A Dictionary of the Dano-Norwegian and English Languages by A. Larsen and The Graphical Law

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Abstract

We study the Dano-Norwegian head entries of A Dictionary of the Dano-Norwegian and English Languages by A. Larsen. We draw the natural logarithm of the number of the Dano-Norwegian language head entries, normalised, starting with a letter vs the natural logarithm of the rank of the letter, normalised/unnormalised. We find that the Dano-Norwegian head entries underlie a magnetisation curve of a Spin-Glass in the presence of little external magnetic field.

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I. INTRODUCTION

Dano-Norwegian is a dialect of the Danish language. In this paper, we study the Dano-Norwegian dialect of the Danish language. We study the Dano-Norwegian head entries as those appear in A Dictionary of the Dano-Norwegian and English Languages by A. Larsen, [1]. Looking for the graphical law, we count one by one all the Dano-Norwegian head entries in the dictionary, [1].

We have started considering magnetic field pattern in [2], in the languages we converse with. We have studied there, a set of natural languages, [2] and have found existence of a magnetisation curve under each language. We have termed this phenomenon as the Graphical Law. Then, we moved on to investigate, [3], into dictionaries of five disciplines of knowledge and found the existence of a curve of magnetisation under each discipline. This was followed by finding of the graphical law in references from [4] to [95].

The planning of the paper is as follows. We give an introduction to the standard curves of magnetisation of Ising model in the section II. In the section III, we describe the analysis of the head entries of the Dano-Norwegian dialect, [1]. Sections IV and V are Acknowledgment and Bibliography respectively.

II. MAGNETISATION

A. Bragg-Williams approximation

Let us consider a coin. Let us toss it many times. Probability of getting head or, tale is half i.e. we will get head and tale equal number of times. If we attach value one to head, minus one to tale, the average value we obtain, after many tossing is zero. Instead let us consider a one-sided loaded coin, say on the head side. The probability of getting head is more than one half, getting tale is less than one-half. Average value, in this case, after many tossing we obtain is non-zero, the precise number depends on the loading. The loaded coin is like ferromagnet, the unloaded coin is like para magnet, at zero external magnetic field. Average value we obtain is like magnetisation, loading is like coupling among the spins of the ferromagnetic units. Outcome of single coin toss is random, but average value we get after long sequence of tossing is fixed. This is long-range order. But if we take a small sequence of tossing, say, three consecutive tossing, the average value we obtain is not fixed, can be anything. There is no short-range order.

Let us consider a row of spins, one can imagine them as spears which can be vertically up or, down. Assume there is a long-range order with probability to get a spin up is two third. That would mean when we consider a long sequence of spins, two third of those are with spin up. Moreover, assign with each up spin a value one and a down spin a value minus one. Then total spin we obtain is one third. This value is referred to as the value of long-range order parameter. Now consider a short-range order existing which is identical with the long-range order. That would mean if we pick up any three consecutive spins, two will be up, one down. Bragg-Williams approximation means short-range order is identical with long-range order, applied to a lattice of spins, in general. Row of spins is a lattice of one dimension.

Now let us imagine an arbitrary lattice, with each up spin assigned a value one and a down spin a value minus one, with an unspecified long-range order parameter defined as above by $L = \frac{1}{N}\Sigma_i \sigma_i$, where σ_i is i-th spin, N being total number of spins. L can vary from minus one to one. $N = N_+ + N_-$, where N_+ is the number of up spins, N_- is the number of down spins. $L = \frac{1}{N}(N_+ - N_-)$. As a result, $N_+ = \frac{N}{2}(1+L)$ and $N_- = \frac{N}{2}(1-L)$. Magnetisation or, net magnetic moment , M is $\mu\Sigma_i\sigma_i$ or, $\mu(N_+ - N_-)$ or, μNL, $M_{max} = \mu N$. $\frac{M}{M_{max}} = L$. $\frac{M}{M_{max}}$ is referred to as reduced magnetisation. Moreover, the Ising Hamiltonian,[96], for the lattice of spins, setting μ to one, is $-\epsilon\Sigma_{n.n}\sigma_i\sigma_j - H\Sigma_i\sigma_i$, where n.n refers to nearest neighbour pairs. The difference $\triangle E$ of energy if we flip an up spin to down spin is, [97], $2\epsilon\gamma\bar{\sigma} + 2H$, where γ is the number of nearest neighbours of a spin. According to Boltzmann principle, $\frac{N_-}{N_+}$ equals $exp(-\frac{\triangle E}{k_B T})$, [98]. In the Bragg-Williams approximation,[99], $\bar{\sigma} = L$, considered in the thermal average sense. Consequently,

$$ln\frac{1+L}{1-L} = 2\frac{\gamma\epsilon L + H}{k_B T} = 2\frac{L + \frac{H}{\gamma\epsilon}}{\frac{T}{\gamma\epsilon/k_B}} = 2\frac{L+c}{\frac{T}{T_c}} \qquad (1)$$

where, $c = \frac{H}{\gamma\epsilon}$, $T_c = \gamma\epsilon/k_B$, [100]. $\frac{T}{T_c}$ is referred to as reduced temperature.

Plot of L vs $\frac{T}{T_c}$ or, reduced magentisation vs. reduced temperature is used as reference curve. In the presence of magnetic field, $c \neq 0$, the curve bulges outward. Bragg-Williams is a Mean Field approximation. This approximation holds when number of neighbours interacting with a site is very large, reducing the importance of local fluctuation or, local order, making the long-range order or, average degree of freedom as the only degree of freedom of the lattice. To have a feeling how this approximation leads to matching between experimental and Ising model prediction one can refer to FIG.12.12 of [97]. W. L. Bragg was a professor of Hans Bethe. Rudolf Peierls was a friend of Hans Bethe. At the suggestion of W. L. Bragg, Rudolf Peierls following Hans Bethe improved the approximation scheme, applying quasi-chemical method.

B. Bethe-peierls approximation in presence of four nearest neighbours, in absence of external magnetic field

In the approximation scheme which is improvement over the Bragg-Williams, [96],[97],[98],[99],[100], due to Bethe-Peierls, [101], reduced magnetisation varies with reduced temperature, for γ neighbours, in absence of external magnetic field, as

$$\frac{ln\frac{\gamma}{\gamma-2}}{ln\frac{factor-1}{factor^{\frac{\gamma-1}{\gamma}}-factor^{\frac{1}{\gamma}}}}=\frac{T}{T_c}; factor=\frac{\frac{M}{M_{max}}+1}{1-\frac{M}{M_{max}}}. \tag{2}$$

$ln\frac{\gamma}{\gamma-2}$ for four nearest neighbours i.e. for $\gamma = 4$ is 0.693. For a snapshot of different kind of magnetisation curves for magnetic materials the reader is urged to give a google search "reduced magnetisation vs reduced temperature curve". In the following, we describe data s generated from the equation(1) and the equation(2) in the table, I, and curves of magnetisation plotted on the basis of those data s. BW stands for reduced temperature in Bragg-Williams approximation, calculated from the equation(1). BP(4) represents reduced temperature in the Bethe-Peierls approximation, for four nearest neighbours, computed from the equation(2). The data set is used to plot fig.1. Empty spaces in the table, I, mean corresponding point pairs were not used for plotting a line.

C. Bethe-peierls approximation in presence of four nearest neighbours, in presence of external magnetic field

In the Bethe-Peierls approximation scheme , [101], reduced magnetisation varies with reduced temperature, for γ neighbours, in presence of external magnetic field, as

$$\frac{ln\frac{\gamma}{\gamma-2}}{ln\frac{factor-1}{e^{\frac{2\beta H}{\gamma}}factor^{\frac{\gamma-1}{\gamma}}-e^{-\frac{2\beta H}{\gamma}}factor^{\frac{1}{\gamma}}}}=\frac{T}{T_c}; factor=\frac{\frac{M}{M_{max}}+1}{1-\frac{M}{M_{max}}}. \tag{3}$$

BW	BW(c=0.01)	BP(4,βH = 0)	reduced magnetisation
0	0	0	1
0.435	0.439	0.563	0.978
0.439	0.443	0.568	0.977
0.491	0.495	0.624	0.961
0.501	0.507	0.630	0.957
0.514	0.519	0.648	0.952
0.559	0.566	0.654	0.931
0.566	0.573	0.7	0.927
0.584	0.590	0.7	0.917
0.601	0.607	0.722	0.907
0.607	0.613	0.729	0.903
0.653	0.661	0.770	0.869
0.659	0.668	0.773	0.865
0.669	0.676	0.784	0.856
0.679	0.688	0.792	0.847
0.701	0.710	0.807	0.828
0.723	0.731	0.828	0.805
0.732	0.743	0.832	0.796
0.756	0.766	0.845	0.772
0.779	0.788	0.864	0.740
0.838	0.853	0.911	0.651
0.850	0.861	0.911	0.628
0.870	0.885	0.923	0.592
0.883	0.895	0.928	0.564
0.899	0.918		0.527
0.904	0.926	0.941	0.513
0.946	0.968	0.965	0.400
0.967	0.998	0.965	0.300
0.987		1	0.200
0.997		1	0.100
1	1	1	0

TABLE I. Reduced magnetisation vs reduced temperature data s for Bragg-Williams approximation, in absence of and in presence of magnetic field, $c = \frac{H}{\gamma\epsilon} = 0.01$, and Bethe-Peierls approximation in absence of magnetic field, for four nearest neighbours .

Derivation of this formula Ala [101] is given in the appendix of [7].
$ln\frac{\gamma}{\gamma-2}$ for four nearest neighbours i.e. for $\gamma = 4$ is 0.693. For four neighbours,

$$\frac{0.693}{ln\frac{factor-1}{e^{\frac{2\beta H}{\gamma}}factor^{\frac{\gamma-1}{\gamma}}-e^{-\frac{2\beta H}{\gamma}}factor^{\frac{1}{\gamma}}}} = \frac{T}{T_c}; factor = \frac{\frac{M}{M_{max}}+1}{1-\frac{M}{M_{max}}}. \tag{4}$$

In the following, we describe data s in the table, II, generated from the equation(4) and curves of magnetisation plotted on the basis of those data s. BP(m=0.03) stands for reduced temperature in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field, H, such that $\beta H = 0.06$. calculated from the equation(4). BP(m=0.025) stands for reduced temperature in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field, H, such that $\beta H = 0.05$. calculated from the equation(4). BP(m=0.02) stands for reduced temperature

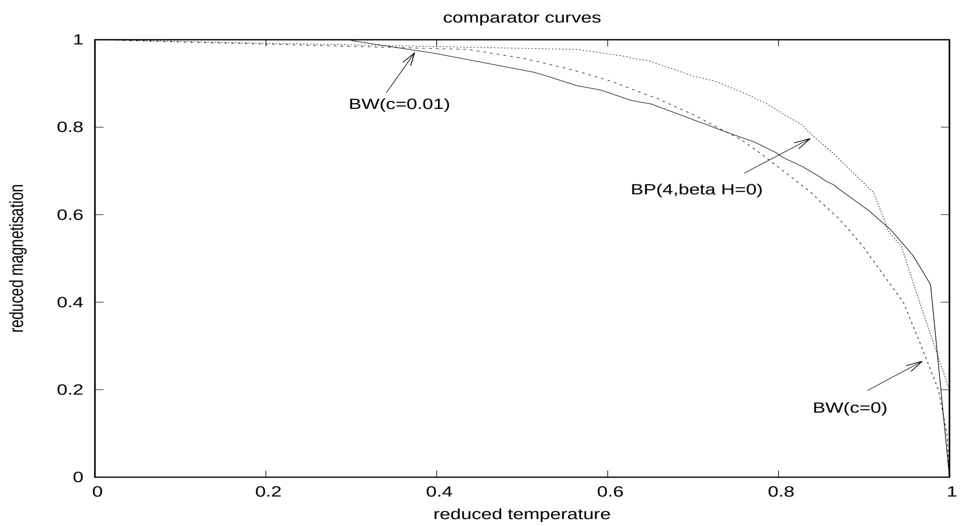

FIG. 1. Reduced magnetisation vs reduced temperature curves for Bragg-Williams approximation, in absence(dark) of and presence(inner in the top) of magnetic field, $c = \frac{H}{\gamma\epsilon} = 0.01$, and Bethe-Peierls approximation in absence of magnetic field, for four nearest neighbours (outer in the top).

in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field, H, such that $\beta H = 0.04$. calculated from the equation(4). BP(m=0.01) stands for reduced temperature in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field, H, such that $\beta H = 0.02$. calculated from the equation(4). BP(m=0.005) stands for reduced temperature in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field, H, such that $\beta H = 0.01$. calculated from the equation(4). The data set is used to plot fig.2. Empty spaces in the table, II, mean corresponding point pairs were not used for plotting a line.

BP(m=0.03)	BP(m=0.025)	BP(m=0.02)	BP(m=0.01)	BP(m=0.005)	reduced magnetisation
0	0	0	0	0	1
0.583	0.580	0.577	0.572	0.569	0.978
0.587	0.584	0.581	0.575	0.572	0.977
0.647	0.643	0.639	0.632	0.628	0.961
0.657	0.653	0.649	0.641	0.637	0.957
0.671	0.667		0.654	0.650	0.952
	0.716			0.696	0.931
0.723	0.718	0.713	0.702	0.697	0.927
0.743	0.737	0.731	0.720	0.714	0.917
0.762	0.756	0.749	0.737	0.731	0.907
0.770	0.764	0.757	0.745	0.738	0.903
0.816	0.808	0.800	0.785	0.778	0.869
0.821	0.813	0.805	0.789	0.782	0.865
0.832	0.823	0.815	0.799	0.791	0.856
0.841	0.833	0.824	0.807	0.799	0.847
0.863	0.853	0.844	0.826	0.817	0.828
0.887	0.876	0.866	0.846	0.836	0.805
0.895	0.884	0.873	0.852	0.842	0.796
0.916	0.904	0.892	0.869	0.858	0.772
0.940	0.926	0.914	0.888	0.876	0.740
	0.929			0.877	0.735
	0.936			0.883	0.730
	0.944			0.889	0.720
	0.945				0.710
	0.955			0.897	0.700
	0.963			0.903	0.690
	0.973			0.910	0.680
				0.909	0.670
	0.993			0.925	0.650
		0.976	0.942		0.651
	1.00				0.640
		0.983	0.946	0.928	0.628
		1.00	0.963	0.943	0.592
			0.972	0.951	0.564
			0.990	0.967	0.527
				0.964	0.513
			1.00		0.500
				1.00	0.400
					0.300
					0.200
					0.100
					0

TABLE II. Bethe-Peierls approx. in presence of little external magnetic fields

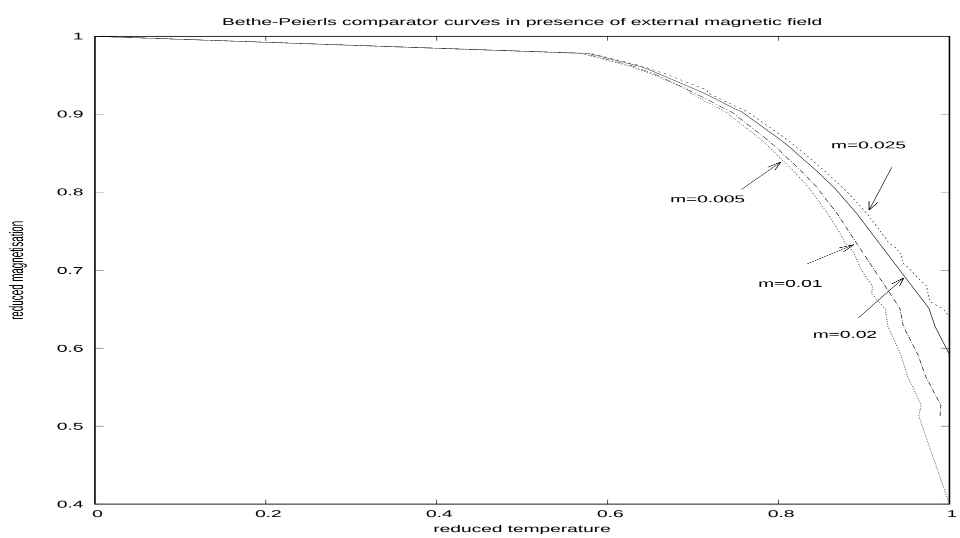

FIG. 2. Reduced magnetisation vs reduced temperature curves for Bethe-Peierls approximation in presence of little external magnetic fields, for four nearest neighbours, with $\beta H = 2m$.

D. Spin-Glass

In the case coupling between(among) the spins, not necessarily n.n, for the Ising model is(are) random, we get Spin-Glass. When a lattice of spins randomly coupled and in an external magnetic field, goes over to the Spin-Glass phase, magnetisation increases steeply like $\frac{1}{T-T_c}$ i.e. like the branch of rectangular hyperbola, up to the the phase transition temperature, followed by very little increase,[105–107], in magnetisation, as the ambient temperature continues to drop.

Theoretical study of Spin Glass started with the paper by Edwards, Anderson,[108]. They were trying to explain two experimental results concerning continuous disordered freezing(phase transition) and sharp cusp in static magnetic susceptibility. This was followed by a paper by Sherrington, Kickpatrick, [109], who dealt with Ising model with interactions being present among all neighbours. The interaction is random, follows Gaussian distribution and does not distinguish one pair of neighbours from another pair of neighbours, irrespective of the distance between two neighbours. In presence of external magnetic field, they predicted in their next paper, [110], below spin-glass transition temperature a spin-glass phase with non-zero magnetisation. Almeida etal, [111], Gray and Moore, [112],finally Parisi, [113], [114] improved and gave final touch, [115], to their line of work. Parisi and collaborators, [116]-[120], wrote a series of papers in postscript, all revolving around a consistent assumption of constant magnetisation in the spin-glass phase in presence of little constant external magnetic field.

In another sequence of theoretical work, by Fisher etal,[121–123], concluded that for Ising model with nearest neighbour or, short range interaction of random type spin-glass phase does not exist in presence of external magnetic field.

For recent series of experiments on spin-glass, the references, [124, 125], are the places to look into.

For an in depth account, accessible to a commoner, the series of articles by late P. W. Anderson in Physics Today, [126]-[132], is probably the best place to look into. For a book to enter into the subject of spin-glass, one may start at [133].

Here, in our work to follow, spin-glass refers to spin-glass phase of a system with infinite range random interactions.

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	R	S	T	U	V	X	Y	Z	AE	O
1615	1876	162	832	622	2092	1023	1084	963	168	2053	1099	1542	568	1434	1450	1043	3266	1670	2047	824	8	35	47	106	65

TABLE III. Dano-Norwegian head entries: the first row represents the English equivalent of the Dano-Norwegian alphabet, the second row represents the corresponding Dano-Norwegian head entries in the dictionary, [1]

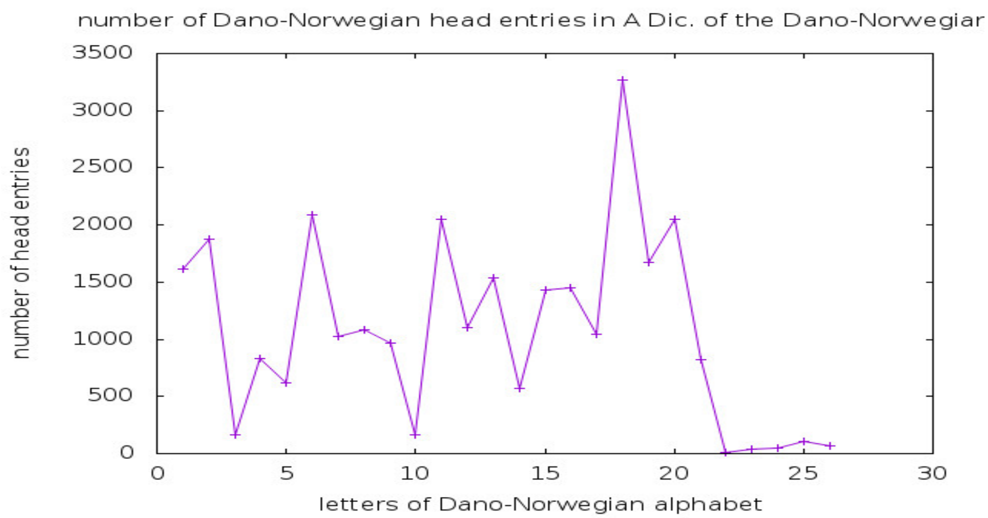

FIG. 3. The vertical axis is number of the Dano-Norwegian head entries, [1], and the horizontal axis is the respective letters. Letters are represented by the sequence number in the alphabet or, dictionary sequence,[1].

III. ANALYSIS OF THE DANO-NORWEGIAN HEAD ENTRIES

The Dano-Norwegian language alphabet is composed of twenty six letters. Counting all the head entries, [1], one by one from the beginning to the end, starting with different letters, we obtain the table, III.

Highest number of head entries, three thousand two hundred sixty six, starts with the letter equivalent of S followed by head entries numbering two thousand ninety two beginning with the letter equivalent of F, two thousand fifty three beginning with the letter equivalent of K etc. To visualise we plot the number of head entries against respective letters in the dictionary sequence, [1], in the figure fig.3.

For the purpose of exploring graphical law, we assort the letters according to the number of head entries, in the descending order, denoted by f and the respective rank, denoted by k. k is a positive integer starting from one. The lowest value of f is eight for the letter X. Hence we attach a limiting number of head entries equal to one. The corresponding limiting rank, k_{lim} or, k_d is twenty seven. As a result both $\frac{lnf}{lnf_{max}}$ and $\frac{lnk}{lnk_{lim}}$ varies from zero to one. Then we tabulate in the adjoining table, IV and plot $\frac{lnf}{lnf_{max}}$ against $\frac{lnk}{lnk_{lim}}$ in the figure fig.4. We then ignore the letter with the highest number of head entries, tabulate in the adjoining table, IV and redo the plot, normalising the lnfs with next-to-maximum lnf_{n-max}, and starting from $k = 2$ in the figure fig.5. This program then we repeat up to $k = 6$, resulting in figures up to fig.9.

k	lnk	lnk/lnk_{lim}	f	lnf	lnf/lnf_{max}	lnf/lnf_{nmax}	lnf/lnf_{2nmax}	lnf/lnf_{3nmax}	lnf/lnf_{4nmax}	lnf/lnf_{5nmax}
1	0	0	3266	8.091	1	Blank	Blank	Blank	Blank	Blank
2	0.69	0.209	2092	7.646	0.945	1	Blank	Blank	Blank	Blank
3	1.10	0.333	2053	7.627	0.943	0.998	1	Blank	Blank	Blank
4	1.39	0.421	2047	7.624	0.942	0.997	0.9996	1	Blank	Blank
5	1.61	0.488	1876	7.537	0.932	0.986	0.988	0.989	1	Blank
6	1.79	0.542	1670	7.421	0.917	0.971	0.973	0.973	0.985	1
7	1.95	0.591	1615	7.387	0.913	0.966	0.969	0.969	0.980	0.995
8	2.08	0.630	1542	7.341	0.907	0.960	0.963	0.963	0.974	0.989
9	2.20	0.667	1450	7.279	0.900	0.952	0.954	0.955	0.966	0.981
10	2.30	0.697	1434	7.268	0.898	0.951	0.953	0.953	0.964	0.979
11	2.40	0.727	1099	7.002	0.865	0.916	0.918	0.918	0.929	0.944
12	2.48	0.752	1084	6.988	0.864	0.914	0.916	0.917	0.927	0.942
13	2.56	0.776	1043	6.950	0.859	0.909	0.911	0.912	0.922	0.937
14	2.64	0.800	1023	6.930	0.857	0.906	0.909	0.909	0.919	0.934
15	2.71	0.821	963	6.870	0.849	0.899	0.901	0.901	0.912	0.926
16	2.77	0.839	832	6.724	0.831	0.879	0.882	0.882	0.892	0.906
17	2.83	0.858	824	6.714	0.830	0.878	0.880	0.881	0.891	0.905
18	2.89	0.876	622	6.433	0.795	0.841	0.843	0.844	0.854	0.867
19	2.94	0.891	568	6.342	0.784	0.829	0.832	0.832	0.841	0.855
20	3.00	0.909	168	5.124	0.633	0.670	0.672	0.672	0.680	0.690
21	3.04	0.921	162	5.088	0.629	0.665	0.667	0.667	0.675	0.686
22	3.09	0.936	106	4.663	0.576	0.610	0.611	0.612	0.619	0.628
23	3.14	0.952	65	4.174	0.516	0.546	0.547	0.547	0.554	0.562
24	3.18	0.964	47	3.850	0.476	0.504	0.505	0.505	0.511	0.519
25	3.22	0.976	35	3.555	0.439	0.465	0.466	0.466	0.472	0.479
26	3.26	0.988	8	2.079	0.257	0.272	0.273	0.273	0.276	0.280
27	3.30	1	1	0	0	0	0	0	0	0

TABLE IV. Dano-Norwegian head entries: ranking, natural logarithm, normalisations

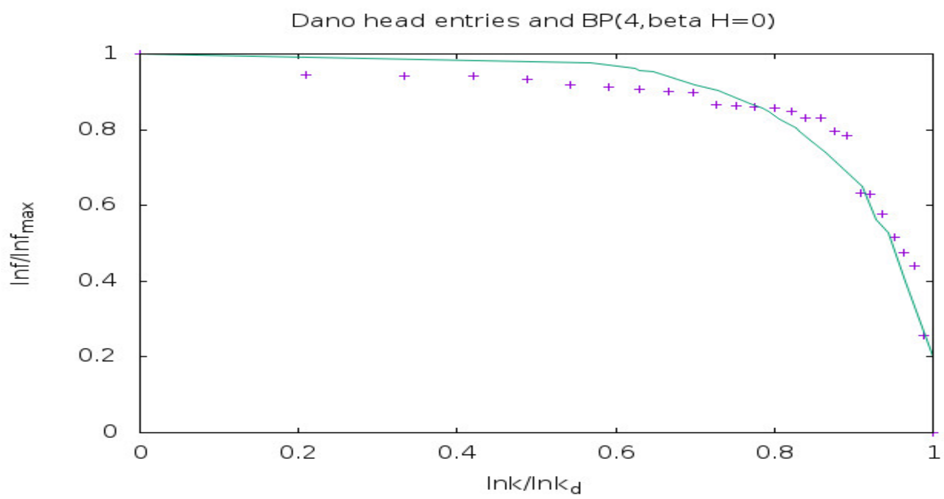

FIG. 4. The vertical axis is $\frac{lnf}{lnf_{max}}$ and the horizontal axis is $\frac{lnk}{lnk_{lim}}$. The + points represent the head entries of the Dano-Norwegian dialect with the fit curve, BP(4,βH = 0), being the Bethe-Peierls curve in the presence of four nearest neighbours and in the absence of external magnetic field, $m = 0$ or, $\beta H = 0$, of the Ising Model.

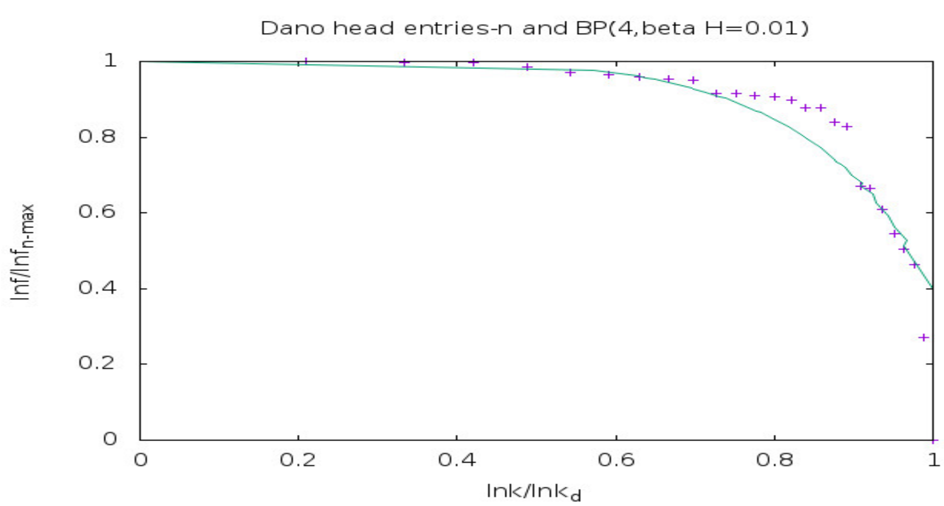

FIG. 5. The vertical axis is $\frac{lnf}{lnf_{n-max}}$ and the horizontal axis is $\frac{lnk}{lnk_{lim}}$. The + points represent the head entries of the Dano-Norwegian dialect with the fit curve, BP(4,βH = 0.01), being the Bethe-Peierls curve in the presence of four nearest neighbours and little external magnetic field, $m = 0.005$ or, $\beta H = 0.01$, of the Ising Model.

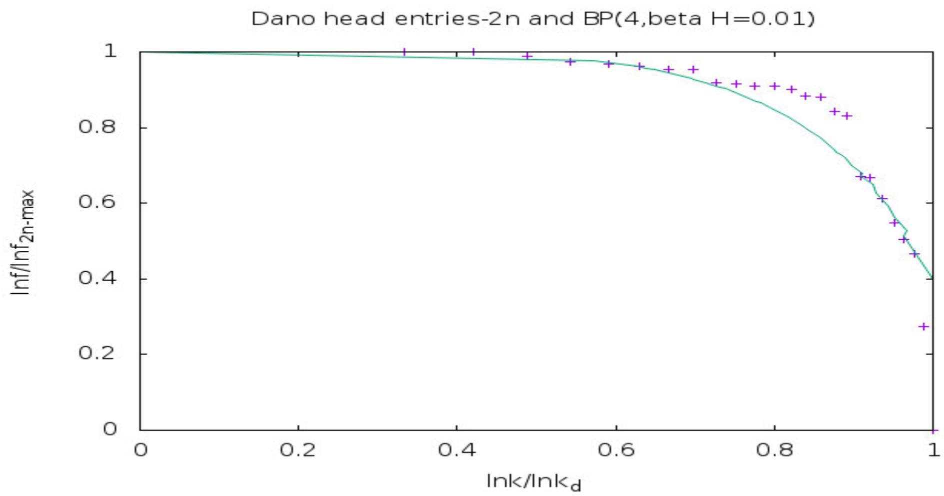

FIG. 6. The vertical axis is $\frac{lnf}{lnf_{2n-max}}$ and the horizontal axis is $\frac{lnk}{lnk_{lim}}$. The + points represent the head entries of the Dano-Norwegian dialect with the fit curve, BP(4,βH = 0.01), being the Bethe-Peierls curve in the presence of four nearest neighbours and little external magnetic field, $m = 0.005$ or, $\beta H = 0.01$, of the Ising Model.

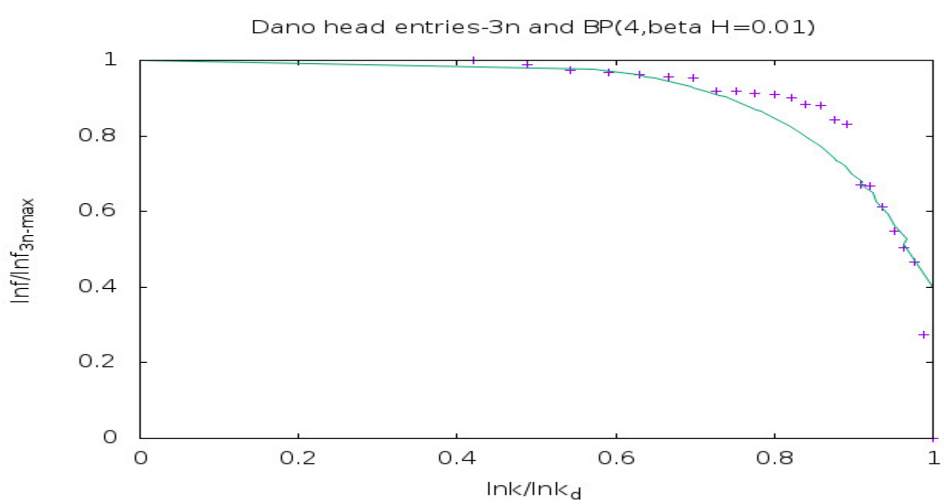

FIG. 7. The vertical axis is $\frac{lnf}{lnf_{3n-max}}$ and the horizontal axis is $\frac{lnk}{lnk_{lim}}$. The + points represent the head entries of the Dano-Norwegian dialect with the fit curve, BP(4,βH = 0.01), being the Bethe-Peierls curve in the presence of four nearest neighbours and little external magnetic field, $m = 0.005$ or, $\beta H = 0.01$, of the Ising Model.

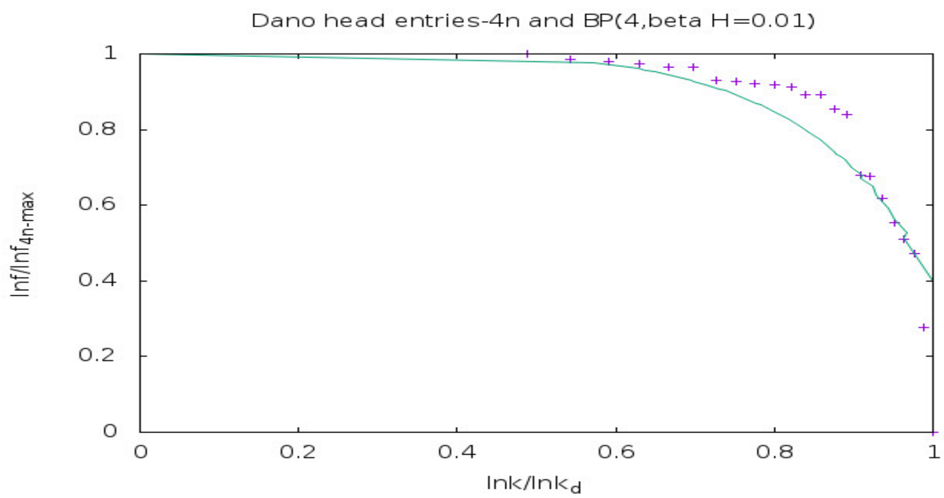

FIG. 8. The vertical axis is $\frac{lnf}{lnf_{4n-max}}$ and the horizontal axis is $\frac{lnk}{lnk_{lim}}$. The + points represent the head entries of the Dano-Norwegian dialect with the fit curve, BP(4,$\beta H = 0.01$), being the Bethe-Peierls curve in the presence of four nearest neighbours and little external magnetic field, $m = 0.005$ or, $\beta H = 0.01$, of the Ising Model.

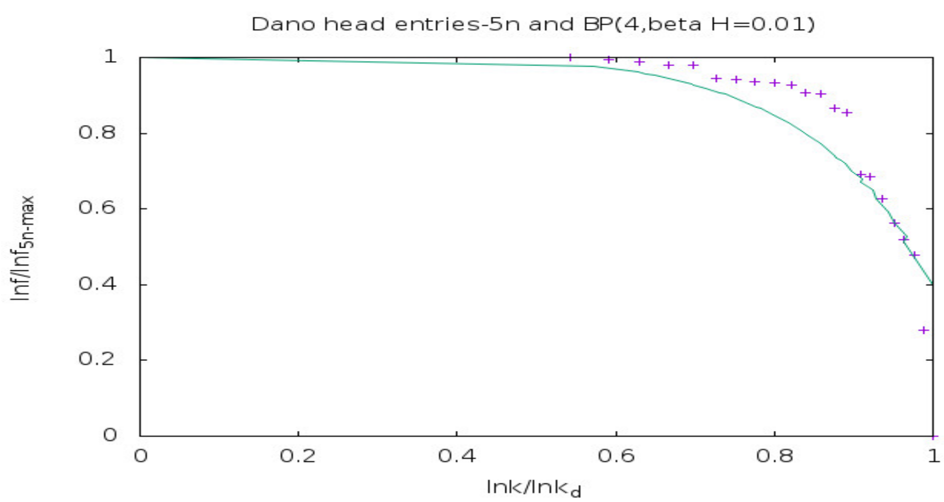

FIG. 9. The vertical axis is $\frac{lnf}{lnf_{5n-max}}$ and the horizontal axis is $\frac{lnk}{lnk_{lim}}$. The + points represent the head entries of the Dano-Norwegian dialect with the fit curve, BP(4,$\beta H = 0.01$), being the Bethe-Peierls curve in the presence of four nearest neighbours and little external magnetic field, $m = 0.005$ or, $\beta H = 0.01$, of the Ising Model.

A. tentative conclusion

Matching of the plots in the figures fig.(4-9), with comparator curves i.e. the magnetisation curves of the Ising Model in various approximations, are with dispersions and dispersions do not reduce over higher orders of normalisations.

To explore for possible existence of spin-glass transition, in the presence of little external magnetic field, $\frac{lnf}{lnf_{r-max}}$ with $r = 0, 1, .., 5$ are drawn against lnk in the figures fig.10-fig.15.

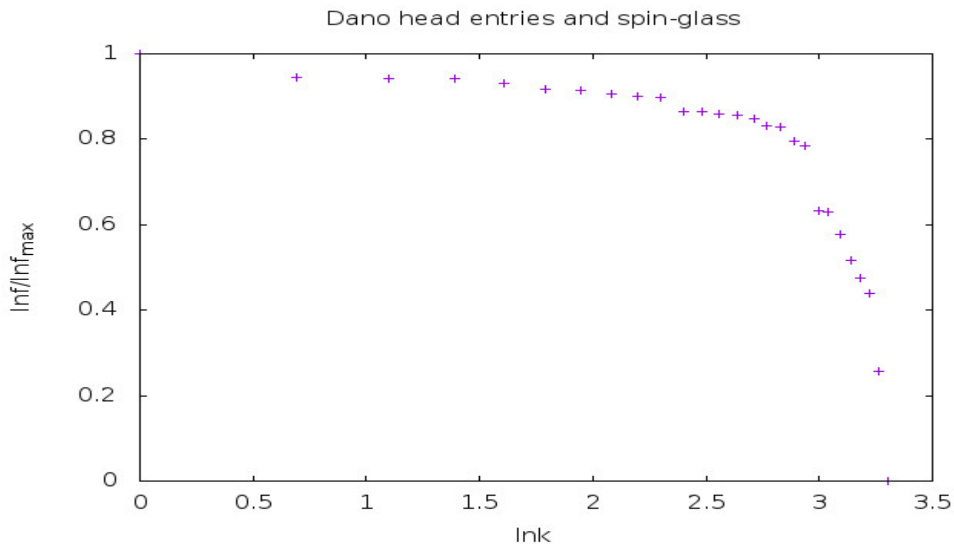

FIG. 10. The vertical axis is $\frac{lnf}{lnf_{max}}$ and the horizontal axis is lnk. The + points represent the head entries of the Dano-Norwegian dialect.

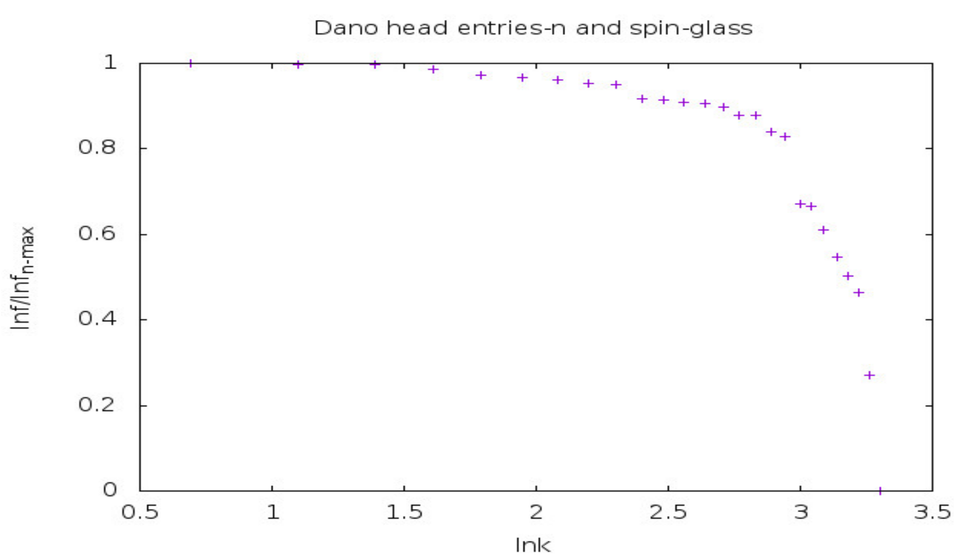

FIG. 11. The vertical axis is $\frac{lnf}{lnf_{n-max}}$ and the horizontal axis is lnk. The + points represent the head entries of the Dano-Norwegian dialect.

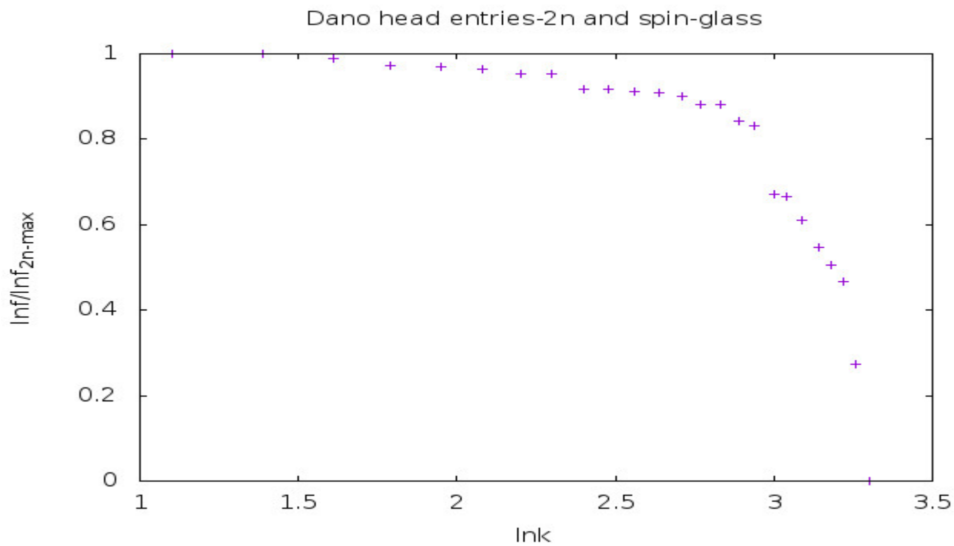

FIG. 12. The vertical axis is $\frac{lnf}{lnf_{2n-max}}$ and the horizontal axis is lnk. The + points represent the head entries of the Dano-Norwegian dialect.

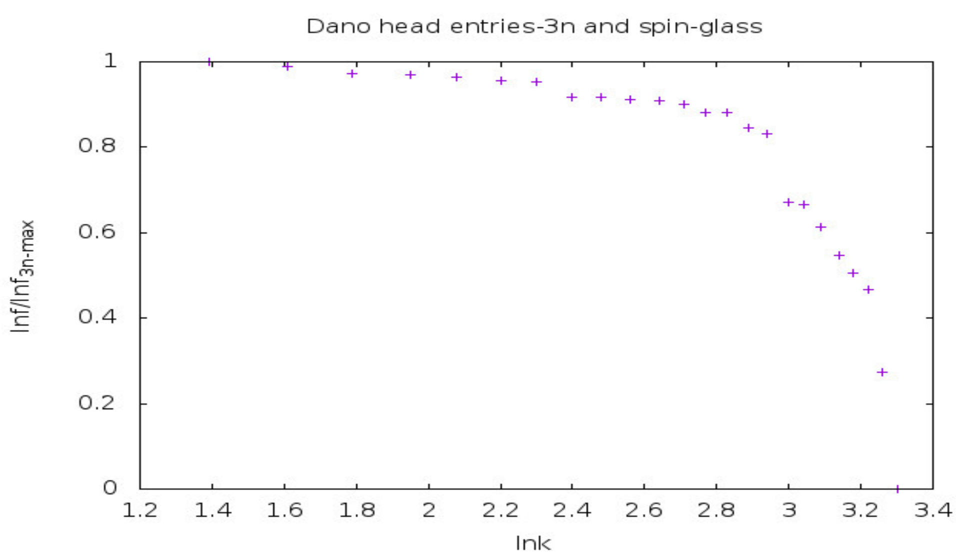

FIG. 13. The vertical axis is $\frac{lnf}{lnf_{3n-max}}$ and the horizontal axis is lnk. The + points represent the head entries of the Dano-Norwegian dialect.

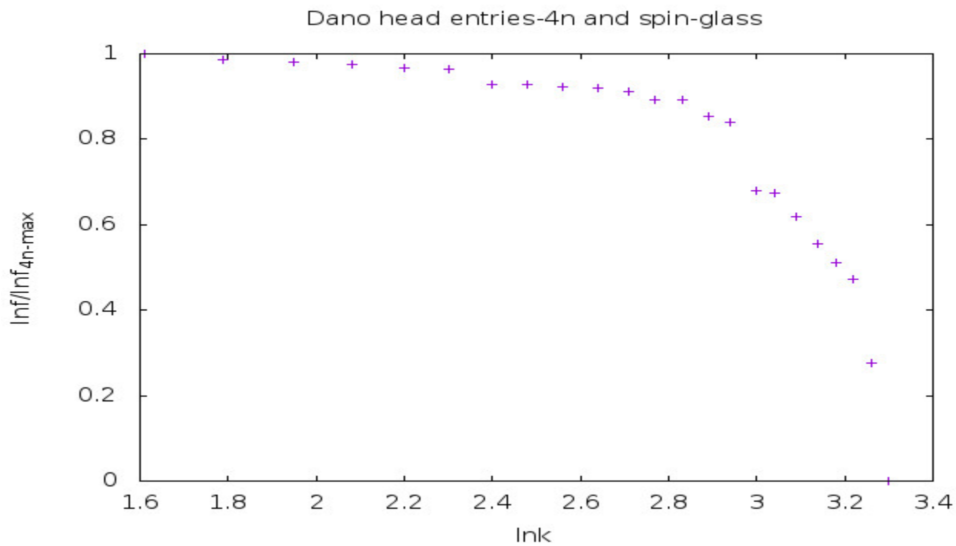

FIG. 14. The vertical axis is $\frac{lnf}{lnf_{4n-max}}$ and the horizontal axis is lnk. The + points represent the head entries of the Dano-Norwegian dialect.

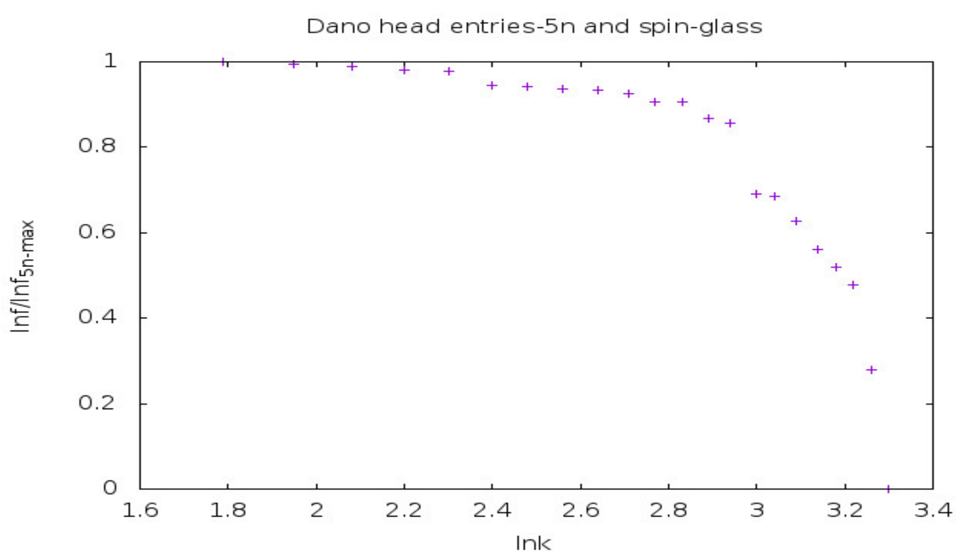

FIG. 15. The vertical axis is $\frac{lnf}{lnf_{5n-max}}$ and the horizontal axis is lnk. The + points represent the head entries of the Dano-Norwegian dialect.

B. conclusion

In the figures Fig.10-Fig.15, the points has a smoothed transition, [120]. Above the transition point(s), the lines are almost horizontal and below the transition point(s), points-line rises sharply, but without the tail part, like the branch of a rectangular hyperbola. Hence, the Dano-Norwegian head entries, [1], are suited to be described by a Spin-Glass magnetisation curve, [105], in the presence of little external magnetic field. Moreover, the associated correspondence is,

$$\frac{lnf}{lnf_{n-max}} \longleftrightarrow \frac{M}{M_{max}},$$
$$lnk \longleftrightarrow T.$$

k corresponds to temperature in an exponential scale, [135].

IV. ACKNOWLEDGMENT

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