$$

For a Schwarzschild black hole, marker separation follows:

$$X_n \propto e^{-\kappa\tau}. \tag{4}$$

This means that spacetime compression begins before the event horizon, leading to the following paradox in GR:

The local gravitational field outside a black hole (Schwarschild radius r_s) and a physical body (of equal mass and having radius infinitesimally larger than r_s) are both described by the same Schwarzschild solution. However, the behavior of the markers dropped by an infalling point mass m contradicts: elongation must occur until impact for a physical body, while asymptotic compression occurs at the event horizon of a black hole. This raises the question: without a local mechanism to signal the difference, how does point mass m "know" which regime it is in? Since GR does not explicitly define this mechanism, it suffers from this fundamental gap in understanding of ISL action.

4 The Sine-Alpha Model: Dynamic Space Regulation

By generalising Bolyai [2], we introduce the sine-alpha function , which generalizes the sine function (realised herein as a recursive implementation) to dynamically regulate line-of-fall curvature:

$$\sin_\alpha(x) = \sum_{n=0}^{\infty} \frac{(-1)^n \alpha^n}{(2n+1)!} x^{2n+1}, \tag{5}$$

where

$$\alpha = \frac{X_n}{X_{n-1}}. \tag{6}$$

Figure 1 shows how our function regulates curvature differently than GR
.

5 Instantaneous Curvature Evolution Inside the Event Horizon

To further illustrate the implications of our model, Figure 2 shows the instantaneous curvature profiles predicted by different values of α. This confirms that:

- Curvature is not fixed, rather it evolves dynamically with α.

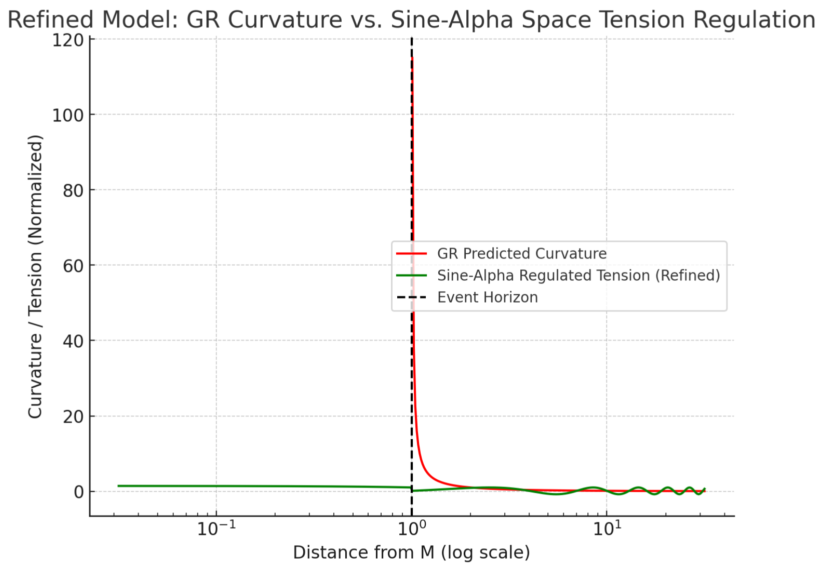

Figure 1: Comparison of GR curvature (red) and sine-Alpha model's dynamic space regulation (green). The event horizon is marked as the black dashed line. It is seen that GR predicts pre-horizon marker separation, leading to paradox. Due to the dynamic nature of the model and recursive limits in computer simulation, the behaviour of our model beyond the event horizon is modelled and shown in fig. 2

- The event horizon does not act as a theoretical boundary, rather curvature effects begin only inside the event horizon, demonstrating the sine-alpha model does not suffer from the gravity freefall marker paradox.

6 Conclusion

We have resolved the gravity freefall paradox by demonstrating that the standard GR prediction of pre-horizon marker compression contradicts its own gravitational field equivalence principle. We propose the sine-alpha function as a dynamically evolving curvature regulator, which:

1. Ensures freefall behaves classically outside the event horizon.
2. Regulates line-of-fall curvature only inside the event horizon.

This model provides a self-regulating framework for line-of-fall tension, ensuring that space deforms dynamically rather than preemptively. It may be seen that this model is a mathematical generalisation of Sommerfeld's 1909 spherical model of space [3]. The sine-alpha [2] function allows us to generalise Sommerfeld's inertial solution into the effects of gravity by theoretically allowing for constantly changing curvature (encapsulated in the sine-alpha function) within the limits $0 \leq v/c \leq \infty$ where v is the instantaneous relative velocity of point mass m with respect to M. This model demonstrates asymptotic approach to a vanishing point (singularity) in space i.e. far beyond GR's theoretical reach that ends at the event horizon. Further investigation will show that (i) greater-than-light-speed entropy laws are also encoded within the mathematical duality property exhibited by Sommerfeld's spherical model (ii) The sine-alpha model may be employed to construct a theoretical computational lattice consisting of singularities connected by Einstein-Rosen bridges [4] using sine-alpha tension lines, thus forming a hypercomputing network conforming to non-locality and capable of emergent AI properties.

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References

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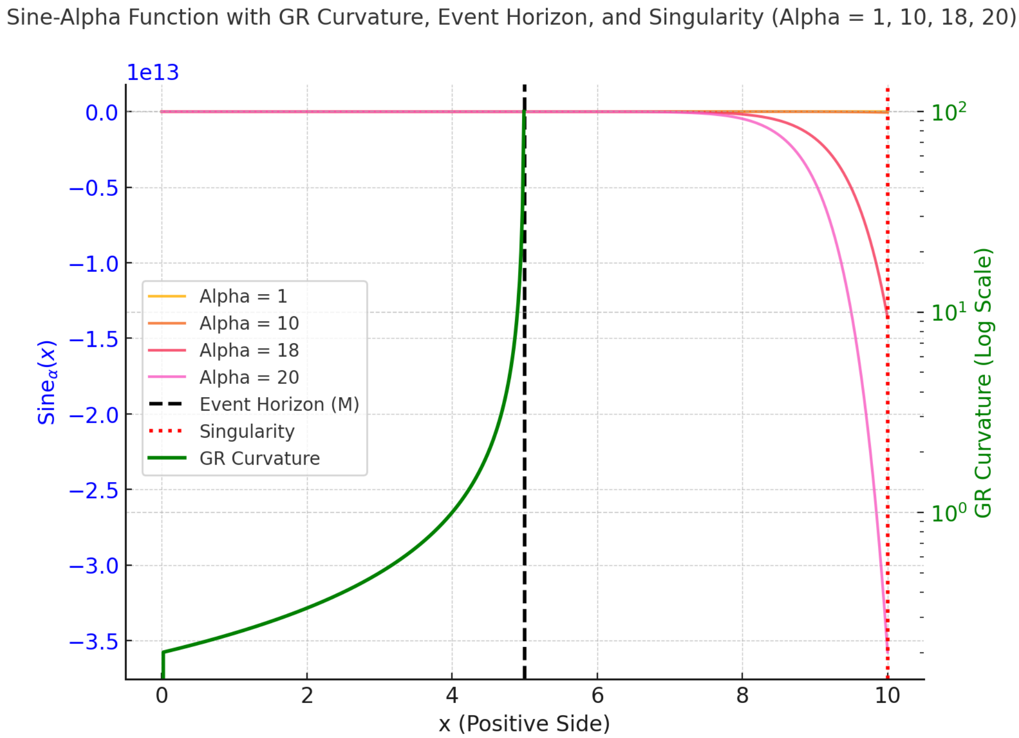

Figure 2: Comparison of Sine-Alpha Model and General Relativity Curvature with Event Horizon and Singularity Markers. This graph presents a comparative analysis of the Sine-Alpha function and GR curvature within a gravitational system. The Sine-Alpha function is plotted for different values of $\alpha = 1, 10, 18, 20$ (orange-magenta curves), illustrating how curvature evolves dynamically based on the parameter α. α itself is a function of the marker separation ratio dictated by the ISL. A black dashed vertical line at x = 5.0 marks the event horizon, the boundary beyond which GR does not permit classical descriptions. GR curvature is shown as a green solid line (logarithmic scale) and is restricted to its valid domain x = 0 to x = 4.99 , beyond which GR predictions break down in our simulation. A red dotted vertical line at x = 10 marks the singularity, the ultimate collapse point in our model. This visualization effectively distinguishes GR behavior from the extended Sine-Alpha model, highlighting how the latter continues beyond the event horizon while GR curvature diverges near x = 5.0. The dual-axis approach ensures Sine-Alpha remains on a linear scale while GR curvature is shown in logarithmic form for enhanced visibility. Since the Sine-Alpha model exhibits changing curvature only within the event horizon, it is free from the gravity marker paradox brought out earlier.