Formulas of the Fine-structure Constant and the Speed of Light in Atomic Units Based on 22/7 and 355/113

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Abstract

In our previous papers, we gave the formulas of the fine-structure constant and the speed of light in atomic units based on 2π-e formula and the natural end of the elements determined by us, i.e., the 112th element Cn*. In this paper, based on these formulas and the two approximate rates of π which are 22/7 and 335/113, we deduce new formulas of the fine-structure constant and the speed of light in atomic units. This is also to answer Richard Feynman's question whether the fine-structure constant is related to π. Besides our previous formulas and explainations, our new additional answer is that it is also amazingly related to the approximate rates 77/2 and 335/113 which were proposed by the ancient Greek and Chinese mathematicians Archimedes (BC 287-212) and Chongzhi Zu (AD 429-500).

Keywords: the fine-structure constant, the speed of light in atomic units, formulas, π, approximate rates, 22/7, 355/113.

1. Definitions of the Fine-structure Constant and Richard Feynman's Comment

The fine-structure constant (α) is a centennial mystery of physics. It was introduced by Arnold Sommerfeld in 1916, it has three definitions as follows.

$$\alpha = \frac{\lambda_e}{2\pi a_0}, \quad \alpha = \frac{2\pi r_e}{\lambda_e}, \quad \alpha = \frac{v_e}{c} = \frac{e^2}{4\pi\varepsilon_0 \hbar c}$$

$$\alpha = \frac{1}{137.035999\cdots} \approx \frac{1}{137.036} \approx \frac{1}{137}$$

The following was Richard Feynman's comment on the fine-structure constant.

It has been a mystery ever since it was discovered more than fifty years ago, and all good theoretical physicists put this number up on their wall and worry about it. Immediately you would like to know where this number for a coupling comes from:

is it related to pi or perhaps to the base of natural logarithms? *Nobody knows. It's one of the greatest damn mysteries of physics: a magic number that comes to us with no understanding by man. You might say the "hand of God" wrote that number, and "we don't know how He pushed his pencil." We know what kind of a dance to do experimentally to measure this number very accurately, but we don't know what kind of dance to do on the computer to make this number come out, without putting it in secretly!* [1]

2. Formulas of the Fine-structure Constant and the Speed of Light in Atomic Units Based on 2π-e Formula and the 112th element

In our previous papers, we gave the formulas of the fine-structure constant and the speed of light in atomic units based on 2π-e formula and the natural end of the elements which was determined by us, i.e., the 112th element Cn* [2-14]. They are listed as follows.

$2\pi - e$ formula:

$$2\pi = (\frac{e}{e^{\gamma_c}})^2 = e^2 \frac{e^2}{(\frac{2}{1})^3} \frac{e^2}{(\frac{3}{2})^5} \frac{e^2}{(\frac{4}{3})^7} \cdots$$

$$(2\pi)_{Chen-k} = (\frac{e}{e^{\gamma_{c-k}}})^2 = e^2 \frac{e^2}{(\frac{2}{1})^3} \frac{e^2}{(\frac{3}{2})^5} \cdots \frac{e^2}{(\frac{k+1}{k})^{2k+1}}$$

Formulas of the fine-structure constant:

$$\alpha_1 = \frac{\lambda_e}{2\pi a_0} = \frac{36}{7(2\pi)_{Chen-112}} \frac{1}{112 + \frac{1}{75^2}} = 1/137.035999037415379188517229528 74$$

Relationships with nuclides:

$${}^{82,83,84}_{36}Kr_{46,47,48} \quad {}^{112}_{48}Cd_{64} \quad {}^{136,137,138}_{56}Ba_{80,81,82} \quad {}^{185,187}_{75}\mathrm{Re}_{110,112} \quad {}^{188}_{76}Os_{112} \quad {}^{209}_{83}Bi^*_{126} \quad {}^{209}_{84}Po^*_{125}$$

$${}^{285}_{112}Cn^*_{173} \quad {}^{312}_{125}Ch^{ie}_{187} \quad {}^{2\cdot157}_{126}Ch^{ie}_{188} \quad {}^{2\cdot173}_{137}Fy^{ie}_{209}$$

$$\alpha_2 = \frac{2\pi r_e}{\lambda_e} = \frac{13(2\pi)_{Chen-278}}{100} \frac{1}{112 - \frac{1}{64\cdot 3\cdot 29}} = 1/137.035999111872962758119209477 93$$

Relationships with nuclides:

$${}^{56}_{26}Fe_{30} \quad {}^{63,65}_{29}Cu_{34,36} \quad {}^{100}_{44}Ru_{56} \quad {}^{112}_{48}Cd_{64} \quad {}^{140,142}_{58}Ce_{82,84} \quad {}^{136,137,138}_{56}Ba_{80,81,82} \quad {}^{157}_{64}Gd_{93} \quad {}^{169}_{69}Tm_{100}$$

$${}^{188}_{76}Os_{112} \quad {}^{223,224}_{87}Fr^*_{136,137} \quad {}^{257}_{100}Fm^*_{157} \quad {}^{278}_{109}Mt^*_{169} \quad {}^{285}_{112}Cn^*_{173} \quad {}^{2\cdot157}_{126}Ch^{ie}_{188} \quad {}^{2\cdot173}_{137}Fy^{ie}_{209} \quad {}^{426}_{169}Ch^{ie}_{257}$$

Formulas of the speed of light in atomic units proposed in our previous papers:

$$c_{au}=\frac{4\pi\varepsilon_0\hbar c}{e^2}=\frac{c}{v_e}=\frac{1}{\alpha_c}=\frac{1}{\sqrt{\alpha_1\alpha_2}}$$

$$=\sqrt{112(168-\frac{1}{3}+\frac{1}{4\cdot 141}-\frac{1}{14\cdot 112(2\cdot 173+1)+13+\frac{7}{72}})}$$

$$=2\sqrt{56(83+\frac{157}{188}-(\frac{1}{8\cdot 141}+\frac{1}{56^2(2\cdot 173+1)+26+\frac{7}{36}}))}$$

$$=137.035999074644171$$

$c_{au}=\frac{1}{\sqrt{\alpha_1\alpha_2}}$ is consitent with Maxwell formula of the speed of light $c=\frac{1}{\sqrt{\mu_0\varepsilon_0}}$

Based on the above formula, we construct the following more precise formulas:

$$c_{au}=\sqrt{112(168-\frac{1}{3}+\frac{1}{4\cdot 141}-\frac{1}{14\cdot 112(2\cdot 173+1)+13+\frac{7}{72+1/50}})}$$

$$=2\sqrt{56(83+\frac{157}{188}-(\frac{1}{8\cdot 141}+\frac{1}{56^2(2\cdot 173+1)+26+\frac{7}{36+1/100}}))}$$

$$=137.0359990746441709683$$

Relationships with nuclides:

$^{56}_{26}Fe_{30}$ $^{83,84}_{36}Kr_{47,48}$ $^{100}_{44}Ru_{56}$ $^{107,109}_{47}Ag_{60,62}$ $^{112}_{48}Cd_{64}$ $^{118,119,120}_{50}Sn_{68,69,70}$ $^{136,137,138}_{56}Ba_{80,81,82}$
$^{140,142}_{58}Ce_{82,84}$ $^{157}_{64}Gd_{93}$ $^{166,167,168}_{68}Er_{98,99,100}$ $^{169}_{69}Tm_{100}$ $^{173}_{70}Yb_{103}$ $^{188}_{76}Os_{112}$ $^{200}_{80}Hg_{120}$ $^{209}_{83}Bi^{*}_{126}$
$^{209}_{84}Po^{*}_{125}$ $^{223,224}_{87}Fr^{*}_{136,137}$ $^{257}_{100}Fm^{*}_{157}$ $^{264,265}_{103}Lr^{*}_{161,162}$ $^{2\cdot 137}_{107}Bh^{*}_{167}$ $^{276}_{108}Hs^{*}_{168}$ $^{285}_{112}Cn^{*}_{173}$ $^{300}_{120}Ch^{ie}_{180}$
$^{2\cdot 157}_{126}Ch^{ie}_{188}$ $^{344,2\cdot 173,348}_{136,137,138}Fy^{ie}_{208,209,210}$ $^{355,357}_{141}Ch^{ie}_{214,216}$ $^{400}_{157}Ch^{ie}_{243}$ $^{420}_{168}Ch^{ie}_{252}$ $^{426}_{169}Ch^{ie}_{257}$ $^{437,438}_{173}Ch^{ie}_{264,265}$

$$c_{au}=\sqrt{112(168-\frac{1}{3}+\frac{1}{4\cdot 141}-\frac{1}{14\cdot 112(2\cdot 173+1)}+\frac{1}{22603891794+\frac{130}{137-\frac{1}{2\cdot 7\cdot 73}}})}$$

$$=137.035999074644170968261216427 08$$

Note: $22603891794=6\cdot 11\cdot 41(128\cdot 13-1)(32\cdot 157-1)$

Relationships with nuclides:

$^{73}_{32}Ge_{41}$ $^{93}_{41}Nb_{52}$ $^{125,130}_{52}Te_{73,78}$ $^{157}_{64}Gd_{93}$ $^{238}_{92}U^{*}_{146}$ $^{240}_{94}Pu^{*}_{146}$ $^{330}_{130}Ch^{ie}_{200}$ $^{370}_{146}Ch^{ie}_{224}$

$$c_{au}=\sqrt{112(168-\frac{1}{3}+\frac{1}{4\cdot 141}-\frac{1}{14\cdot 112(2\cdot 173+1)+13}+\frac{1}{3045979902845-36/91})}$$

$$=137.03599907464417096826121642708$$

Note: $3045979902845=5(2\cdot 27\cdot 31\cdot 103-1)(2\cdot 3\cdot 17\cdot 47\cdot 67)$

Relationships with nuclides:

$^{27}_{13}Al_{14}$ $^{91}_{40}Zr_{51}$ $^{103}_{45}Rh_{58}$ $^{155}_{64}Gd_{91}$ $^{165}_{67}Ho_{98}$ $^{173}_{70}Yb_{103}$ $^{231,232}_{91}Pa^{*}_{140,141}$ $^{264,265}_{103}Lr^{*}_{161,162}$ $^{437,438}_{173}Ch^{ie}_{264,265}$

3. Formulas of the Fine-structure Constant and the Speed of Light in Atomic Units Based on 22/7, 355/113 and the 112th Element

Based on the above formulas of the fine-structure constant and the speed of light in atomic units and the approximate rates of π which are 77/2 and 335/113, we deduce new formulas of the fine-structure constant and the speed of light in atomic units as follows.

$$\alpha_1 = \frac{36}{7(2\pi)_{Chen-112}} \frac{1}{112 + \frac{1}{75^2}}$$

$$= 1/137.035999037415379188517229 52874$$

Use $2\pi \approx 44/7$ to replace $(2\pi)_{Chen-112}$:

$$\alpha_1 = \frac{36}{44} \frac{1}{112 + \frac{16}{7 \cdot 19} + \frac{1}{2 \cdot 3 \cdot 44 \cdot 61}} = 1/137.035999037$$

$$\alpha_1 = \frac{36}{44} \frac{1}{112 + \frac{16}{7 \cdot 19} + \frac{1}{2 \cdot 3 \cdot 44 \cdot 61 - \delta_1}}$$

$$= 1/137.035999037415379188517229 52874$$

$$\delta_1 = \frac{8}{3 \cdot 19 + \frac{1}{9 \cdot 17 + \frac{20}{3 \cdot 19 + \frac{5}{4 \cdot 3 \cdot 17 - \frac{2}{3 \cdot 5 \cdot 13 - \frac{17}{137}}}}}}$$

Relationships with nuclides:

$${}^{78}_{34}Se_{44} \quad {}^{83,84}_{36}Kr_{47,48} \quad {}^{100,101}_{44}Ru_{56,57} \quad {}^{107,109}_{47}Ag_{60,62} \quad {}^{112}_{48}Cd_{64} \quad {}^{137}_{56}Ba_{81}$$

$${}^{146}_{61}Pm^{*}_{85} \quad {}^{209}_{83}Bi^{*}_{126} \quad {}^{223,224}_{87}Fr^{*}_{136,137} \quad {}^{2\cdot137}_{107}Bh^{*}_{167} \quad {}^{285}_{112}Cn^{*}_{173} \quad {}^{2\cdot173}_{137}Fy^{ie}_{209}$$

$$\alpha_2 = \frac{13(2\pi)_{Chen-278}}{100} \frac{1}{112 - \frac{1}{64 \cdot 3 \cdot 29}}$$

$$= 1/137.035999111872962758119209 47793$$

Use $2\pi \approx 710/113$ to replace $(2\pi)_{Chen-278}$:

$$\alpha_2 = \frac{13}{100} \frac{710}{113} \frac{1}{112 - \frac{8}{\frac{44 \cdot 19}{7} - \frac{1}{7}} + \frac{16}{\frac{3 \cdot 44 \cdot 71(2 \cdot 7 \cdot 83 + 1)}{7}})}$$

$$= 1/137.035999111873$$

$$\alpha_2 = \frac{13}{100}\frac{710}{113}\frac{1}{112 - \frac{8}{\frac{44\cdot 19}{7} - \frac{1}{7}} + \frac{16}{\frac{3\cdot 44\cdot 71(2\cdot 7\cdot 83+1)}{7} + \delta_2})}$$

=1/137.035999111872962758119209477 93

$$\delta_2 = \frac{7}{113 + \frac{1}{60 + \frac{20}{157 - \frac{1}{3\cdot 126 - 26/87}}}}$$

Relationships with nuclides:

$^{56}_{26}Fe_{30}$ $^{69,71}_{31}Ga_{38,40}$ $^{85,87}_{37}Rb_{48,50}$ $^{100}_{44}Ru_{56}$ $^{107,109}_{47}Ag_{60,62}$ $^{112}_{48}Cd_{64}$ $^{113}_{49}In_{64}$

$^{120,122}_{50}Sn_{70,72}$ $^{126}_{52}Te_{74}$ $^{137}_{56}Ba_{81}$ $^{143}_{60}Nd_{83}$ $^{157}_{64}Tm_{93}$ $^{166,168}_{68}Er_{98,100}$ $^{169}_{69}Tm_{100}$

$^{175,176}_{71}Lu_{104,105}$ $^{188,189}_{76}Os_{112,113}$ $^{209}_{83}Bi^{*}_{126}$ $^{223,224}_{87}Fr^{*}_{136,137}$ $^{257}_{100}Fm^{*}_{157}$ $^{273}_{107}Bh^{*}_{166}$

$^{278}_{109}Mt^{*}_{169}$ $^{285}_{112}Cn^{*}_{173}$ $^{287}_{113}Nh^{*}_{174}$ $^{2\cdot 157}_{126}Ch^{ie}_{188}$ $^{2\cdot 173}_{137}Fy^{ie}_{209}$ $^{400}_{157}Ch^{ie}_{243}$ $^{6\cdot 71}_{169}Ch^{ie}_{257}$

In the above formulas of α_1 and α_2, the rate 44/7 and the factors 44 and 71 appear miraculously. In the formula of δ_1, there is the typical factor 137. And in the formula of δ_2, there are the typical factors 113, 126 and 157. So we suppose these formulas should be reasonable and precise.

$$c_{au} = \frac{c}{v_e} = \frac{1}{\alpha_c} = \frac{1}{\sqrt{\alpha_1\alpha_2}}$$

$$= \frac{10}{6}\sqrt{\frac{7}{13}\frac{44}{7}\frac{113}{710}}(112 + \frac{1}{\frac{2\cdot 3\cdot 44}{7} - \frac{1}{7}} - \frac{8}{\frac{3\cdot 5\cdot 44(2\cdot 83+1)(4\cdot 83-1)}{7}})$$

$$= 137.0359990746442$$

$$c_{au} = \frac{c}{v_e} = \frac{1}{\alpha_c} = \frac{1}{\sqrt{\alpha_1\alpha_2}}$$

$$= \frac{10}{6}\sqrt{\frac{7}{13}\frac{44}{7}\frac{113}{710}}(112 + \frac{1}{\frac{2\cdot 3\cdot 44}{7} - \frac{1}{7}} - \frac{8}{\frac{3\cdot 5\cdot 44(2\cdot 83+1)(4\cdot 83-1)}{7} + \delta_c})$$

$$= 137.035999074644170968261216427 08$$

$$\delta_c = \frac{2}{41 + \frac{1}{16\cdot 61 - \frac{1}{4\cdot 9\cdot 29 + 1/4}}}$$

Relationships with nuclides:

$^{56}_{26}Fe_{30}$ $^{63,65}_{29}Cu_{34,36}$ $^{69,71}_{31}Ga_{38,40}$ $^{82,83,84}_{36}Kr_{46,47,48}$ $^{85,87}_{37}Rb_{48,50}$ $^{93}_{41}Nb_{52}$ $^{100}_{44}Ru_{56}$

$^{107,109}_{47}Ag_{60,62}$ $^{112}_{48}Cd_{64}$ $^{113}_{49}In_{64}$ $^{116,120,122}_{50}Sn_{66,70,72}$ $^{126}_{52}Te_{74}$ $^{137}_{56}Ba_{81}$ $^{140,142}_{58}Ce_{82,84}$

$^{141}_{59}\mathrm{Pr}_{82}$ $^{146}_{61}Pm^{*}_{85}$ $^{155,157}_{64}Tm_{91,93}$ $^{166,167,168}_{68}Er_{98,99,100}$ $^{169}_{69}Tm_{100}$ $^{175,176}_{71}Lu_{104,105}$

$^{188,189}_{76}Os_{112,113}$ $^{208}_{82}Pb_{126}$ $^{209}_{83}Bi^{*}_{126}$ $^{223,224}_{87}Fr^{*}_{136,137}$ $^{231,232}_{91}Pa^{*}_{140,141}$ $^{257}_{100}Fm^{*}_{157}$

$^{273,2\cdot 137}_{107}Bh^{*}_{166,167}$ $^{285}_{112}Cn^{*}_{173}$ $^{287}_{113}Nh^{*}_{174}$ $^{2\cdot 157}_{126}Ch^{ie}_{188}$ $^{2\cdot 173}_{137}Fy^{ie}_{209}$ $^{355}_{141}Ch^{ie}_{214}$ $^{6\cdot 71}_{169}Ch^{ie}_{257}$

In the above formulas of c_{au}, the rate 44/7 appears miraculously, so we suppose they should be reasonable and precise. It is worth noting that the factors 6, 10 and 14 (112=8×14) should correspond to the senary, decimal and forteenary number systems respectively [15], the factor 112 corresponds to the natural end of elements, i.e., the 112th element $_{112}Cn^{*}$, the factor 83 corresponds to the end of stable elements and the start of radioactive elements, i.e., the 83th element $_{83}Bi^{*}$, and the factor 137 corresponds to the end of hydrogen-like elements which was proposed by physicist Feynman, i.e., the 137th ideal extended element $_{137}Fy^{ie}$.

As (44/7)(113/710)=1.0004≈1, the above formulas could be simplified as follows.

$$c_{au}=\frac{c}{v_e}=\frac{1}{\alpha_c}=\frac{1}{\sqrt{\alpha_1\alpha_2}}$$
$$=\frac{1}{6}\sqrt{\frac{7}{13}}(56\cdot 20+\frac{1}{2}-\frac{1}{2\cdot 59}+\frac{1}{2\cdot 3\cdot 5\cdot 7\cdot 19\cdot 131})$$
$$=137.0359990746442$$

$$c_{au}=\frac{c}{v_e}=\frac{1}{\alpha_c}=\frac{1}{\sqrt{\alpha_1\alpha_2}}$$
$$=\frac{1}{6}\sqrt{\frac{7}{13}}(56\cdot 20+\frac{1}{2}-\frac{1}{2\cdot 59}+\frac{1}{2\cdot 3\cdot 5\cdot 7\cdot 19\cdot 131-\delta_c})$$
$$=137.035999074644170968261216427 08$$

$$\delta_c=\cfrac{3}{92+\cfrac{1}{30+\cfrac{27}{34+\cfrac{1}{79+\cfrac{3}{34+1/21}}}}}$$

Relationships with nuclides:

$^{20}_{10}Ne_{10}$ $^{28}_{14}Si_{14}$ $^{40}_{20}Ca_{20}$ $^{54,56}_{26}Fe_{28,30}$ $^{59}_{27}Co_{32}$ $^{58,60}_{28}Ni_{30,32}$ $^{64,66,68}_{30}Zn_{34,36,38}$

$^{78,80}_{34}Se_{44,46}$ $^{79,81}_{35}Br_{44,46}$ $^{91,92}_{40}Zr_{51,52}$ $^{100}_{44}Ru_{56}$ $^{105}_{46}Pd_{59}$ $^{118}_{50}Sn_{68}$ $^{131}_{54}Xe_{77}$

$^{135,137}_{56}Ba_{79,81}$ $^{155,156}_{64}Gd_{91,92}$ $^{197}_{79}Au_{118}$ $^{210}_{85}At^{*}_{125}$ $^{231}_{91}Pa^{*}_{140}$ $^{235,238}_{92}U^{*}_{143,146}$

It is worth noting that the factors 56 and 20 in the above simplified formulas are the most stable and much stable numbers in atomic nucleus according to our previous paper [16], and they should relate to chirality or our hands. However, the correlation of the factors in the above simplified formulas are not as good as that in the former formulas in terms of their relationships with nuclides. In other words, the former formulas containing 44/7 and 710/113 are more meaningful. So we suppose the former formulas should be more reasonable. This also implies that the rates 44/7 and 710/113 are necessary in the formulas.

4. Discussion and Conclusion

The ancient Greek mathematician Archimedes (BC 287-212) calculated the value of π to be between 223/71 and 22/7 and proposed 22/7 as approximate value of π in practical calculation. The ancient Chinese mathematician Chongzhi Zu (AD 429-500) calculated the value of π to be between 3.1415926 and 3.1415927 and proposed 22/7 and 355/113 as approximate rates of π. The mathematicians in later generations deduced the following precise formulas about 22/7 and 355/113 in 1944 and 2005 respectively [17].

$$\pi = \frac{22}{7} - \int_0^1 \frac{x^4(1-x)^4}{1+x^2}\,dx$$

$$\pi = \frac{355}{113} - \frac{1}{3164}\int_0^1 \frac{x^8(1-x)^8(25+816x^2)}{1+x^2}\,dx$$

The above two formulas could be expressed in 2π format as follows.

$$2\pi = \frac{44}{7} - 2\int_0^1 \frac{x^4(1-x)^4}{1+x^2}\,dx$$

$$2\pi = \frac{710}{113} - \frac{1}{2\cdot 7\cdot 113}\int_0^1 \frac{x^8(1-x)^8(25+16\cdot 3\cdot 17x^2)}{1+x^2}\,dx$$

These precise formulas indicate that 22/7 and 355/113 or 44/7 and 710/113 are not only the approximate rates of π or 2π, but also the scientific simulation of π or 2π. In other words, they could stands for π or 2π and should have scientific meanings. And after so many years from Archimedes and Chongzhi Zu's eras, we have found what their real scientific meanings are and have applied them to construct reasonable formulas of the fine-structure constant and the speed of light in atomic units which are extremely important in physics.

Besides our previous formulas and explanations, this is another answer to Richard Feynman's question whether the fine-structure constant is related to π. Our answer is yes but in a very unexpected, subtle and reasonable way, especially related to 22/7 and 355/113.

Reference

1. Richard P. Feynman (1985). QED: The Strange Theory of Light and Matter. p. 129.
2. E-preprint: vixra.org/abs/2002.0203
3. E-preprint: vixra.org/abs/2002.0020
4. E-preprint: vixra.org/abs/2012.0107
5. E-preprint: vixra.org/abs/2102.0162
6. E-preprint: vixra.org/abs/2103.0088
7. E-preprint: vixra.org/abs/2104.0053
8. E-preprint: vixra.org/abs/2106.0042
9. E-preprint: vixra.org/abs/2106.0151
10. E-preprint: vixra.org/abs/2308.0168
11. E-preprint: vixra.org/abs/2407.0038
12. E-preprint: vixra.org/abs/2409.0044
13. E-preprint: vixra.org/abs/2501.0003
14. E-preprint: vixra.org/abs/2501.0095
15. E-preprint: vixra.org/abs/2501.0055
16. E-preprint: vixra.org/abs/2312.0055
17. https://mathworld.wolfram.com/PiFormulas.html