The game show problem

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abstract
The game show problem aka Monty Hall problem [1], originated when Craig Whitaker posed a question of a winning strategy for a modified 3 door game show to Marilyn Savant who wrote articles for Parade magazine.
Her 1990 response was to switch doors when given the option. [2] [3]
The debate of probability of success as 2/3 vs 1/2 has continued until today.
This paper reveals errors in her response.

the basic game

The basic game consists of 3 doors, one concealing a car and two concealing goats. A single player makes a random guess of which door contains the car. The measure of a successful event is the ratio of (win a car)/(all possible guesses). Thus the player has a 1 in 3 chance of winning a car. There is no option.

the modified game

Whitaker's modified game includes participation of the host, in a sequence of 3 events.

1. Player makes their 1st guess.
2. Host opens 1 door revealing a goat, eliminating it from the game, per the game rules.
3. Host offers a 2nd guess to the player to stay with their 1st guess or switch to the remaining closed door.

Whitaker asks, "Is it to your advantage to switch your choice of doors?"
Marilyn Savant's 1990 response including a table.[3]
Yes; you should switch. The first door has a 1/3 chance of winning, but the second door has a 2/3 chance.

		DOOR 2	DOOR 3	RESULT
GAME 1	AUTO	GOAT	GOAT	Switch and you lose.
GAME 2	GOAT	AUTO	GOAT	Switch and you win.
GAME 3	GOAT	GOAT	AUTO	Switch and you win.

GAME 4	AUTO	GOAT	GOAT	Stay and you win.
GAME 5	GOAT	AUTO	GOAT	Stay and you lose.
GAME 6	GOAT	GOAT	AUTO	Stay and you lose.

Sample of the general game.

Game rules

1. The host cannot open the door 1st selected by the player.
2. The host cannot open a door containing the car.

There are 3 distinct prizes, a car 'c' and 2 prizes 'a' and 'b' of lesser value.
There are 3!=6 possible distributions of the prizes per door.
Since all 3 doors must have the same possible distributions of prizes, it's only necessary to simulate for 1 door, thus the player always chooses door 1.
The host is restricted to only open 1 door per game.
If the host opened 2 doors in one game, the car location is known, the game ends, and the player is denied their option!

g	1	2	3	op
1	a	b	c	c
2	b	a	c	c
3	a	c	b	c
4	b	c	a	c
5	c	a	b	b
6	c	b	a	a

locations

g	1	2	3	op
1	a	b	c	c
2	b	a	c	c
3	a	c	b	c
4	b	c	a	c
5	c	a	b	b
6	c	a	b	a
7	c	b	a	a
8	c	b	a	b

choices

The tables show all possible games for locations on the left, and choices on the right.
When door 1 contains 'c', the host has 2 games of play vs. 1 game when door 1 does not contain 'c'.
The win ratio for locations is stay 2/6 vs switch 4/6.

The win ratio for choices is stay 4/8 vs switch 4/8.
If g1 and g2 are substituted for a and b, the general game becomes the problem game.
Savant's interpretation was based on location, and failure to recognize the fact of distinct prizes, g1 and g2, and retaining the 2/3 probability when there are only 2 choices for the players 2nd optional guess.

The key factor is possible choices, not possible locations.

reference

[1} The American Statistician, August 1975, Vol. 29, No. 3
[2] game show problem, Wikipedia Sep 2024
[3] Marilyn vos Savant,
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