Proof that *e* is Rational

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ABSTRACT: In this short paper, I used Joseph Fourier's proof that *e* is irrational to prove that it is actually a *rational* number.

Proof That *e* Is Rational

Assume $e = \frac{r}{n}$, where *r* and *n* are very large positive integers.

$$\frac{r}{n} = e$$

Multiply both sides by *n!* to obtain

$$r(n-1)! = n!e$$

The left side is an integer, while the right side could be an integer if and only if *n* is a very large positive integer

$$n!e = n!\sum_{k=0}^{\infty}\frac{1}{k!} = \sum_{k=0}^{n}\frac{n!}{k!} + \sum_{k=n+1}^{\infty}\frac{n!}{k!}$$

The first sum on the right is an integer. The second sum x

$$x = \sum_{k=n+1}^{\infty}\frac{n!}{k!} = \frac{n!}{(n+1)!} + \frac{n!}{(n+2)!} + \ldots$$

Now take the limit

$$x = \lim_{n\to\infty}\left(\frac{1}{n+1} + \frac{1}{(n+1)(n+2)} + \ldots\right)$$

$$x = \lim_{n\to\infty}\left(\frac{1}{n} + \frac{1}{n^2} + \frac{1}{n^3} + \ldots\right) = \lim_{n\to\infty}\left(\frac{1}{n-1}\right) = 0$$

Since $x = 0$

$$n!e = \sum_{k=0}^{n}\frac{n!}{k!} + x$$

$$n!e = \text{integer} + 0 = \text{integer}$$

and from

$$e = \frac{r}{n}$$

r must also be a very large positive integer

$$e = \lim_{r,n\to\infty}\frac{r}{n}$$

Therefore, *e* is a rational number that is the ratio of two very large positive integers.

$$e = \frac{271828182\ldots d}{100000000\ldots 0} = 2.71828182\ldots d$$

d is the last decimal digit of *e*.

REFERENCE: **https://en.wikipedia.org/wiki/Proof_that_e_is_irrational**