Conditional Negation of The abc Conjecture

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Abstract. In this paper, the abc conjecture is negated under certain conditions.
Keyword. the abc conjecture.
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The abc conjecture is a very famous and difficult problem. In this paper, we got some results, I hope it can help solve the abc conjecture completely.

The abc conjecture. let $\varepsilon > 0$, a, b, c are three nonzero pairwise coprime integers such that $a = b + c$, then

$$max\left(|a|, |b|, |c| \right) \le C(\varepsilon) \left(rad\left(abc \right) \right)^{1+\varepsilon}$$

Where $rad(N) = \prod_{p|N} p$, $C(\varepsilon)$ is a positive constant related to ε .

See page483 of the references [1]

In this paper, we prove the following theorem.

Theorem Let M be any positive integer and $M \ge 10$,

then when $C(\varepsilon) \le \varepsilon^{-1+\frac{2}{M}} C_0$, the abc conjecture is wrong.

where C_0 is any given positive constant. it has nothing to do with ε .

Lemma. Let k and m are the positive integers,

If $(\mathrm{k}, \mathrm{m}) = 1$, then $k^{\varphi(\mathrm{m})} \equiv 1 \ (\mathrm{mod\ m})$.

where $\varphi(\mathrm{m})$ be Euler's function.

See page 25 of the references [2].

Now, we begin our proof.
Let M be any positive integer and $M \ge 10$.

In the Lemma, we take $k = 2, \quad m = p^M$, the prime number $p \ge 8C_0$,

where C_0 is any given positive constant in the Theorem.

By the Lemma, we have

$2^{\varphi(\mathrm{p}^M)} \equiv 1 \ (\mathrm{mod}\ p^M)$, namely $2^{\varphi(\mathrm{p}^M)} = 1 + np^M$

we write $a=2^{\varphi(\mathrm{p}^M)}$, $b=1$, $c=np^M$, then $a=b+c$

Evident $(a,b)=\left(2^{\varphi(p^M)},1\right)=1$, $(c,b)=(np^M,1)=1$.

Because $a=1+c$, therefore $(a,c)=1$.

By the abc conjecture, we have

$c\le C(\varepsilon)\left(rad\left(abc\right)\right)^{1+\varepsilon}$, namely $np^M\le C(\varepsilon)\left(rad\left(2^{\varphi(p^M)}np^M\right)\right)^{1+\varepsilon}$,

According to the definition of $rad(N)$, we have

$rad\left(2^{\varphi(p^M)}np^M\right)\le 2np$, therefore $np^M\le C(\varepsilon)\left(2np\right)^{1+\varepsilon}$,

namely $np^M\le C(\varepsilon)2^{1+\varepsilon}n^{1+\varepsilon}p^{1+\varepsilon}$, $p^{M-1}\le\varepsilon^{-1+\frac{2}{M}}C_0 2^{1+\varepsilon}n^{\varepsilon}p^{\varepsilon}$.

Because $n\le p^{-M}2^{\varphi(p^M)}\le p^{-2}2^{p^M}$, therefore $n^{\varepsilon}\le p^{-2\varepsilon}2^{\varepsilon p^M}$,

we take $\varepsilon=p^{-M}$, then $n^{\varepsilon}\le 2p^{-2\varepsilon}$, $\varepsilon^{-1+\frac{2}{M}}=p^{M-2}$,

Therefore $p^{M-1}\le p^{M-2}C_0 2^{2+\varepsilon}p^{-\varepsilon}$, $p\le 4C_0$

Previously, we already assumed that $p\ge 8C_0$, hence the contradiction.

This completes the proof of the theorem.

REFERENCES

[1] Henri Cohen, Number Theory Volume II: Analytic and Modern Tools Springer Science + Business Media, LLC 2007

[2] Hua Loo Keng, Introduction to Number Theory, Springer-Verlag Berlin Heidelberg New York, 1982.

Negation of The Strong abc Conjecture

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Abstract. In this paper, we negate the strong abc conjecture.

Keyword. the abc conjecture.

The abc conjecture is a very famous and difficult problem. In this paper, we give a weak result. Hope to help solve the abc conjecture.

The abc conjecture. let $\varepsilon > 0$, if a, b and c are three nonzero pairwise coprime integers such that $a = b + c$, then

$$max\left(|a|,|b|,|c| \right) \le C(\varepsilon)\left(rad\left(abc \right) \right)^{1+\varepsilon}$$

Where $rad(N) = \prod_{p|N} p$, $C(\varepsilon)$ is a positive constant related to ε.

See page483 of the references [1]

Under the same conditions, we define

the strong abc conjecture. $max\left(|a|,|b|,|c| \right) \le C\left(rad\left(abc \right) \right)^{1+\varepsilon}$

Where C is a positive constant, it has nothing to do with ε.

In this paper, we prove the following theorem.

Theorem the strong abc conjecture is wrong.

Lemma. Let k and m are the positive integers,

If $(\mathrm{k}, \mathrm{m}) = 1$, then $k^{\varphi(\mathrm{m})} \equiv 1 \ (\mathrm{mod\ m})$.

where $\varphi(\mathrm{m})$ be Euler's function.

See page 25 of the references [2].

Below, we give the proof of the theorem.

In the Lemma, we take $k = 2$, $m = p^2$, the prime number $p \ge 10C$.

where C is a positive constant in the strong abc conjecture.

By the Lemma, we have

$2^{\varphi(\mathrm{p}^2)} \equiv 1 \ (\mathrm{mod}\ p^2)$, namely $2^{\varphi(\mathrm{p}^2)} = 1 + np^2$

we write $a = 2^{\varphi(\mathrm{p}^2)}$, $b = 1$, $c = np^2$, then $a = b + c$

Evident $(a,b) = (2^{\varphi(p^2)}, 1) = 1$, $(c,b) = (np^2, 1) = 1$.

Because $a = 1 + c$, therefore $(a,c) = 1$.

By the strong abc conjecture, we have

$c \le C\left(rad\left(abc \right) \right)^{1+\varepsilon}$, namely $np^2 \le C\left(rad\left(2^{\varphi(p^2)} np^2 \right) \right)^{1+\varepsilon}$, According to the definition of $rad(N)$, we have

$rad\left(2^{\varphi(p^2)} np^2\right) \le 2np$, therefore $np^2 \le C\left(2np\right)^{1+\varepsilon}$,

namely $np^2 \le C2^{1+\varepsilon} n^{1+\varepsilon} p^{1+\varepsilon}$, $p \le C2^{1+\varepsilon} n^{\varepsilon} p^{\varepsilon}$.

Because $n \le p^{-2} 2^{\varphi(p^2)} \le p^{-2} 2^{p^2}$, therefore $n^{\varepsilon} \le p^{-2\varepsilon} 2^{\varepsilon p^2}$,

we take $\varepsilon = p^{-2}$, then $n^{\varepsilon} \le 2p^{-2\varepsilon}$,

therefore $p \le C\, 2^{2+\varepsilon} p^{-\varepsilon} \le 4C$,

Previously, we already assumed that $p \ge 10C$, hence the contradiction.

This completes the proof of the theorem.

REFERENCES

[3] Henri Cohen, Number Theory Volume II: Analytic and Modern Tools Springer Science + Business Media, LLC 2007

[4] Hua Loo Keng, Introduction to Number Theory, Springer-Verlag Berlin Heidelberg New York, 1982.