The game show problem

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abstract
The game show problem aka Monty Hall problem [1], originated when Craig Whitaker posed a question of a wining strategy for a modified 3 door game show to Marilyn Savant who wrote articles for Parade magazine.
Her 1990 response was to switch doors when given the option. [2] [3]
The debate of probability of success as 2/3 vs 1/2 has continued until today.
This paper reveals errors in her response.

The logic of the game, with the player never knowing where the car is located until after their 2nd choice, precludes them from defining a basis for a strategy. It's always a random guess. Whitaker's question involved 1 car and 2 goats. Since the goats exist simultaneously, they must have separate identities g1 and g2. The host opening a door to reveal a goat only informs the player of where the car is NOT located. My analysis is more general with Whitaker's description a special case, and excludes any form of deception.

In the graphic, there are 3 distinct prizes, a car 'c' and 2 prizes 'a' and 'b' of lesser value. There are 3!=6 possible arrangements of the prizes per door. Since all 3 doors must have the same possible arrangements of prizes, it's only necessary to simulate for 1 door, thus the player always chooses door 1. The host is restricted to only open 1 door not containing a car and not chosen by the player, thus door 2 or door 3.
If the host opened 2 doors in one game, the car location is known, the game ends, and the player is denied their option!
A game consists of 3 events:
e1. player chooses 1 of 3 doors, which is door 1 for this example.
e2. host opens 1 of 2 doors in a left to right sequence.
e3. host allows player a 2nd choice of 1 of 2 closed doors (1 or 'op').
Whitaker asks, is there an advantage to stay with door 1 or switch to 'op'.
Without any more detail, e3 determines the ratio of (win a car)/(games played) as 1/2. There are not 3 choices when choosing 1 of 2 doors in the same game!
There are always 2 closed doors remaining, 1 with a car and 1 with no car.
The probability is the same as flipping a coin for H or T, averaging 1/2.

The graphic shows all possible sequences of play.
When door 1 contains 'c', the host has 2 sequences of play vs. 1 sequence when door 1 does not contain 'c'.
The player wins a car 4/8 games on average, with or without the option.
Failure to recognize the fact of distinct prizes resulting in 2 possible sequences of choices when the car is behind door 1 is the 1st error. Believing that abstract and misleading probabilities have an independent existence is the 2nd error.
Consider that no door contains 1/3 of a prize. Using real world 'possibilities', the values for each door would be a variation of {0, 0, 1}. 1 meaning the door contains a car, and 0 meaning the door does not contain a car. The value for the set of doors is 1 independent of number of doors. I.e. adding more doors for the host to open only makes for a prolonged and boring game.
The key factor is possible choices, not possible locations.
This is also a study comparing abstraction vs reality. The location of the car is fixed and certain for the host. The player's knowledge of its location is uncertain. Thus for the player, probability distributes its location uniformly over all doors.

1	2	3	host		op
a	b	c	2		c
a	c	b		3	c
b	a	c	2		c
b	c	a		3	c
c	a	b	2		b
				3	a
c	b	a	2		a
				3	b

reference

[1} The American Statistician, August 1975, Vol. 29, No. 3
[2] game show problem, Wikipedia Sep 2024
[3] Marilyn vos Savant,
https://web.archive.org/web/20130121183432/http://marilynvossavant.com