A Small Contribution to Ross-Littlewood Paradox

Marko V. Jankovic

Institute of Electrical Engineering "Nikola Tesla", Belgrade, Serbia
Swiss Rockets Serbia, Belgrade, Serbia

Abstract In this paper Ross-Littlewood paradox is going to be analyzed. Two new experiments were proposed and it will be argued that number of balls at the end of experiment is infinite.

1 Introduction

Here, we are going to analyze two elementary experiments with enumerated balls and boxes (or vases) [1, 2], known as Ross-Litllewood paradox (RLP). In this section, the analysis of the RLP that was presented in [3] is going to be repeated (obviously, in [3] author was not aware of existence of RLP).

First experiment: Imagine that we have infinite number of balls with all natural numbers written on them exactly once, that are placed in the source box (SB) that has the size equal to the number of natural numbers, and that we have another, experimental box (EB) of proper size.

In the moment 1 minute before midnight, we are going to move balls with numbers 1 to 10 on them from SB to EB, and remove from the EB the ball with number 10 on it. In the moment ½ minute before midnight, balls with numbers 11-20 are transferred from SB to EB, and ball with number 20 on it is removed from EB. We continue the process at the moments $1/2^n$, $n \in N$, $n > 1$, minute before midnight – transfer the balls with numbers form n*10+1 to (n+1)*10 from SB to EB, and remove ball with number (n+1)*10 on it from EB.

Now, we can try to answer the following question: What is the number of the balls in EB at midnight (this is slightly imprecise question, but it can be easily rectified)? The answer is obvious and everybody will answer that the number of balls in EB is infinite.

Second experiment: Imagine, again, that we have infinite number of balls with all natural numbers written on them exactly once that are placed in the source box (SB) that has the size equal to the number of natural numbers, and that we have another, experimental box (EB) of proper size.

In the moment 1 minute before midnight, we are going to move balls with numbers 1 to 10 on them from SB to EB, and remove from the EB the ball with number 1 on it. In the moment ½ minute before midnight, balls with numbers 11-20 are transferred from SB to EB, and the ball with number 2 on it is removed from EB. We continue the process at the moments $1/2^n$, $n \in N$, $n > 1$, minute before midnight – transfer the balls with numbers from $n*10+1$ to $(n+1)*10$ from SB to EB, and remove ball with number $(n+1)$ on it from EB.

Now, we can try, again, to answer the following question: What is the number of the balls in EB at midnight? Again, the answer looks quite obvious and everybody will answer that number is infinite. However, if you are asked to give an example of the ball with any specific number on it, that is still in the EB, you will not be able to do it. The reason is quite obvious – for any number you choose, you can specify a moment in time in which the ball with that number on it, has been removed from the EB. That can lead to the conclusion that there is a chance that the number of balls in EB, at the "end" of the process is zero, although in every moment, clearly, 9 new balls were put in the EB. Two new experiment, proposed in the next section, will contribute to the understanding that the number of the balls in the EB at the end of the process is infinite.

So, numbers on the balls could lead to different conclusions for two experiments that are equivalent. If we remove the numbers from balls – in both experiments that were previously analysed, the process was following – put 10 balls in the EB and then remove one – or very simplified, put nine balls in the EB in every step. If there is no collapse of elementary reasoning (CER), it can be safely concluded that number of the balls in the EB at midnight is infinite. Actually, it is not difficult to be seen that two previously mentioned experiments are the special cases of a more general experiment in which in every step ten balls are put in the EB and one of the existing balls in the EB is removed completely randomly [2].

It is interesting to notice that experiment 2, can be seen as the process that represents an algorithm for removal of the first 1/10 of the infinite number of natural numbers from the set of infinite number of natural numbers. What can be interesting to notice is that we can actually define shallow and deep infinity (or even mathematical black hole) – however, this discussion is out of the scope of this paper.

In order to see why one possible conclusion in the second experiment is wrong, two new experiments are going to be analyzed.

2 Additional experiments

In this section, two new experiments, similar to previous ones, are going to be presented.

Third experiment: Imagine that we have infinite number of balls with all natural numbers written on them exactly once, that are placed in the source box (SB) that has the size equal to the number of natural numbers, and that we have another, experimental box (EB) of proper size.

In the moment 1 minute before midnight, we are going to move ball with number 1 on it from SB to EB, and remove from the EB the ball with number 1 on it. In the moment ½ minute before midnight, ball with number 2 on it is transferred from SB to EB, and ball with number 2 on it is removed from EB. We continue the process at the moments $1/2^n$, $n \in N, n > 1$, minute before midnight – transfer the ball with number n+1 on it from SB to EB, and remove ball with number n+1 on it from EB.

Now, we can try to answer the following question: What is the number of the balls in EB at midnight (what is expected number of balls in the EB as time approaches midnight)? The answer is obvious and everybody will answer that number of balls in EB is zero. If we assume here that the idea that it is possible to establish bijection between natural numbers and natural numbers that are powers of 2, in this experiment all natural numbers were put in EB and removed from EB. One thing is quite clear – in this case there is no potential for a paradox.

<u>Forth experiment</u>: Here, the second experiment is going to be slightly changed. Imagine, again, that we have infinite number of balls with reciprocals of all natural numbers written on them exactly once, that are placed in the source box (SB) that has the size equal to the number of natural numbers, and that we have another, experimental box (EB) of proper size.

In the moment 1 minute before midnight, we are going to move balls with reciprocals of numbers 1 to s ($s \in N, s > 1$) on them from SB to EB, and remove from the EB the ball with number 1 on it. In the moment ½ minute before midnight, balls with reciprocals of numbers (s+1) to (2s) are transferred from SB to EB, and the ball with reciprocal of number 2 on it is removed from EB. We continue the process at the moments $1/2^n$, $n \in N$, $n > 1$, minute before midnight – transfer the balls with reciprocals of numbers from $n*s+1$ to $(n+1)*s$ from SB to EB, and remove ball with reciprocal of number (n+1) on it from EB.

Now, we can try, again, to answer the following question: What is the number of the balls in EB at midnight? Here, like in the experiment 2, we can conclude that the number of balls in the EB at midnight can be zero if we cannot specify any number that is still inside the EB. However, in this case we have an opportunity to analyze the problem from different point of view..

Here we are going to analyze the problem from a lightly different perspective: we are going to calculate the sum of the numbers that are in the box at the “end” of the process. Let's denote the with S(s) following sum:

$$S(s) = 1+1/2+...+1/s-\mathbf{1}+1/(s+1)+1/(s+2)+...+1/(2s)-\mathbf{2}+...$$

We can see that S(s) represents the sum off all numbers left in the EB at midnight (or sum of all numbers in EB as time approaches midnight). It is known that S(s) = ln(s) (see e.g. the

Mathologer's video: The best A-A≠0 paradox). So, since S(s) is not zero for $s > 1$, we can safely conclude that EB at the end of the process cannot be empty. Actually, the number of balls in EB at the end of the process is not zero but infinite, which is quite reasonable from the point of view that we put s-1 new balls in the EB in every moment (the other way to conclude it is that ln (s), $s > 1$, is an irrational number, so number of balls at the end must be infinite). We can also see that the conclusion in the Third experiment is also supported, since S(1) = ln(1) = 0. It also can be concluded that probabilistic approach that was used in [2] was not implemented precisely and leads toward wrong conclusion.

However, if you are asked to give an example of the ball with any specific number on it, that is still in the EB, you will not be able to do it. The reason is quite obvious – for any number you choose, you can specify a moment in time in which the ball with that number on it has been removed from the EB. So, how can we justify the answer that there is an infinite number of balls in the EB when for every ball with specific number on it we can define a moment in time when that ball is removed from the EB? In order to answer that question properly, clearly, there is a necessity for a proper definition of infinite numbers and that is quite out of the scope of this paper.

Conclusion

An analysis of Ross-Littlewood paradox was performed. Two new experiment were analzsed and they enabled us to conclude that the number of balls in the EB at the end of the experiment is infinite. Also, from the analysis that was performed, it seems that, anyway, it is necessary to give a proper definition of infinite natural numbers, at least to answer properly to all challenges that are posed by RLP.

References

[1] A. Ross. (1941). Imperatives and Logic. Theoria, vol. 7, pp. 53-71.

[2] S. Ross. (2010). A First Course in Probability, Pearson Prentice Hall. (Eight Edition, Chapter 2, Example 6a, p. 46).

[3] M. Jankovic. (2021) Proof of Twin Prime Conjecture (Together with the Proof of Polignac's conjecture for Cousin Primes). hal-02549967v9.