Direction of Zero Vector - General Logical Contradictions on Undefined Objects -

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Abstract: First, as a natural extension of the paper [2], we introduce a natural definition for the direction of general zero vectors, which may not have been previously considered. Second, in connection with the direction of zero vectors, we point out some general logical contradictions related to undefined objects.

For the new journal publication [3], we stated:

The spirit of the journal is: fundamental, beautiful and impactful for humanity. Love, passion and fairness are important in the journal.

Key Words: Division by zero, division by zero calculus, $1/0 = 0/0 = z/0 = \tan(\pi/2) = \log 0 = 0$, $[(z^n)/n]_{n=0} = \log z$, $\arg 0 = 0$, direction of zero vector, general contradictions on undefined objects.

AMS Mathematics Subject Classifications: 00A05, 00A09.

1 Introduction

First, we introduce the direction of general zero vectors as a natural extension of the paper [2]. Second, in connection with the direction of zero vectors, we point out some general contradictions related to undefined objects.

2 On the zero vector on the plane

Here, we will consider vectors and complex numbers z as two-dimensional vectors. We state that the direction of the zero vector is zero. However, the precise meaning is $\arg 0 = 0$.

The direction of zero $z = 0$ exists as in other vectors z.

This definition, along with its natural motivation and many applications, has been established in prior works [1, 6] as a natural extension of division by zero and division by zero calculus for functions, as stated in the cited references.

Note the simple facts:

In the well-known formula

$$\log z = \log |z| + \arg z, \tag{2.1}$$

we have

$$\log 1 = \log 1 + \arg 1,$$

and

$$\log 0 = \log 0 + \arg 0.$$

Therefore, we have

$$\arg 1 = \arg 0 = 0.$$

Here, if we assume that the identity (2.1) is extensible with any definition of $\log 0$ by a number at $z = 0$, then we can obtain the result

$$\arg 0 = 0.$$

Note, furthermore, that in the identity

$$\arg \overline{z} = -\arg z,$$

if the function $\arg z$ is extensible to the origin as an odd function, then the value $\arg 0$ must be zero.

In addition, note that in the formula

$$\arg z = \arctan \frac{y}{x},$$

for $x = y = 0$ we have, from $0/0 = 0$, that

$$\arg 0 = 0.$$

For different complex numbers α, β, γ,

$$\arg \frac{\alpha - \gamma}{\beta - \gamma}$$

represents the angle $\angle \alpha\gamma\beta$.

If $\alpha = \beta$, then

$$\arg 1 = 0.$$

If $\beta = \gamma$, then

$$\arg 0 = 0.$$

If $\alpha = \gamma$, then

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If $\alpha = \beta = \gamma$, then

$$\arg 0 = 0.$$

We used the division by zero

$$\frac{1}{0} = \frac{0}{0} = 1.$$

If $\arg 0 = 0$ is not defined, then the following formulas would be nonsensical for the case $a = 0$:

$$a = |a| \exp(i \arg a),$$

$$|a^z| = |a|^x \exp(-y \arg a), z = x + iy,$$

and

$$\arg a^z = y \log |a| + x \arg a.$$

For this Section, see also [2, 6] for details.

2.1 The direction of general zero vectors

We are interested in the direction of zero vectors in general dimensions.

In order to state the representation precisely, we shall consider vectors as elements of a separable Hilbert space. Then, we consider the representation of vectors $\mathbf{v}$ in terms of a fixed complete orthonormal system $\{\mathbf{e_j}\}_\mathbf{j}$ as

$$\mathbf{v} = \sum_j v_j \mathbf{e}_j.$$

Then, the vector $\mathbf{v}$ and the coefficients $\{v_j\}$ correspond one to one onto ℓ^2.

Statement: *We shall define the direction of* $\mathbf{v}$ *by the coefficients* $\{v_j\}$*, which are determined up to a positive scalar multiplication of* $\{v_j\}$ *and the zero vector is represended by all* $v_j = 0$*. Therefore, the direction of the zero vector may be considered zero in this sense.*

Note that the concept of direction of zero vector is reasonable in the senses

$$\mathbf{v} + \mathbf{u} = \sum_j v_j \mathbf{e}_j + \sum_j u_j \mathbf{e}_j = \sum_j (v_j + u_j)\mathbf{e}_j$$

and

$$\mathbf{v} - \mathbf{v} = \sum_j (v_j - v_j)\mathbf{e}_j = \sum_j 0\mathbf{e}_j = \mathbf{0}.$$

3 General logical contradictions on undefined objects

Logical Problem: *If we do not give the definition of direction of zero vector, in the fundamental equation*

$$\mathbf{v} + \mathbf{0} = \mathbf{v},$$

we have the logical contradiction that by the addition of the zero vector, which has no defined direction, we obtain the same direction as $\mathbf{v}$.

Indeed, in the above identity, we cannot specify the direction of vectors.

This contradiction is similar to the fact that the identity

$$\frac{1}{\sqrt{x}} - \frac{1}{\sqrt{x}} + x = x$$

is not valid at $x = 0$, because the terms are undefined at $x = 0$.

4 Open problem

However, we can still consider the open problem:

Open problem: *As in two dimensions, can we find a natural formulation such that the direction of the zero vector is zero in general dimensions?*

Indeed, in the 2 dimensional case, zero direction was given by the pleasant and intuitive sense $\arg 0 = 0$.

5 On zero

Why Can't We Divide by Zero?

The common beliefs about the impossibility of division by zero, the indeterminate discussions, the reasons for the inability, and the computational troubles – all these are well-known. Mathematicians would think it's absurd to debate about them since they are understood in a few seconds.

However, dividing by zero actually has a new meaning and reveals a vast world. When it comes to dividing by zero, there is another interpretation. This is the new meaning of division by zero. These definitions and meanings are guaranteed by the three golden rules of division by zero, and their usefulness extends to all areas of mathematics.

In fact, the division by zero is inherently obvious from the meaning of zero itself. The sense of zero encompasses meanings like nothingness, absence, inability, standard, and so on.

To divide by zero means not dividing at all. Thus, there is no number to be allocated, resulting in zero. Look at the simple essentials:

On Division by Zero Calculus With 8 Figures
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"The direction of zero vectors is zero" comes from the standered meaning of zero for the direction.

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