New proof of dark numbers by means of the thinned out harmonic series

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Abstract: It is shown that not all numbers can be expressed and communicated such that the receiver knows what the sender has meant. We call them dark numbers.

Proof: The harmonic series diverges. Kempner has shown in 1914 that when all terms containing the digit 9 are removed, the serie converges.

That means that the removed terms, all containing the digit 9, diverge. Same is true when all terms containing the digit 8 are removed. That means all terms containing the digits 8 and 9 simultaneously diverge. We can continue and remove all terms containing 1, 2, 3, 4, 5, 6, 7 in the denominator without changing this result because the corresponding series are converging. So the remaining terms carry the divergence. That means that only the terms containing all these digits simultaneously constitute the diverging series.

But that is not the end! We can remove any number, like 2025, and the remaining series will converge. For proof use base 2026 where 2025 is a digit. This extends to every definable number, i.e. every number that can be communicated such that the receiver knows what the sender has meant. Therefore the diverging part of the harmonic series is constituted only by terms containing a digit sequence of all defined numbers. No defined number exists which must be left out.

All series splitted off in this way are converging and therefore their always finite sum is finite too (every defined number belongs to a finite initial segment of the natural numbers). The divergence however remains. It is carried only by terms which are dark and greater than all digit sequences of all defined numbers – we can say of all defin*able* numbers because when numbers later are defined, they behave in the same way.

This is a proof of the huge set of undefinable or dark numbers.