The Apex Proof to Fermat's Last Theorem

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Abstract-Hypothesis:

This paper shall show with Diophantine equations, which will be shown to be derived from $A^P + B^P = C^P$, which is analyzed more quickly as $A^P + B^P + C^P = 0$, C being negative. That the 3 base variables A, B and C are congruent, expressed formulaically:

A = B = C Mod P   *for Sophie Germane Case 1, when P >= 5*
                                Thus establishing that A + B + C ≠ 0 Mod P

as well as:

A = B Mod P   *for Sophie Germane Case 2, when P >= 3*
                                Thus establishing that A + B + C ≠ 0 Mod P

A + B + C = 0 Mod P of course being one of the prerequisite equations for FLT, based on Sophie Germain's first Axiom. And this prototypical formula is quite easily established by Fermat's Little Theorem.

The solution will involve establishing two factors for each of the key variables A, B and C, which will be denoted by subscripts.

While FLT was proved quite some time ago by Wiles/Taylor, it remains out of reach for the vast majority of mathematicians, due to the need of a strong background in modularity theory for *elliptic* curves, and other arcane branches of Number Theory. Thus most mathematicians are hoping for a proof that is a little easier to comprehend using Diophantine equations. This paper is intended to satisfy that need.

I have tried hard to making the writing light and entertaining. Writing this paper was like writing a book, a tremendous amount of blood, sweat and tears went into it's construction. Thousands of hours of math work. Do not feel the need to try to rush thru it, three subsequent readings of perhaps an hour each should allow complete absorption of this creative work of mathematics art.

The basic formula $A^P + B^P = C^P$, is non-symmetrical in presentation. This exposition on FLT, for the most part makes use of the symmetrical presentation in the form $A^P + B^P + C^P = 0$, with C being considered to be a negative integer value. This approach method was also used by Euler, who was the first recorded mathematician to prove the case P = 3 for Fermat's Last theorem.

In my earlier 9th proof attempt, which I wrote up several months ago, I used a metaphor of climbing Mount Everest liberally throughout the proof in various places, and I will reuse much of that proof in this new document. I hope you find the reading of this proof entertaining and sparkling. Or at least you may find it more entertaining and sparkling than your average Diophantine proof

you may find on arXiv. For quite certainly, it is highly conceivable that others could have discovered a similar proof years before, but due to an inability to promote their ideas to the world at large, a proof would have gone unnoticed. Note, mathematics manipulation is only a way to pass the time for me, my true skills lie in music creation and engineering, thus you may find my notations somewhat arcane, for which I apologize in advance.

Basic knowledge regarding the exponent value. For any case of $A^N + B^N = C^N$, where N is >=3, it is relatively easy to show that it is only necessary to prove FLT for prime number exponents. Additionally, it is only necessary to prove FLT for A, B and C being coprime for obvious reasons. For even number value exponents, any that are composite and have an odd number factor will be provable by the odd number having a prime number factor, and if N = 4, 8, 16, 32 etcetera, Fermat's proof for N =4 by Infinite Descent serves as the simple basis of a proof. I will not elaborate on the above statements in this paragraph, as the proofs are very simple and can be viewed on a 1000 different web portals.

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Addendum:
   A) Foundational Knowledge, Expansions of Fermat's Little Theorem
   B) References and Suggested reading
   C) Email Contact information
   D) Individuals who have assisted me in my quest, who are worthy of my mention

Change Log located at the end of the paper.

Conventions used in this Paper:

Please note that instead of using the congruence operator of 3 parallel lines, I will instead be using a standard equality operator, for all modulus equations, as was the practice used regularly in the somewhat distant past. This will save me considerable mouse clicks during the creation of this document.

The abbreviation FLT will be used to indicate Fermat's Last Theory.

In the last 20 years of working on this theory, I have become accustomed to using a Symmetrical Form of the presentation of FLT, as follows: $A^P + B^P + C^P = 0$, this form has the benefit of reducing the amount of analysis when dealing with a symmetrical problem such as FLT. It should be mentioned the first Mathematician to seriously do some work on this problem other than Pierre Fermat himself was Leonard Euler, and he wrote his proof for the case N = 3 in the Symmetrical form as well. At times I may switch over to the non-symmetrical standard form of $A^P + B^P = C^P$, when the NSF (*non-Symmetrical Form*) may yield better clarity in an explanation.

Finally, the variables A, B and C are broken down into factors A_1, A_2, B_1, B_2, C_1 and C_2. The subscripts help to organize the factoring and memorizing of these 6 variables.

FOUNDATION THEORY, Necessary to Gain Basic Skills to understanding Fermat's Last Theorem

Note, there is a certain amount of repetition in this section, and some of the final forms referred to as "Presentation of D", may be not actually be required to be absorbed for a clear understanding of the two final SGC (*Sophie Germain Case*) proofs, but are of interest in gaining a solid foothold into the fundamentals, none-the-less. The first 12 pages are presented in a Classroom Lesson type presentation style, with use of metaphor to enhance the reading experience.

These next few pages will give the basic equational tools and gear necessary for climbing to the peak of the Mount Everest of math problems. Note the Himalaya's peaks are many and this Sherpa can only explore a limited number of them. I have found two routes to the summit, from which an inspiring view and feeling well being may spring. The climb is not without ardor, and to try to push to quickly to the summit may find one out of breath, and a fuzzy mind. Thus it is essential to accumulate these basic equational tools and commit them to memory. In further documents in this proof, the level of detail that will be expressed DEPENDS on a deep internal mathematics absorption of this foundational base.

At the completion of this portion of the proof we will be at Base Camp, and prepared to ascend to the heights of Everest.

The starting point will be defining the problem. It is normally defined as follows:

$$X^N + Y^N = Z^N$$

E4a

With X, Y and Z being positive integer values, and N being an integer value >= 3. That there exist no possible solutions.

A proof for the case for N = 4 was shown by Fermat in a margin of his copy of Arithmetica, and later published by his son, after his death. Adjacent to the short detailed proof which makes use of the technique of Infinite Descent, is a comment that there are no solutions for any other higher exponent than 2, and that the margin of the paper is to small to hold this proof. Hard to say one way or another if he had a rock solid proof.

Anyway moving on, if N is any power of 2 >= 4 the proof would also hold, based upon simple algebraic use of exponent rules. Using similar reasoning, we can prove that any odd number exponent which is a composite number, will also hold true, if we can establish a proof for either of the factors for that composite number. And of course any even number which is a product of an odd prime number or odd composite number will also be "covered" by a proof for prime numbers which are >=3.

Based upon the above, and my personal preferences, we may rewrite the starting point equation as:

$$A^P + B^P = C^P$$

E4b

In this presentation, the exponent P represents a prime number >=3, and A, B and C as coprime integers.
The fundamental reasoning that A, B and C are considered as coprime, is that if A and B had a common factor, then C would also, and then we could remove this factor from all 3 variables, and rewrite.

Again based upon personal preference we may rewrite the equation in the symmetrical form as:

$$A^P + B^P + C^P = 0$$

E4c

In this presentation, we presume one of the 3 variables A, B and C must be negative. For convenience sake we will assume that C has a negative value. It should be noted that Euler was the first mathematician to find a proof for the case P = 3, and his proof used the symmetrical form. In other words, good historical precedent to proceed along this approach vector to the solution.

At this point maybe good to throw in some philosophy (*OH NOOOOOOO!*) Oh yes, consider the following.

This proof could also be for two negative numbers and one positive number, and be equally valid. And if we conveniently ignore the trivial solution aspect, the potential values and polarities of **negative, zero and positive** sort of make up a spectrum analogy of the human race coloration and sexual orientation. (*Note, this paper may be burned in "Fahrenheit 451ish fashion" in some*

fundamentalist republic provinces, and produce lots of heat, and additional CO_2 for our sky.) So much for my comedic relief, back to reality.

Sophie Germain around the year 1800 was working on a number of mathematical and physics problems, her work on Fermat's Last Theorem has had a profound effect on the understanding of the underlying aspects of the problem. And her definition of Case 1 and Case 2 analysis of the famous equation is a starting point in understanding the two fundamental analysis approaches which must be employed.

Case 1, is when **none** of the integer variables A, B or C contains a factor of P.

Case 2, is when **one** of the integer variables A, B or C contains a factor of P.

Other than this simple branching aspect of the proof definition, no other aspects of Sophie Germain's extensive work on Fermat's Last Theory are utilized, in this exposition.

FACTORING $A^P + B^P + C^P = 0$

Consider $G^P + H^P$ and $G^P - H^P$ each consists of two factors as follows:

$G^P + H^P = (G + H)(G^{P-1} - G^{P-2}H + G^{P-3}H^2 - \ldots\ldots + G^2H^{P-3} - GH^{P-2} + H^{P-1})$ **E5a**
Note, alternating sign polarities in factor 2

$G^P - H^P = (G - H)(G^{P-1} + G^{P-2}H + G^{P-3}H^2 + \ldots\ldots + G^2H^{P-3} + GH^{P-2} + H^{P-1})$ **E5b**
Note, same polarities in factor 2

Note, writing out the above right side factor 2 is time consuming to write, so as a shortcut, we may consider using the following functions instead:

$f_a(G, H, P) = (G^{P-1} - G^{P-2}H + G^{P-3}H^2 - \ldots\ldots + G^2H^{P-3} - GH^{P-2} + H^{P-1})$ **E5c**
(f_a being the additive function factor of $G^P + H^P$)

$f_s(G, H, P) = (G^{P-1} + G^{P-2}H + G^{P-3}H^2 + \ldots\ldots + G^2H^{P-3} + GH^{P-2} + H^{P-1})$ **E5d**
(f_s being the subtractive function factor of $G^P - H^P$)

While working in the symmetrical presentation of Fermat's Last Theory I do not show
the subscript "a" or "s", since all factoring work is from an additive point of view.

We may now expand the presentation form for Sophie Germain Case 1, using the above factoring
Concepts.

Please bear in mind that G + H, may only divide once into $G^N + H^N$, and that for SGC1 there can be no common factors that exist between G + H and f_a(G, H, P). This is shown in Lemma T3 on page 12. Regarding SGC2, this T3 Lemma also shows that if G + H contains one or more P factors then f_a(G, H, P) must contain exactly one factor of P.

$A_1{}^P A_2{}^P + B_1{}^P B_2{}^P + C_1{}^P C_2{}^P = 0$ (*Specific to SGC1*)   **E6a**

where $A_1{}^P = -(B + C)$ and $A_2{}^P = f(B, C, P)$

and $B_1{}^P = -(A + C)$ and $B_2{}^P = f(A, C, P)$

and $C_1{}^P = -(A + B)$ and $C_2{}^P = f(A, B, P)$

Similarly, we may expand the presentation for Sophie Germain Case 2:

$A_1{}^P A_2{}^P + B_1{}^P B_2{}^P + \mathbf{P_1{}^P} C_1{}^P C_2{}^P = 0$ (*Specific to SGC2*)   **E6b**

where $A_1{}^P = -(B + C)$ and $A_2{}^P = f(B, C, P)$

and $B_1{}^P = -(A + C)$ and $B_2{}^P = f(A, C, P)$

and $\mathbf{P^{P-1}} C_1{}^P = -(A + B)$ and $\mathbf{P} C_2{}^P = f(A, B, P)$

At this point, I suppose a simple presentation that can be written out on a blackboard for the class is needed. Let's look at the simpler case of SGC1 first, for P=5.

$$A^5 + B^5 + C^5 = 0 = (A+B)(A^4 - A^3B + A^2B^2 - A^3B + B^4) + C^5$$ **E6c**

and we could rewrite this as $(A+B)(A^4 - A^3B + A^2B^2 - A^3B + B^4) = -C^5$ **E6d**

The above form looks pretty basic, of course if we used the typical non-symmetrical presentation form instead of $-C^5$ we would simply have C^5. At this point you may wonder, why deal with a symmetrical form at all, which has positive and negative integer variables. Well, when the algebraic juggling gets super complex, using a somewhat simpler form helps to keep the polarity errors from creeping in to the analysis. Of course at this point in the exposition, everything is pretty simple. When we get to the trinomial expansion of $(A + B + C)^P$, the symmetrical form starts to look more appealing.

Binomial Expansion of (a+b)^P

When (a+b)^P goes thru binomial expansion, the expanded form may be presented/condensed as:

a^P + P (f(a,b)) + b^P *(with P (f(a,b)) representing the sum of all center terms)*   **E7a**

Basically, all of the center term coefficients will have a prime factor of P.

This may be understood by absorbing the basic standard formula for Binomial Expansion which is noted to the right:

Maybe a little too abstract? Let's try a few prime exponent examples to add light to the concept.
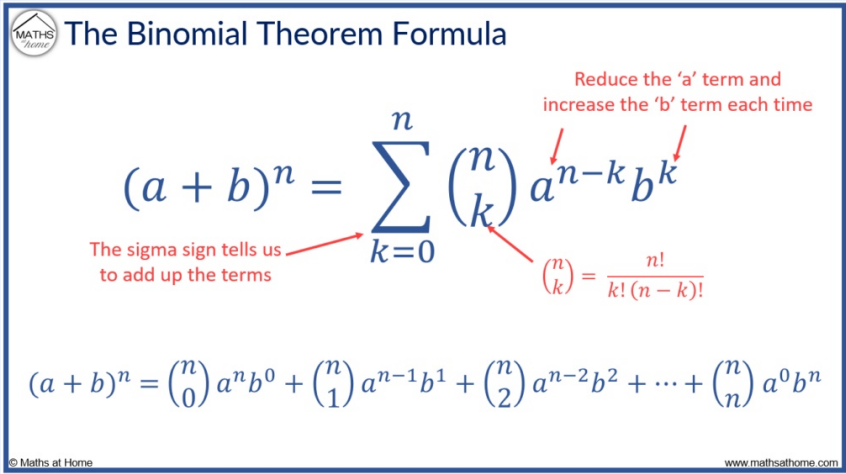
(a+b)^3 = a^3 + 3a^{2}b + 3 ab^2 + b^3 **E7b**
(a+b)^5 = a^5 + 5a^{4}b + 10a^{3}b^2 + 10a^{2}b^3 + 5ab^4 + b^5 **E7c**

If you study the coefficient formula for a bit (*shown in Red Text above*), it will make sense, that all of the center term coefficients must have a prime factor of P, since a prime factor of n occurs in the numerator and can not occur in the denominator for all center term coefficients.

The Binomial Theorem Formula

Reduce the 'a' term and increase the 'b' term each time

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$$

The sigma sign tells us to add up the terms

$$\binom{n}{k} = \frac{n!}{k!\,(n-k)!}$$

$$(a+b)^n = \binom{n}{0} a^n b^0 + \binom{n}{1} a^{n-1} b^1 + \binom{n}{2} a^{n-2} b^2 + \cdots + \binom{n}{n} a^0 b^n$$

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Below is Pascal's triangle from Wiki which shows all of the term coefficients up to exponent 7:
(*It's a classic math diagram!*) The center term coefficient prime factors are obvious for 3, 5 and 7.

```
              1
            1   1
          1   2   1
        1   3   3   1
      1   4   6   4   1
    1   5  10  10   5   1
  1   6  15  20  15   6   1
1   7  21  35  35  21   7   1
```

Trinomial Expansion of $(A+B+C)^P$

Now for Trinomial Expansion, pretty much the same applies, but we will now have to start thinking somewhat geometrically, but with supportive algebraic logic.

$(A + B + C)^3 =$ *(first diagrams, exponent = 3)*   **E8a**

$(A + B + C)^5 =$ *(following diagram, exponent = 5)*   **E8b**

$$C^3$$

$$3AC^2 + 3BC^2$$

$$3A^2C + 6ABC + 3B^2C$$

$$A^3 + 3A^2B + 3AB^2 + B^3$$

All Multiples of 3
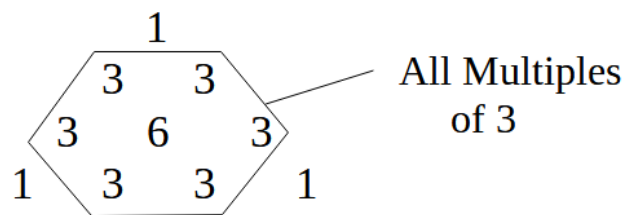
$$C^5$$

$$5AC^4 + 5BC^4$$

$$10A^2C^3 + 20ABC^3 + 10B^2C^3$$

$$10A^3C^2 + 30A^2BC^2 + 30AB^2C^2 + 10B^3C^2$$

$$5A^4C + 20A^3BC + 30A^2B^2C + 20AB^3C + 5B^4C$$

$$A^5 + 5A^4B + 10A^3B^2 + 10A^2B^3 + 5AB^4 + B^5$$

NOTE, all of the coefficients (*shown in brown text*) for the P=5 trinomial expansion are divisible by 5.

For the general case of any prime number equal to 3 or greater this must also be true, since the center terms of the Binomial expansion are all multiples of the prime exponent factor, when expanded.

From the above rather un-artistic graphics we can gain a foothold into Trinomial expansion coefficients, that they all appear to be multiples of the prime exponent.

Formulaically expressed as:

$(A + B + C)^P = A^P + B^P + C^P + P\,(f(A,B,C,P))$ **E9a**
Where $P\,(f(A,B,C,P))$ is a unique positive integer value function representing the sum of all center terms.

Thus we observe the 3 corner terms have coefficients of 1, and all of the center coefficients are multiples of prime exponent value P.

The graphical view is nice, maybe algebraically you may understand that since all non-corner *perimeter* binomial expansions have factors of prime P, when we can multiply any horizontal binomial center row coefficients by the outer perimeter angled vertical row coefficients then all interior term coefficients must also contain a factor of prime P.

Perhaps at this point a more tangible proof of the center none-perimeter coefficients is needed. Supposing we rewrite the starting point equation in this analysis as follows:

$(A + B + C)^P = ((A+B) + C)^P$ and next simply apply Binomial Expansion to (A+B) and C.   **E9b**

In this case, if we consider P = 5, and the second row from the bottom, we will see that the coefficient elements will all be multiples of 5. Then once we expand (A+B), all of these coefficients will be multiplied by the factor 5. QED.

Since the summation of A^P, B^P and C^P is supposedly zero, we may now remove the 3 corner elements from the isosceles matrix.

With the 3 Corner Values of A^P, B^P and C^P removed, we find that all remaining elements are divisible by P, additional a careful analysis of a typical binomial expansion shows that the sum of the center terms are also divisible by a + b, therefore we can now show that the expansion of $(A + B + C)^P$ has the following 4 factors:
      P   (A+B)   (B+C)   and   (C+A)

And bearing in mind the previous work from page 6: $A+B = -C_1^P$, $B+C = -A_1^P$, $C+A = -B_1^P$ **E9c**

Then based upon the knowledge that (A + B + C) must have an initial value which can be raised to the P exponent to $(A + B + C)^P$, we may determine that (A + B + C) must have an alternate form of:

$$A + B + C = P\,A_1\,B_1\,C_1\,K_0$$ **E9d**

with K_0 being an arbitrary integer value which is related to the remaining factor of the division of $(A+B+C)^P$ by $P(A+B)(B+C)(C+A)$

For the case P = 3, K_0 is easily determined for SGC2 and SGC1. However for higher order prime exponents the computation of K_0 as a formula derived from A, B and C becomes more and more difficult as the exponent P increases. Yet we do not need to know the exact value of K_0, only that it is an integer if there would exist a counter-example solution to FLT.

Additionally, the various presentations of A + B + C may be given a single variable designation of **D** to simplify reference to this important variable in the FLT analysis.

Restating:

$$D = A + B + C = P\,A_1\,B_1\,C_1\,K_0 \qquad \text{E10a}$$

Still there are many more Presentations of D, which we will be required to be fluent in, as we forge our way to Base Camp.

<u>Presentations of D:</u>

Perhaps the **most important presentation of D** is as follows, thru substitution:

$$A + B + C \;=\; \frac{(A + B) + (B + C) + (A + C)}{2} \;=\; \frac{C_1^P \;+\; A_1^P \;+\; B_1^P}{-\,2} \qquad \text{E10b}$$

(*Note, above form specific to SGC1*)

Although the -2 in the denominator of the far right presentation, appears out of place, it's required to be a negative. Not too hard to show that, if you go back to the beginning of the proof.

This particular form is instrumental to the final proof for SGC2 since it is factorable, and after factoring new transforms are possible which lead directly to the actual proofs, which will be explored in later sections of this document.

These forms can also be expressed in relation to SGC2 as:

$$A + B + C \;=\; \frac{(A + B) + (B + C) + (A + C)}{2} \;=\; \frac{P^{P-1}C_1^P \;+\; A_1^P \;+\; B_1^P}{-\,2} \qquad \text{E11a}$$

It may be noted that this form is less factorable, than the form for SGC1, however $A_1^P + B_1^P$ can be factored!

And there yet remain a few more forms of D, which will be useful gear as we approach Base Camp:

$A_1^P = -(B + C)$ $\quad\quad\quad$ $A + (B + C) = A - A_1^P$ $\quad\quad\quad$ **Similar substitutions for B and C arrive at:** $\quad\quad\quad$ **E11b**

$A + B + C = A - A_1^P = B - B_1^P = C - C_1^P$ $\quad\quad\quad$ *This form for **SGC1*** $\quad\quad\quad$ **E11c**

$\quad\quad\quad\quad$ *and*

$A + B + C = A - A_1^P = B - B_1^P = C - \mathbf{P^{P-1}}C_1^P$ $\quad\quad\quad$ *This form for **SGC2*** $\quad\quad\quad$ **E11d**

Now these last forms have a use of proving some detail about A_2, B_2 and C_2 for SGC1 as follows:

$A - A_1^P = A_1 (A_2 - A_1^{P-1})$ $\quad\quad\quad\quad\quad\quad$ *Of course same considerations for B and C* $\quad\quad\quad$ **E11e**

Based upon a complete understanding of Fermat's Little Theorem, we can show that:

$A^P = A \text{ Mod } P$ $\quad\quad$ and less well expounded: $A^{P-1} = 1 \text{ Mod } P$ $\quad\quad\quad$ **E11f**

From the above we can prove for SGC1 that A_2, B_2 and $C_2 = 1 \text{ Mod } P$, and for SGC2 if we assume C has the factor P then A_2 and $B_2 = 1 \text{ Mod } P$ and C_2 is an undefined Modulus of P, which is not $0 \text{ Mod } P$.

Below supporting lemma was written abut 18 months ago, and demonstrates that no common factors can exist between A_1 and A_2 other than P, and similarly for variables B and C. It also shows that if P is a factor of A_1, then it must also be a factor of A_2. Below simple Axioms are demonstrated by the T3 Lemma.

$\quad\quad$ **Axiom 1:** *with the precondition that J+K ≠ 0 Mod P, with P being an odd prime number,* $\quad\quad\quad\quad\quad\quad$ *SGC1*
$\quad\quad$ $J^P + K^P$ is divisible by J+K, and can not be divisible by any factors within J, K, J+K or P.

$\quad\quad$ **Axiom 2:** *with the precondition that J+K = 0 Mod P, with P being an odd prime number,* $\quad\quad\quad\quad\quad\quad$ *SGC2*
$\quad\quad$ $J^P + K^P$ is divisible by J+K, but can not be divisible by any factors within J, K or J+K, besides P.

$\quad\quad$ **Axiom 3:** *with the precondition that J+K = 0 Mod P, with P being an odd prime number,* $\quad\quad\quad\quad\quad\quad$ *SGC2*
$\quad\quad$ When $J^P + K^P$ is divided by J+K the result can only contain a single factor of P, any other
$\quad\quad$ possible factors of P, must be contained within J+K.

Further information regarding this Lemma is explained in the "Elucidation on the T3 Lemma" paper.

Binomial Expansion &
Subduction of $J^P + K^P$

It is generally well known in number theory, proper factoring of $J^P + K^P$, and limits of prime cofactors when J and K are coprime. However this common knowledge is repeated below in a somewhat abbreviated form. I use the term Subduction here, as an indication of the application of subtractive and deductive reasoning processes.

And obviously, the same method of proof would apply to $J^P - K^P$

Similar to the form on pages 1 to 4, $J^{P-1} - J^{P-2}K + J^{P-3}K^2 \ldots K^{P-1}$ is simply represented by $f(J,K)$.

For the case P=5 as an example, it is given

$J^P + K^P$ Factors Into:
$$(J+K)(J^4 - J^3K + J^2K^2 - JK^3 + K^4)$$
However $(J+K)$ can not have any prime co-factor within $(J^4 - J^3K + J^2K^2 - JK^3 + K^4)$ except P as follows,

If attempting to divide J+K into $(J^4 - J^3K + J^2K^2 - JK^3 + K^4)$,   *(this detailed on pg 6 to right)*

J+K Long Division   Coefficients only shown

	1	-1	1	-1	1
Subtr J³(J+K)* 1	1	1			

	0	-2			
Subtr J²K(J+K)* -2		-2	-2		

		0	3		
Subt JK²(J+K)* 3			3	3	

			0	-4	
Subt K3(J+K)* -4				-4	-4

				0	5

Here the remainder (*AKA residue*) is $5K^4$. Similarly, by successive J+K factor subtraction (*long division*), the remaining may be shown alternately as $5J^4$ or $5J^2K^2$.

The remainder is not fully divisible into J+K.

However it is easy to show any prime cofactors would need to exist between J+K and (*with symmetrical form*) $5J^2K^{2.}$,

Thus $\dfrac{5J^2K^2}{J+K}$ would have to have these cofactors.

The only cofactor can be P (*or 5 in this case*).
J^2 and K^2 can not contain any cofactors to J+K, by reciprocity.
Such that $\dfrac{J+K}{JK}$ can not have any cofactors since

it can be rewritten/understood that K is stated to be relatively prime (*coprime*) to J.

Then due to the simplicity of the subduction process:

$$\frac{PJK}{J+K}$$ may only have a single cofactor of P.

Thus J^P+K^P can only be factored as:

       Case 1: $(J+K) \cdot f(J,K)$ with no common factor P
Or   Case 2: $(J+K) \cdot f(J,K)$ with a common factor P

With $f(J,K)$ only able to contain a single factor of P

Detailed example of long division by J+K shown below, for clarity of understanding:

$$J^4 - J^3K + J^2K^2 - JK^3 + K^4 \ / \ (J + K)$$

$J^4 - J^3K + J^2K^2 - JK^3 + K^4$
$- \ J^3 (J+K)$
_____
   $- 2J^3K + J^2K^2$
   $+ \ 2J^2K \ (J+K)$ *(note, -1 * -1 = +1)*
_____
       $3J^2K^2 - JK^3$
       $- \ 3JK^2 \ (J+K)$
_____
           $- 4JK^3 + K^4$
           $+ \ 4K^3 \ (J+K)$ *(note, -1 * -1 = +1)*
_____
             $5K^4$

Thus showing that P, in this case 5, is the only remainder when divided by J + K, similarly if dividing right to left the remainder will be $5J^4$, and if dividing symmetrically from both ends simultaneously, the result will be $5J^2K^2$. In all 3 cases, the only possible cofactor to J +K is 5 in essence P.

This T3 Lemma is fundamentally written to show that there are no possible common factors between A_1, A_2, B_1, B_2, C_1 and C_2 except the possibility of a factor of P.

I coined the term "Subduction" as being Subtraction/Deduction combined.

It should be somewhat obvious from the above analysis that if $J^P + K^P$ can not have a *single* factor of P, since both factors of it must contain a factor of P. Of course J + K could contain multiple factors of P, but f_A(J,K,P) may only contain a single factor of P.

The long division presented above, dividing J + K into f_A(J,K,P), can be done from left to right, right to left or may simultaneously be approached from both left and right sides. Although it is clearly intuitively obvious that J+K can not divided into f_A(J,K,P) with the exception of factor P, this Lemma drives the point home using Long Division.

My first writeup on this in my NoteBook was for the case P = 7, with the Long division approached from both left and right sides simultaneously. Quite naturally, the residue was $7J^3K^3$.

BASE CAMP REVIEW:

<u>Presentations of D for SGC1</u>   **E13a**

$$D = A + B + C = P A_1 B_1 C_1 K_0 = \frac{(A + B) + (B + C) + (A + C)}{2} = \frac{C_1^P + A_1^P + B_1^P}{-2}$$

$$D = A + B + C = A - A_1^P = A_1(A_2 - A_1^{P-1}) = B - B_1^P = B_1(B_2 - B_1^{P-1}) = C - C_1^P = C_1(C_2 - C_1^{P-1})$$

$A_2 = B_2 = C_2 = 1 \bmod P$ (see **E11f**)

<u>Presentations of D for SGC2,</u> ($C = PC_1C_2$)   **E13b**

$$D = A + B + C = P A_1 B_1 C_1 K_0 = \frac{(A + B) + (B + C) + (A + C)}{2} = \frac{\mathbf{P^{P-1}} C_1^P + A_1^P + B_1^P}{-2}$$

$$A + B + C = A - A_1^P = A_1(A_2 - A_1^{P-1}) = B - B_1^P = B_1(B_2 - B_1^{P-1}) = C - \mathbf{P^{P-1}} C_1^P = \mathbf{P} C_1(C_2 - \mathbf{P^{P-2}} C_1^{P-1})$$

$A_2 = B_2 = 1 \bmod P$ (see **E11f**)

The Apex Proof

Now that you have persevered through an arduous climb of historic proportions, struggled thru a labyrinth of abstruse equations, and finally reached the plateau where we may climb the final ascent, it is clear your strong determination to succeed in climbing Mount Everest is ever-shining.

Pierre Fermat himself, if were here today, would be proud of you. The final 100 meters of ascent will take us to the apex.

We will start the analysis at $P = 5$.

$$D = A+B+C = A - A_1^5 = B - B_1^5 = C - C_1^5 = 5A_1B_1C_1K_0 \qquad \textbf{E14a}$$

$$A - A_1^5 = B - B_1^5 \qquad \textbf{E14b}$$

$$A_1^5 - B_1^5 = A - B \qquad \textbf{E14c}$$

$$(A_1 - B_1)(A_1^4 + A_1^3B_1 + A_1^2B_1^2 + A_1B_1^3 + B_1^4) = A - B = A_1A_2 - B_1B_2 \qquad \textbf{E14d}$$

$$A_1^4 + A_1^3B_1 + A_1^2B_1^2 + A_1B_1^3 + B_1^4 = \frac{A_1A_2 - B_1B_2}{A_1 - B_1} \qquad \textbf{E14e}$$

Evaluation of the RHS above equation. $\qquad \dfrac{A_1A_2 - B_1B_2}{A_1 - B_1} \qquad \textbf{E14f}$

We may surmise with a quick inspection that in order for $A_1 - B_1$ to be divisible into $A_1A_2 - B_1B_2$, if $A_2 = B_2$ then the denominator would divided into the numerator, however in this case A and B would not be coprime. In a more general sense,

$(A_1 - B_1)(A_2 + B_2 + X)$ can be shown to be equal to $\textbf{A}_1\textbf{A}_2 - \textbf{B}_1\textbf{B}_2 + (A_1(B_2 + X) - B_1(A_2 + X))$, thus if: $A_1(B_2 + X) - B_1(A_2 + X) = 0 \qquad \textbf{E14g}$

we can use this form to approach the apex proof.

Thus, in order for $\quad \dfrac{A_1A_2 - B_1B_2}{A_1 - B_1} \quad$ to be divisible by $A_1 - B_1$, it is necessary that $(A_1(B_2 + X) - B_1(A_2 + X)) = 0 \qquad \textbf{E14h}$

As stated earlier in the Base Camp Foundation, A_2, B_2 and C_2 for SGC1 must all be of the form 1 Mod P. And this is certainly obvious, with a fundamental understanding of the form of Fermat's Last Theorem.

$$A_1(B_2 + X) - B_1(A_2 + X) = 0 \qquad\qquad \textbf{E15a}$$

$$A_1(B_2 + X) = B_1(A_2 + X)$$

Since A_1 must be coprime to B_1: $\qquad A_1 = A_2 + X \qquad$ and $\qquad B_1 = B_2 + X \qquad\qquad$ **E15b**

Now we can solve for X: $\quad X = A_1 - A_2 = B_1 - B_2 \qquad\qquad$ **E15c**

Let's remove X and rearrange: $\quad A_1 - B_1 = A_2 - B_2 \qquad\qquad$ **E15d**

From here we see the big affect of the A_1 coprimeness to B_1 step above, which now shows that since $A_2 - B_2$ is 0 Mod P, then it must also be true that A_1 is congruent to B_1, which if we now rotate the 3 variables A, B and C and present in SGC1 form, we get:

$$A_1 = B_1 = C_1 \text{ Mod P} \qquad\qquad \textbf{E15e}$$

All 3 variables are congruent (*same modulus of P*) thus analyzing from the basic presentation of D:

$A_1A_2 + B_1B_2 + C_1C_2 = 0$ Mod 5, we can see a dilemma, since A2, B2 and C2 are equal to 1 Mod 5, there can be no solution to the D equation, if P is >= 5. Only for the case of P = 3 is there any imperfection in the analysis, in SGC1. And this specific exception will be proved later on in this paper, on a future rewrite.

For SGC1: Reductio Ad Absurdum P=5, and by logical extension all other primes greater than 5.

Now for SGC2 we will find if the factor of P resides in C, that $A_1 = B_1$ Mod P, and we will see that for the D basis equation:

$A_1A_2 + B_1B_2 + C_1PC_2 = 0$ Mod P, that A and B are congruent, then this leads to $A_1A_2 + B_1B_2 \neq 0$ Mod P, and we can surmise then that for SGC2 that the proof will stand for all prime exponents >= 3.

For SGC2: Reductio Ad Absurdum P=3, and by logical extension all other primes greater than 3.

ADDENDUM

-A- STATEMENTS of EXPANSIONS of FERMAT'S LITTLE THEOREM:

A^P = A Mod P, is a typical way of writing Fermat's Little Theorem, it therefore thru induction it holds that A^{P-1} = 1 Mod P.
And now since A^0 = 1 Mod P and A^{P-1} = 1 Mod P, we can determine the periodicity which is P-1, thus we may write

$$A^{K(P-1)+1} = A \text{ Mod } P$$

If we look at a simplified case of P = 5, we can understand that A Mod P will occur at N = 0, 5, 9, 13, 17 … as K is incremented. The best way to attain great clarity of this concept is to observe some "output" from a few Libre Office worksheets, presented below:

Modulus of Prime Number 3
Periodicity is 3 - 1

N	0	1	2
N = 13	0	1	2
N = 12	0	1	1
N = 11	0	1	2
N = 10	0	1	1
N = 9	0	1	2
N = 8	0	1	1
N = 7	0	1	2
N = 6	0	1	1
N = 5	0	1	2
N = 4	0	1	1
N = 3	0	1	2
N = 2	0	1	1
N = 1	0	1	2
N = 0	0	1	1

Modulus of Prime Number 5
Periodicity is 5 - 1

N	0	1	2	3	4
N = 13	0	1	2	3	4
N = 12	0	1	1	1	1
N = 11	0	1	3	2	4
N = 10	0	1	4	4	1
N = 9	0	1	2	3	4
N = 8	0	1	1	1	1
N = 7	0	1	3	2	4
N = 6	0	1	4	4	1
N = 5	0	1	2	3	4
N = 4	0	1	1	1	1
N = 3	0	1	3	2	4
N = 2	0	1	4	4	1
N = 1	0	1	2	3	4
N = 0	0	1	1	1	1

Modulus of Prime Number 7
Periodicity is 7 - 1

N	0	1	2	3	4	5	6
N = 13	0	1	2	3	4	5	6
N = 12	0	1	1	1	1	1	1
N = 11	0	1	4	5	2	3	6
N = 10	0	1	2	4	4	2	1
N = 9	0	1	1	6	1	6	6
N = 8	0	1	4	2	2	4	1
N = 7	0	1	2	3	4	5	6
N = 6	0	1	1	1	1	1	1
N = 5	0	1	4	5	2	3	6
N = 4	0	1	2	4	4	2	1
N = 3	0	1	1	6	1	6	6
N = 2	0	1	4	2	2	4	1
N = 1	0	1	2	3	4	5	6
N = 0	0	1	1	1	1	1	1

Modulus of Prime Number 11
Periodicity is 11 - 1

N = 21	0	1	2	3	4	5	6	7	8	9	10
N = 20	0	1	1	1	1	1	1	1	1	1	1
N = 19	0	1	6	4	3	9	2	8	7	5	10
N = 18	0	1	3	5	9	4	4	9	5	3	1
N = 17	0	1	7	9	5	3	8	6	2	4	10
N = 16	0	1	9	3	4	5	5	4	3	9	1
N = 15	0	1	10	1	1	1	10	10	10	1	10
N = 14	0	1	5	4	3	9	9	3	4	5	1
N = 13	0	1	8	5	9	4	7	2	6	3	10
N = 12	0	1	4	9	5	3	3	5	9	4	1
N = 11	0	1	2	3	4	5	6	7	8	9	10
N = 10	0	1	1	1	1	1	1	1	1	1	1
N = 9	0	1	6	4	3	9	2	8	7	5	10
N = 8	0	1	3	5	9	4	4	9	5	3	1
N = 7	0	1	7	9	5	3	8	6	2	4	10
N = 6	0	1	9	3	4	5	5	4	3	9	1
N = 5	0	1	10	1	1	1	10	10	10	1	10
N = 4	0	1	5	4	3	9	9	3	4	5	1
N = 3	0	1	8	5	9	4	7	2	6	3	10
N = 2	0	1	4	9	5	3	3	5	9	4	1
N = 1	0	1	2	3	4	5	6	7	8	9	10
N = 0	0	1	1	1	1	1	1	1	1	1	1

Modulus of Prime Number 13
Periodicity is 13 - 1

N = 25	0	1	2	3	4	5	6	7	8	9	10	11	12
N = 24	0	1	1	1	1	1	1	1	1	1	1	1	1
N = 23	0	1	7	9	10	8	11	2	5	3	4	6	12
N = 22	0	1	10	3	9	12	4	4	12	9	3	10	1
N = 21	0	1	5	1	12	5	5	8	8	1	12	8	12
N = 20	0	1	9	9	3	1	3	3	1	3	9	9	1
N = 19	0	1	11	3	4	8	7	6	5	9	10	2	12
N = 18	0	1	12	1	1	12	12	12	12	1	1	12	1
N = 17	0	1	6	9	10	5	2	11	8	3	4	7	12
N = 16	0	1	3	3	9	1	9	9	1	9	3	3	1
N = 15	0	1	8	1	12	8	8	5	5	1	12	5	12
N = 14	0	1	4	9	3	12	10	10	12	3	9	4	1
N = 13	0	1	2	3	4	5	6	7	8	9	10	11	12
N = 12	0	1	1	1	1	1	1	1	1	1	1	1	1
N = 11	0	1	7	9	10	8	11	2	5	3	4	6	12
N = 10	0	1	10	3	9	12	4	4	12	9	3	10	1
N = 9	0	1	5	1	12	5	5	8	8	1	12	8	12
N = 8	0	1	9	9	3	1	3	3	1	3	9	9	1
N = 7	0	1	11	3	4	8	7	6	5	9	10	2	12
N = 6	0	1	12	1	1	12	12	12	12	1	1	12	1
N = 5	0	1	6	9	10	5	2	11	8	3	4	7	12
N = 4	0	1	3	3	9	1	9	9	1	9	3	3	1
N = 3	0	1	8	1	12	8	8	5	5	1	12	5	12
N = 2	0	1	4	9	3	12	10	10	12	3	9	4	1
N = 1	0	1	2	3	4	5	6	7	8	9	10	11	12
N = 0	0	1	1	1	1	1	1	1	1	1	1	1	1

Now Let's consider the composite number 5 x 7 = 35

You may note that periodicity is the product of (5 - 1) and (7 – 1) divided by 2, which is 12. And that for the 12th and 24th rows that the Modulus of 35 is only 1 if the input parameter A is coprime to both 5 and 7.

N	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	0
37	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	0
36	1	1	1	1	15	1	21	1	1	15	1	1	1	21	15	1	1	1	1	15	21	1	1	1	15	1	1	21	1	15	1	1	1	1	0
35	1	18	12	9	10	6	28	22	4	5	16	3	27	14	15	11	33	2	24	20	21	8	32	19	30	31	13	7	29	25	26	23	17	34	0
34	1	9	4	11	30	1	14	29	16	25	11	9	29	21	15	16	4	4	16	15	21	29	9	11	25	16	29	14	1	30	11	4	9	1	0
33	1	22	13	29	20	6	7	8	29	20	1	27	13	14	15	1	27	8	34	20	21	22	8	34	15	6	27	28	29	15	6	22	13	34	0
32	1	11	16	16	25	1	21	1	11	30	16	11	1	21	15	11	16	16	11	15	21	1	11	16	30	11	1	21	1	25	16	16	11	1	0
31	1	23	17	4	5	6	28	22	9	10	11	33	27	14	15	16	3	32	19	20	21	8	2	24	25	26	13	7	29	30	31	18	12	34	0
30	1	29	29	1	15	1	14	29	1	15	1	29	29	21	15	1	29	29	1	15	21	29	29	1	15	1	29	14	1	15	1	29	29	1	0
29	1	32	33	9	10	6	7	8	4	5	16	17	13	14	15	11	12	23	24	20	21	22	18	19	30	31	27	28	29	25	26	2	3	34	0
28	1	16	11	11	30	1	21	1	16	25	11	16	1	21	15	16	11	11	16	15	21	1	16	11	25	16	1	21	1	30	11	11	16	1	0
27	1	8	27	29	20	6	28	22	29	20	1	13	27	14	15	1	13	22	34	20	21	8	22	34	15	6	13	7	29	15	6	8	27	34	0
26	1	4	9	16	25	1	14	29	11	30	16	4	29	21	15	11	9	9	11	15	21	29	4	16	30	11	29	14	1	25	16	9	4	1	0
25	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	0
24	1	1	1	1	15	1	21	1	1	15	1	1	1	21	15	1	1	1	1	15	21	1	1	1	15	1	1	21	1	15	1	1	1	1	0
23	1	18	12	9	10	6	28	22	4	5	16	3	27	14	15	11	33	2	24	20	21	8	32	19	30	31	13	7	29	25	26	23	17	34	0
22	1	9	4	11	30	1	14	29	16	25	11	9	29	21	15	16	4	4	16	15	21	29	9	11	25	16	29	14	1	30	11	4	9	1	0
21	1	22	13	29	20	6	7	8	29	20	1	27	13	14	15	1	27	8	34	20	21	22	8	34	15	6	27	28	29	15	6	22	13	34	0
20	1	11	16	16	25	1	21	1	11	30	16	11	1	21	15	11	16	16	11	15	21	1	11	16	30	11	1	21	1	25	16	16	11	1	0
19	1	23	17	4	5	6	28	22	9	10	11	33	27	14	15	16	3	32	19	20	21	8	2	24	25	26	13	7	29	30	31	18	12	34	0
18	1	29	29	1	15	1	14	29	1	15	1	29	29	21	15	1	29	29	1	15	21	29	29	1	15	1	29	14	1	15	1	29	29	1	0
17	1	32	33	9	10	6	7	8	4	5	16	17	13	14	15	11	12	23	24	20	21	22	18	19	30	31	27	28	29	25	26	2	3	34	0
16	1	16	11	11	30	1	21	1	16	25	11	16	1	21	15	16	11	11	16	15	21	1	16	11	25	16	1	21	1	30	11	11	16	1	0
15	1	8	27	29	20	6	28	22	29	20	1	13	27	14	15	1	13	22	34	20	21	8	22	34	15	6	13	7	29	15	6	8	27	34	0
14	1	4	9	16	25	1	14	29	11	30	16	4	29	21	15	11	9	9	11	15	21	29	4	16	30	11	29	14	1	25	16	9	4	1	0
13	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	0
12	1	1	1	1	15	1	21	1	1	15	1	1	1	21	15	1	1	1	1	15	21	1	1	1	15	1	1	21	1	15	1	1	1	1	0
11	1	18	12	9	10	6	28	22	4	5	16	3	27	14	15	11	33	2	24	20	21	8	32	19	30	31	13	7	29	25	26	23	17	34	0
10	1	9	4	11	30	1	14	29	16	25	11	9	29	21	15	16	4	4	16	15	21	29	9	11	25	16	29	14	1	30	11	4	9	1	0
9	1	22	13	29	20	6	7	8	29	20	1	27	13	14	15	1	27	8	34	20	21	22	8	34	15	6	27	28	29	15	6	22	13	34	0
8	1	11	16	16	25	1	21	1	11	30	16	11	1	21	15	11	16	16	11	15	21	1	11	16	30	11	1	21	1	25	16	16	11	1	0
7	1	23	17	4	5	6	28	22	9	10	11	33	27	14	15	16	3	32	19	20	21	8	2	24	25	26	13	7	29	30	31	18	12	34	0
6	1	29	29	1	15	1	14	29	1	15	1	29	29	21	15	1	29	29	1	15	21	29	29	1	15	1	29	14	1	15	1	29	29	1	0
5	1	32	33	9	10	6	7	8	4	5	16	17	13	14	15	11	12	23	24	20	21	22	18	19	30	31	27	28	29	25	26	2	3	34	0
4	1	16	11	11	30	1	21	1	16	25	11	16	1	21	15	16	11	11	16	15	21	1	16	11	25	16	1	21	1	30	11	11	16	1	0
3	1	8	27	29	20	6	28	22	29	20	1	13	27	14	15	1	13	22	34	20	21	8	22	34	15	6	13	7	29	15	6	8	27	34	0
2	1	4	9	16	25	1	14	29	11	30	16	4	29	21	15	11	9	9	11	15	21	29	4	16	30	11	29	14	1	25	16	9	4	1	0
1	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35
N																																			

It's quite mind numbing I suppose. But we can understand the basics of Composite number Exponential Modulus when simply inspecting the above table, and we can thru induction extend these concepts to other composite scenarios.

-B- References and Suggested Reading

George Gamow, "One Two Three, Infinity", 1959
A plain look at the outer-universe, the inner-universe, the expansion of space time, and infinity. Out-of-print, for quite a few years now, good luck finding a copy.

References:

[1] Weisstein, Eric W. "Fermat's Last Theorem". MathWorld. (mathworld.wolfram.com/FermatsLastTheorem.html)

[2] Wikipedia, "Fermat's Last Theorem", (en.wikipedia.org/wiki/Fermat's_Last_Theorem)

[3] Edwards, Harold M. "Fermat's Last Theorem", Scientific American Vol. 239, No. 4 (October 1978), pp. 104-123 (www.jstor.org/stable/24955826)

[5] Hanna Kagele, "Sophie Germain, The Princess of Mathematics and Fermat's Last Theorem"
        https://www.gcsu.edu/sites/files/page-assets/node-808/attachments/kagele.pdf

[6] Colleen Alkalay-Houlihan, "Sophie Germain and Special Cases of Fermat's Last Theorem"
        https://www.math.mcgill.ca/darmon/courses/12-13/nt/projects/Colleen-Alkalay-Houlihan.pdf

Mathematicians thru history whose work is foundational to this exposition.
Wikipedia Links:

Diophantus

Euclid

Pythagoras of Samos
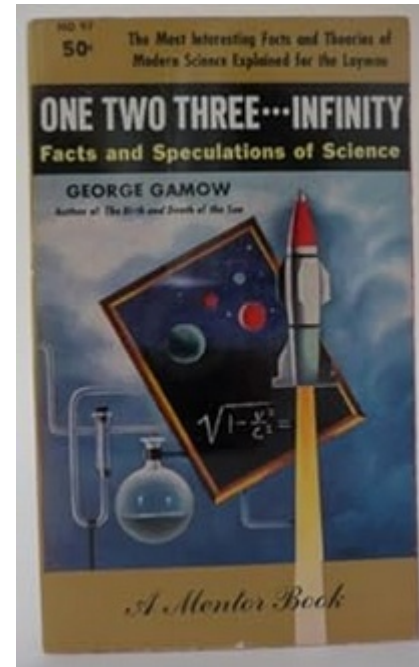
Al-Khwarizmi

Pierre Fermat

Blaise Pascal

Leonard Euler

Sophie Germain

-C- For the near future, I may be contacted by email at: D.Ross.Randolph345@Gmail.com   Feel free to establish contact.
I can assist you with further explanation/clarification of any murky areas within the proof.

-D- Individuals who have assisted me in my quest, who are worthy of my mention

Quoran, Will Jadson of Brazil: A mathematician enthusiast, who has derived an interesting limited case proof to FLT, which is presented in a web page dangling off the sitemap.

Quoran, David Smith of Gloucestershire, UK : Excellent trained mathematician with an inherent curiosity, who was quite central in my proof analysis in the summer of 2024. With simplicity, he demonstrated a fundamental modularity concept, which I needed to absorb, at a deep grey matter cellular level.

Reddit, Edderiofer and Xhiw usernames, unidentified and well intentioned individuals.

arXiv, Giulio Morpurgo, a retired Physicist and Statistician from the EU, who was the first commentator re my early work

CHANGE LOG:

March 3, 2025- A new proof origin is started using Trinomial expansion of S – (A+B).

March 4, 2025 – Added proof for the general case any exponent P.

March 22, 2025 – Resolved previous errors, new theme based upon 5^∞ modulus.

March 23, 2025 – Equation numbering in **light blue bold text** added to pages 4 thru 15.

March 29, 2025 – Fixed a spelling error in Sophie Germain's name.

April 1, 2025 – Axioms added on page 11 for the T3 Lemma.

April 3, 2025 – Major cleanup of the Apex proof section, and a new high clarity Abstract added to page 1.

April 11, 2025 – Cleaned up the old Addendum A, Fermat's Little Theorem presentation, which had 2 small errors.

April 14, 2025 – Big clean up pgs 11,13,14 &15 regarding clarity of A_2, B_2 and C_2 and added Presentations of D Review. Removed most of the fog in the structural explanation/organization towards the end of the proof, starting from the T3 Lemma.

April 17, 2025 - Added a references section to Addendum B, six references for Fermat's Last Theorem and Sophie Germain.

Miscellanea Index
Origin April 5, 2025

Symmetrical versus non-symmetrical variable approach to FLT

The important fact here is that the first published proof for the case P =3 was by Leonard Euler, and he used the symmetrical form:

$A^3 + B^3 + C^3 = 0$, read that somewhere on the www in the last year. Of course the math appears a little more complex when it comes to arriving at the proper polarity/sign when factoring the equations, and it's logical to assign the negative number to C.

The things to be aware of, when dealing with the symmetrical form is that you are not going to take any square roots, where the answer might branch to a positive and a negative value. Additionally, if for instance J + K could equal 1 or -1, then the factoring will be substantially different than if J and K are both positive values.

Once the proof is written in the symmetrical form, transcribing into a non-symmetrical form of $A^P + B^P = C^P$ is not to terribly difficult. But this proof does run out to around 20 pages, so it's going to be some tedious work, and the page count may end up increasing by 2 or 3.

It would sort of be nice if some high school student with an extra 8 hours on his hands would give it a try. Send me an email and I'll send off the Libre Office files to you, if you feel inspired.

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Congruence equality operator and redundancy to the Mod operator
Read a few web postings regarding the congruence equality operator (*3 horizontal line equal sign, instead of the normal 2 horizontal line equal sign*), and apparently it used to be more common to simply use the 2 line equality operator, in the past. Nowadays, you will probably only see the 3 line congruence equality operator. (*Is operator event the right word??*)
Back in the 1960's as a young child, there were 2 horizontal lines for the congruence equality operator, and we learned about integers, modularity, the Goldbach conjecture, and why such as easy to state conjecture would be so hard to prove. A few days later, something else.

Anyway, per the web, whether 2 lines or 3 lines, the exact same meaning, providing the Mod operator is over there on the right-hand side of the equation. where it always is placed.

And you try typing formulas and commentary into Libre Office for hours on end, and try searching for that special congruence symbol. Yeah, I don't have the cash-ola to shell out for a spiffy new Math paper writing software package, or the motivation to learn how to navigate it's various menus. HEY, there's the equal sign right on the keyboard, under the "+" symbol, fast and easy, my kind of girl. Whoops, hope my wife doesn't see this. hehehehe

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Elucidation on the T3 Lemma
Sept 1, 2024   Updated Mar 30th, 2025

There are several cornerstones to the creation of these congruence proofs to Fermat's Last Theory, or Theorem. The following are necessary basics to internalize the structural integrity of the proof:

a) Binomial Expansion, b) Trinomial Expansion, c) Fermat's Little Theorem and d) the T3 Lemma. Of course understanding the modulus operator also a key, and if you've done any computer programming, and maybe even studied how computers work, the modulus operator is pretty much droll stuff, and shouldn't need much explanation.

I'll go into some detail on the T3 Lemma is in this exposition.

My T3 Lemma, uses techniques available in my grade school days in the 1960's. Basic Technique is Long Division, applied to algebraic variables. There are other approaches to showing the same proof for this Lemma, as detailed towards the end of this paper.

In this Lemma the exponent variable is given the symbolic letter P, as is the case in most of the rest of the sections on this website, and the two base coprime variables are J and K. All are integers.

One interesting aspect is that if you analyze the structure the proof is the same regardless of whether an addition or subtraction operator is used between J^P and K^P, if we simply multiply K times negative 1, the proof is the same then. This proof uses an addition operator for the initial condition, being $(J^P + K^P)$. We will determine the factors, and what may not be the factors from this starting point equation.

Below are the 3 Diophantine Axioms we will be proving with this mathematical work.

> **Axiom 1:** *with the precondition that J+K ≠ 0 Mod P, with P being an odd prime number,*
> $J^P + K^P$ is divisible by J+K, and can not be divisible by any factors within J, K, J+K or P.

> **Axiom 2:** *with the precondition that J+K = 0 Mod P, with P being an odd prime number,*
> $J^P + K^P$ is divisible by J+K, and can not be divisible by any factors within J, K or J+K, besides P.

> **Axiom 3:** *with the precondition that J+K = 0 Mod P, with P being an odd prime number,*
> When $J^P + K^P$ is divided by J+K the result can only contain a single factor of P, any other possible factors of P, must be contained within J+K.

The T3 Lemma presented below is from my first FLT paper from Feb 2023.

Binomial Expansion & Subduction of $J^P + K^P$

It is generally well known in number theory, proper factoring of $J^P + K^P$, and limits of prime cofactors when J and K are coprime. However this common knowledge is repeated below in a somewhat abbreviated form. I use the term Subduction here, as an indication of the application of subtractive and deductive reasoning processes.

And obviously, the same method of proof would apply to $J^P - K^P$

Similar to the form on pages 1 to 4, $J^{P-1} - J^{P-2}K + J^{P-3}K^2 \ldots K^{P-1}$ is simply represented by $f(J,K)$.

For the case P=5 as an example, it is given

$J^P + K^P$ Factors Into:
$$(J+K)(J^4 - J^3K + J^2K^2 - JK^3 + K^4)$$
However $(J+K)$ can not have any prime co-factor within $(J^4 - J^3K + J^2K^2 - JK^3 + K^4)$ except P as follows,

If attempting to divide J+K into $(J^4 - J^3K + J^2K^2 - JK^3 + K^4)$, *(this detailed on pg 6 to right)*

```
 J+K Long Division            Coefficients only shown
                            1    -1    1    -1    1
 Subtr J³(J+K)* 1           1     1
                            ---------
                            0    -2
 Subtr J²K(J+K)* -2              -2    -2
                                -----------
                            0     3
 Subt JK²(J+K)* 3                 3     3
                                 -----------
                                 0    -4
 Subt K3(J+K)* -4                      -4    -4
                                      ------------
                                       0     5
```

Here the remainder (*AKA residue*) is $5K^4$. Similarly, by successive J+K factor subtraction (*long division*), the remaining may be shown alternately as $5J^4$ or $5J^2K^2$.

The remainder is not fully divisible into J+K.

However it is easy to show any prime cofactors would need to exist between J+K and (*with symmetrical form*) $5J^2K^2$,

Thus $\dfrac{5J^2K^2}{J+K}$ would have to have these cofactors.

The only cofactor can be P (*or 5 in this case*).
J^2 and K^2 can not contain any cofactors to J+K, by reciprocity.
Such that $\dfrac{J+K}{JK}$ can not have any cofactors since

it can be rewritten/understood that K is stated to be relatively prime (*coprime*) to J.

Then due to the simplicity of the subduction process:

$\dfrac{PJK}{J+K}$ may only have a single cofactor of P.

Thus J^P+K^P can only be factored as:

Case 1: $(J+K) \cdot f(J,K)$ with no common factor P
Or   Case 2: $(J+K) \cdot f(J,K)$ with a common factor P

With $f(J,K)$ only able to contain a single factor of P

Detailed example of long division by J+K shown below, for clarity of understanding:

$$J^4 - J^3K + J^2K^2 - JK^3 + K^4 \ / \ (J + K)$$

$$J^4 - J^3K + J^2K^2 - JK^3 + K^4$$
$$- \ J^3 \ (J+K)$$

$$- \ 2J^3K + J^2K^2$$
$$+ \ 2J^2K \ (J+K) \qquad\qquad (note, -1 * -1 = +1)$$

$$3J^2K^2 - JK^3$$
$$- \ 3JK^2 \ (J+K)$$

$$- \ 4JK^3 + K^4$$
$$+ \ 4K^3 \ (J+K) \qquad (note, -1 * -1 = +1)$$

$$5K^4$$

Thus showing that P, in this case 5, is the only remainder when divided by J + K, similarly if dividing right to left the remainder will be $5J^4$, and if dividing symmetrically from both ends simultaneously, the result will be $5J^2K^2$. In all 3 cases, the only possible cofactor to J +K is 5 in essence P.

Now while during the origination of the T3 Lemma, I believed it would be easy for people to "see", this was not the case. It may be because long division is not taught to 9 year olds in school anymore? Or it was just poorly written?, probably the second reason.

In either case, hopefully this new long exposition adds some clarity. A good source of understanding for me has always been to take a pencil and a piece of paper, and work thru the problem in my own way. Which is what I would advise any reader who gets this far to do. Of course if you have a PhD in advanced mathematics you are probably well aware of factoring this exponential form.

David Smith, a Quoran and amateur mathematician from the UK pointed out that the T3 Lemma "needs some work". Sort of non-professional math professor looking. Well I'm an engineer, and number theory, is really just a hobby of mine. But Dave S pointed out a few supporting proofs of the T3 Lemma, listed below.

Apparently this concept is reviewed in "An Introduction to the Theory of Numbers" (*Niven, Zuckerman & Montgomery*) in section 1.2 on problem 51:

Show that if $\gcd(a, b) = 1$ and p is an odd prime, then

$$\gcd\left(a + b, \frac{a^p + b^p}{a + b}\right) = 1 \text{ or } p$$

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Additionally noted by Dave S is a response from Brian Sittinger, a PhD in mathematics from the University of California:

If $\gcd(a, b) = 1$ and p is an odd prime, how would you prove that

$$\gcd\left(\frac{a^p + b^p}{a + b}, a + b\right) \neq 1$$

if and only if $p | (a + b)$?

Brian Sittinger

PhD in Mathematics, University of California, Santa Barbara (Graduated 2006) · Upvoted by
Yair Livne, Master's Mathematics, Hebrew University of Jerusalem (2007) · 4y

Hence, reducing modulo $a - b$ yields

$$\frac{a^p - b^p}{a - b} \equiv pa^{p-1} \equiv pb^{p-1} \bmod a - b.$$

Therefore, we obtain

$$\gcd\left(\frac{a^p - b^p}{a - b}, a - b\right) = \gcd(a - b, pa^{p-1}, pb^{p-1})$$
$$= \gcd(a - b, \gcd(pa^{p-1}, pb^{p-1}))$$
$$= \gcd(a - b, p \gcd(a^{p-1}, b^{p-1}))$$
$$= \gcd(a - b, p \gcd(a, b)^{p-1})$$
$$= \gcd(a - b, p \cdot 1), \text{ since } \gcd(a, b) = 1$$
$$= \gcd(a - b, p).$$

Since this holds true for any integers a and b as long as $\gcd(a, b) = 1$, by replacing b
with $-b$, noting that $\gcd(a, -b) = 1$, and p being an odd prime, we obtain

$$\gcd\left(\frac{a^p + b^p}{a + b}, a + b\right) = \gcd(a + b, p).$$

The desired claim directly follows from this last statement.

Anyway, I guess that pretty much covers the T3 Lemma from various perspectives.

One of the most important points as I introduced on page 1, is that if the analysis of a Fermat Last Theorem riddle is based upon Sophie Germain's Case 1 condition, that we can predict to a certain extent the factors of $J^P + K^P$, using Axiom 1 and Axiom 2.

And if the FLT scenario we are analyzing is based upon Sophie Germain Case 2 conditions, then Axiom 3, Axiom 4 and Axiom 5 will allow us some good solid insight as to what these factors can be.

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T5 Lemma
September 27, 2024

This Lemma will demonstrate that for coprime variable G and H where $G^P + H^P = (G + H)(G^{P-1} - G^{P-2}H + G^{P-3}H^2 - \ldots\ldots + G^2H^{P-3} - GH^{P-2} + H^{P-1})$
where G +H does not have a factor of P, that $G^{P-1} - G^{P-2}H + G^{P-3}H^2 - \ldots\ldots + G^2H^{P-3} - GH^{P-2} + H^{P-1}$
may only be composed of prime factors of type 1 Mod 2P.

We will simplify presentation of $G^{P-1} - G^{P-2}H + G^{P-3}H^2 - \ldots\ldots + G^2H^{P-3} - GH^{P-2} + H^{P-1}$ as function f(G, H, P).

Proof:

We will examine two cases,
      1) Prime Factors of the type 1 Mod 2P, and
      2) Prime Factors not of the type 1 Mod 2P and P itself

Case 1,
Consider the example for P = 5, prime factors of the type 1 Mod 2P will be 11, 31, 41, 61, 71, 101…

Based upon Fermat's Little Theorem, f(G, H, P) must be equal to 1 Mod P, and all prime factors are of the type 1 Mod 2P then the product of these factors will also be 1 Mod P.

Case 2,

If we consider the example for P = 5, any prime factors not of the type 1 Mod 2P or P itself,

Suppose that f(G, H, P) could have a prime factor of 7 and another prime factor of 3. When multiplied they are 21, which is 1 Mod 2P. In this case is it possible to show that 3 and 7 could be possible prime factors of f(G, H, P)?

Let us look at prime factor 7 first, and at $G^{25} + H^{25}$.

Since $G^{25} + H^{25}$ can be factored as ($G^5 + H^5$)f(G^5, H^5, 5) we can assume if $G^5 + H^5$ has a factor of 7 that $G^{25} + H^{25}$ will have a factor of 7.

However, analyzing Fermat's Little Theorem for P = 7, shows that 25 = 4(7-1) + 1 so that $G^{25} + H^{25}$ = G + H Mod 7, thus if $G^{25} + H^{25}$ contains a factor of 7, it would also be true that G + H would have a factor of 7. This proves that 7 can not be a factor of f(G, H, 5), (*and logical application of Lemma T3*).

Using similar logic we can show that the prime number 3 can not be a factor of f(G, H, 5).

Any factor which is not of the form 1 Mod 2P can be shown to not be a possible factor of f(G, H, P), in a similar fashion.

And any prime factor of the form 1 Mod 2P can be shown to not be able to be factored into G + H.

An algebraic equation can be determined from the above methodology to find integer X, where:

$G^X + H^X$ can be used to prove that:

f(G, H, P) can not have a prime factor P_2 where $P_2 \neq$ 1 Mod P and is not P itself.

For the time being I will leave it to the somewhat advanced Number theorist who may be studying this Lemma to derive this algebraic formula tying the value of P to the value of P_2, thru the determination of the value of exponent X.

Please feel free to email me the formula, should you find thew 60 minutes needed to derive it, and test it for a few simple cases.

D. Ross Randolph   D.Ross.Randolph@gmail.com

PS+++ 2024-9-28

We can determine that X <= $(P_2 - 1)(2P)$, as $P_2 - 1$ is the frequency of one ring, and $2P$ is the frequency of the other ring.

Similar to the Totient concept, due to the 2 factor in both rings we can remove one 2 and show X <= $(P_2 - 1)(P)$

Below I have reversed P and P_2, showing P = 7 and P_2 = 5 to demonstrate determination of X.

Below are my charted results:

	P = 5, P_2 = 7	P=7, P_2 = 5
Equivalence 1	X = 25	X = 21
Equivalence 2	X = 55	X = 49
Equivalence 3	X = 85	X = 77
Periodicity	6x5 = 30	4x7 = 28

This can be shown for any two odd primes where $P_2 \neq 1$ Mod 2P

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T9 Lemma, an infinity of primes congruent to 1 Mod P1

Orig Jan 28, 2025

We may show that there are an infinitude of factors of the form 1 Mod P, for any prime number P as follows:

$$A^P + B^P = (A+B)(A^{P-1} - A^{P-2}B \ldots\ldots - AB^{P-2} + B^{P-1})$$

and Per Lemma T5, all factors of the equation $(A^{P-1} - A^{P-2}B \ldots\ldots - AB^{P-2} + B^{P-1})$ will be of the form 1 Mod P, for the following set of conditions:

P is a positive prime >= 3
A+B is Coprime to P
A is Coprime to B

As an example with some small prime numbers:

$(11^5 + 1^5) / (11 + 1) = 13421$, single prime factor

$(13^5 - 7^5) / (13-7) = 72611$,   11 x 41 x 131, 3 prime factors

Easy to see factors of 1 Mod 5, in decimal form as above. Lets use P = 7 as below:

$(3^7 + 2^7) / (3 + 2) = 463$, single prime factor = (7 x 66) + 1

$(6^7 - 5^7) / (6 - 5) = 201811 = 29$ x 6959, 29 = (7 x 4) + 1, 6959 = (7 x 994) + 1

As mentioned Lemma T5 proves that the only factors possible must be of the form 1 Mod P, with P being the exponent input.

Thus if we were perhaps uncertain if there was an infinitude of factors of the form 1 Mod 586577,
we could simply apply an algorithm as follows:

(*Note, 586577 and 586429 are both somewhat large prime values*)

$(586429^{586577} + 1) / (586429 + 1)$

All of the factors of this equation will be far larger than 586429.

If we were to then take the largest of these prime factors, which we will call X, and then iterate the algoritm as follows:

$(X^{586577} + 1) / (X + 1)$, we will then find a new group of congruent factor(s), we may again take the largest factor of the group, and repeat this algoritm infinitely many times, thus showing there are infinite prime numbers of the form   1 Mod 586577.

Thus there are an infinite number of primes of the form 1 Mod P, for any prime value P.

Both Lemma T3 and Lemma T5, are necessary tools which form the foundation of the proof.

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in 1837 this was shown in a more generalized form as follows:

Dirichlet's theorem, also called the Dirichlet prime number theorem, states that for any two positive coprime integers *a* and *d*, there are infinitely many primes of the form *a + nd*, where *n* is also a positive integer.

https://en.wikipedia.org/wiki/Dirichlet%27s_theorem_on_arithmetic_progressions

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N=4 and N=3 Proofs Foreword

As noted by math historians Fermat is credited with determining there can be no solution to the equation $X^4 + Y^4 = Z^4$ by the method of infinite descent. An alternate method for N=4 is presented at the short 1 page paper location below.

The N = 4 Proof

Note, the N=4 proof suffices to prove FLT for any exponent which is a power of 2.

Euler is generally credited with being the first mathematician to prove that there was no solution to the equation $X^3 + Y^3 = Z^3$ about a hundred years later. His proof uses the sq rt of -3, as a figurative fulcrum. I do not have a brief proof for SGC2 for this segment.

The N = 3 Proof *(specific to SGC1 initial conditions)*

Note, these two papers use a congruence distillation process in the presentation, based upon a technique I developed after working with the amateur mathematician Dave Smith from the UK about 8 months ago, as well as Edderiofer roughly about 6 months ago.

In order to edify and clarify to the reader of this section, the basis of understanding needed to absorb these 2 very short proofs, I have created a short explanation of a key technique, which although simple in concept, may be beneficial for you to study.

DEECA, Diophantine Exponent Equation Congruence Analysis
Mar. 28, 2025   D.Ross.Randolph   Last Edit Mar. 28. 205

Use of symbolic math language allows us to condense relatively complex math formulas into easier to absorb structures:

The exponent nature of the Fermat equation requires a simple way of inspecting and analyzing the solution proof, which focuses on proving non-congruence of the modulus of a prime number raised to a specific power. Convoluted language, an example will be more clear.

Some basics:

$0 = (0 \bmod P^\infty)$ Very unusual. Since we will base our analysis's on the number times P may be a factor of various terms, we simply are indicating that we can divide zero by P, an infinite number of times.

Now for some more practical examples, with the three K variables all being coprime to exponent P:
Example 1:

$$K_1 P^{17} + K_2 P^{15} = K_3 P^2$$

We could also rewrite this as: $K_1 P^{17} + K_2 P^{15} - K_3 P^2 = 0$

We can solve and demonstrate Reductio Ad Absurdum by dividing all 3 terms by P^2, which will leave us with the following equation:

$$K_1 P^{15} + K_2 P^{13} - K_3 = 0$$

Therefore, we may simplify and write the original equation as:

$(0 \bmod P^{17}) + (0 \bmod P^{15}) - (0 \bmod P^2) \neq (0 \bmod P^\infty)$

Which is perhaps, somewhat easier to absorb. And since (0 Mod P^2) may appear to be ambiguous, as perhaps the zero on the LHS would be also divisible by P^3, we may apply a modified modulus operator, I refer to as **_Limited_** Modulus operator, symbolically: **Mod$_L$**

[(0 Mod P^2) ≠ (0 Mod P^3)] can now be simply indicated as (0 Mod$_L$ P^2)

And now rewrite more precisely: (0 **Mod$_L$** P^{17}) + (0 **Mod$_L$** P^{15}) – (0 **Mod$_L$** P^2) ≠ (0 Mod P^∞)

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Now we contrast this factor of P examination, with the below restructured problem which has a twist.

Example 2:

$K_1P^{23} + K_2P^7 – K_3P^7 = 0$ Where $K_2 ≠ K_3$

(0 Mod P^{23}) + (0 Mod P^7) – (0 Mod P^7) ≠ (0 Mod P^∞)

In this second example, we run into a dilemma, "Does $K_2 = K_3$ Mod P ?

Because if $K_2 = K_3$ Mod P , then $K_2P^7 – K_3P^7 = 0$ Mod P^8, and it could also equal 0 Mod P^9.

So if we are unaware of the congruence relationship between K_2 and K_3, we will be in a quandary.

We could say that $K_2P^7 – K_3P^7$ is equal to 0 Mod P^7 , 0 Mod P^8 , 0 Mod P^9 ….

We just will never be sure, in many cases.

What we can be assured of is that the sum cannot have a factor less than P^7.

So going back to this example:

$K_1P^{23} + K_2P^7 – K_3P^7 = 0$ Where $K_2 ≠ K_3$

If the statement is true as written $K_2P^7 – K_3P^7$ must have a factor of P^{23} to P^∞ ,

However if $K_1P^{23} + K_2P^7 - K_3P^7 \neq 0$ Where $K_2 \neq K_3$

Then it must be true that $K_2P^7 - K_3P^7$ summation has a P factor between P^7 and P^∞.

Thus it is important to recognize when analyzing FLT, that in an equation like the following:

$(0 \text{ Mod } P^{23}) + (0 \text{ Mod } P^7) - (0 \text{ Mod } P^7) \text{ ??? } (0 \text{ Mod } P^\infty)$

That we have an indistinct logic state, that we can not ascertain the existence of a true or false statement without the intimate knowledge of the variables K_2 and K_3, which is typically not known precisely, in many instances.

So now we can write a few Axioms, relating to this simple exponent treatment logic analysis:

Axiom 1: (from example 1)

If a DEEPA (*Diophantine Exponent Equation Congruence Analysis*) equation contains 3 or more factors, and the lowest exponent of the base factor has no like exponential terms, the summed terms will only be divisible by the base term raised to the lowest exponent value.

Axiom 2: (from example 2)

If a DEEPA equation contains 3 or more factors, and the lowest exponent value of the base factor has two (*or more*) like variables raised to that same exponent value, the summed terms will have a base term raised to an unknown exponent value which is equal or greater than the lowest exponent value.

With a secure, and intuitive understanding of these two concepts, interpreting the FLT proofs become much easier to understand.

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N=4 Proof for Fermat's Last Theorem

Mar 24, 2025   D. Ross Randolph

"*It is impossible to separate a cube into two cubes, a fourth power into two fourth powers, or generally, any power above the second into two powers of the same degree*", Fermat wrote this in the margin of his copy of an ancient Greek math book written by Diophantus, titled Arithmetica.

This proposition was first stated as a theorem by Pierre de Fermat around 1637. And it is likely that Fermat used the method of Infinite Descent to prove this statement for N=4.

This method shown below based on indivisibility
of powers of the integer 2.

Can $X^4 + Y^4 = Z^4$ have a finite solution?

X and Y odd integers, Z even integer
$(X^2 + Y^2)^2 - 2X^2Y^2 = Z^4$
$(X^2 + Y^2)^2 = 0 \bmod 4$
$2X^2Y^2 \quad = 0 \bmod 2 \neq 0 \bmod 4$
$Z^4 \qquad = 0 \bmod 16$
(0 Mod 4) + (0 Mod 2) = (0 Mod 2) ≠ (0 Mod 16)

Z and Y odd integers, X even integer
$(Z^2 - Y^2)^2 + 2Z^2Y^2 = X^4$
$(Z^2 - Y^2)^2 \ = 0 \bmod 4$
$2Z^2Y^2 \quad = 0 \bmod 2 \neq 0 \bmod 4$
$X^4 \qquad = 0 \bmod 16$
(0 Mod 4) + (0 Mod 2) = (0 Mod 2) ≠ (0 Mod 16)

Reductio Ad Absurdum

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N=3 Proof for Fermat's Last Theorem, for Sophie Germain Case 1

Mar 25, 2025　　　　　　　　D. Ross Randolph　　　　　　　　Updated April 1, 2025

Can $A^3 + B^3 + C^3 = 0$ (*C being negative*) have a finite solution?

Sophie Germain Case 1, none of the coprime variables have a factor of 3
$A+B+C = 0$ Mod 3　(by virtue of Fermat's Little Theorem)
$(A+B+C)^3 = A^3 + B^3 + C^3 + 3(A+B)(B+C)(C+A)$

　　　　$(A+B+C)^3$　　　　　　　　　　　　　　$(A+B)(B+C)(C+A)$　　　*(8 terms)*

　　　　　　　C^3　　　　　　　　　　　　　　　　C^3
　　　　　　　1　　　　　　　　　　　　　　　　**0**
　　　　　3　　**3**　　　　　　　　　　　**1**　　**1**
　　　3　　**6**　　**3**　　　　　　　**1**　　**2**　　**1**
　1　　**3**　　**3**　　**1**　　　　**0**　　**1**　　**1**　　**0**
A^3　　　　　　　　　B^3　　　A^3　　　　　　　　　B^3

From the above trinomial expansion diagrams which *only show* the coefficients, we can easily conceptualize, that if we multiply $(A+B)(B+C)(C+A)$ by 3, and then add the corner coefficients for
A^3, B^3 and C^3 that the resulting diagram will be equal to $(A+B+C)^3$.

Next,

If $A^3 + B^3 + C^3 = 0$ then,
$(A+B+C)^3 = 3(A+B)(B+C)(C+A)$

$(A+B+C)^3 = 0$ Mod 27
$3(A+B)(B+C)(C+A) = 0$ Mod 3 $\neq 0$ Mod 9

0 Mod 27 $\neq$ 0 Mod 3　　　　　　**Reductio Ad Absurdum**

　　　　　　　　　88888888888888888888888　　888888888888888888888　　　888888888888888888888

It should be noted, that regardless of the number of factors of P within A+B+C, *it is self-evident that the proof will be solid.*

Additionally, the *Apex proof on page 15 equation E15e* proves FLT for Sophie Germain Case 1, for the general case of any odd prime exponent >= 5, but it can not cover the case of P = 3, thus the need for the above proof to cover all possible odd prime exponents.

Regarding the potential extension of this SGC1 proof for n=3 to the general case. Previously I had erroneously considered that this would be possible, however a recent careful analysis showed that not to be possible, in any simplistic fashion.

<u>Summary Remark:</u>
While it is not known if Fermat had determined a solution for N=3, in several letters sent to his math nerd contemporaries, he asked the question, do you know how to solve the problem of 2 cubes summing to the value of a single cube.

Now this could be interpreted two ways. (*Note this is historical conjecture.*)
  a) Fermat wanted to know of such a method.
  b) Fermat was quizzing his friends to see if they had determined a solution which Fermat already knew of.

In support of b), in these letters he also asked a few of his math nerd friends if they knew a method for the proof of N = 4, when indeed he had already found a proof by the method of Infinite Descent.

<p style="color:red; text-align:center; font-weight:bold">8888888888888888888888888888888888888888888888888888888888888888</p>

TTFOG (To Touch the Face of God)
Origin: Mar 16, 2024

<u>ABSTRACT:</u>
As an engineer, I often find the need to research former scientific developments to understand resolutions to a present problem. Some 10 years back, while trying to correct an audible noise issue with a medical product, which needed to be less than approx 15 or 20 dBa level, I perchance stumbled across the London, Edinburgh and Dublin Philosophical Magazine and Journal of Science publication pages from the 1847 with the topic of Magneto-Striction explained well by Joule.

This download, also contained all article in the Journal from January to June of that year. While skimming thru the various articles in the index, I noted that a Reverend Bronwin, had written extensively on various advanced calculus topics for that esteemed journal. As I read thru a few of his musings, I could not help but make the connection between the learned religious education from the 1800's with the typical sociological religious education exposure to present day philosophers. This made me perhaps question my viewpoint of god/God and the everlife, and the topic of mathematics, which I had heretofore only considered as "the queen of the sciences", and an entirely earthly mentality type of intricate logical exposition.

Perhaps, my view of mathematics changed at this point, to one more philosophical, and adaptable, as to discovering "what already exists" in other realms, but has not yet reached our own realm, mired in the various earthly dependent aspects of our short existence on this planet and plane.

So, I am throwing down the philosophical gauntlet so to speak, and associating this proof with the everafter, and since this proof is somewhat ephemeral, what could possibly be a more logical approach?

Are you sleepy, do you care? Have I got your attention?

In engineering terms or math geometry, a "face" is a surface beyond which we do not know the precise contents. For instance in engineering the "face" may be the outer surface of the housing, which may contain various controls, so when we consider God, we may realize, we do not know what is on the other side of this impermeable membrane of another plane of existence. But is it completely impermeable? Maybe any ideas, concepts or universal understanding can only penetrate thru this barrier in an extremely slow manner. So the allegory is now known, and perhaps it is accurate, that all we do not know and all of the future, is behind an impermeable face/surface.

This proof is of a category, which I define as ephemeral, difficult to prove, difficult to absorb, wispy and cloud-like. Thus the Himalayan expedition to Mount Everest analogy, truly needed for knowledge absorption, must be modified to a back in time "airship" analogy from the mid 1800's, when an airship required a lighter than air gas to remain afloat, and a fearless captain to pilot this ship, I ladies and gentlemen, will be your Captain on this flight to the ethereal reaches of the outer atmosphere, where all people will appears as ants, and the largest edifices created by the human race will appear as small toys.

If I start to write in a "steam punk" style, please understand it is part of the voyage, with no extra cost to you as a passenger. Only just recently, has steam punk crossed my mental horizons, and I am still somewhat enamored of the concept.

Terminology abbreviations:
FLT, Fermat's Little Theorem
FBT, Fermat's Big Theorem, AKA *Fermat's Last Theory*

INTRODUCTION:
After I first became aware of FBT in 1978, after the publication of an article by Scientific American on the topic. I first derived the n=4 proof, by Infinite Descent, a day or two afterwards and decided I wanted to understand the n=3 proof, and I spent a little time on it. Over the next 40 year period, I dabbled with the proof, and found that FLT seemed to help clarify the workings of some of the various formulaic configurations which I was working on. *In reality,* I worked on it, simply to pass the time, especially if I was in a period of great stress in my life. I found math was a simple and relaxing activity, even as a small boy, I was usually far ahead of the classroom studies for algebra and geometry, often spending 2 or 3 hours of time solving a quadratic equation, (*before I knew there was a thing/name referred to as a "quadratic equation"*). So when I noted years ago that FLT could be adapted to composite numbers, but was not particularly useful for solving FBT, I simply put this metaphysical math tool on the side, and forget about it the next day.

Then for some odd reason, 15 months ago time expenditure for FBT grew exponentially (*play on words, hehehe*), at some point on my 5th attempt at summiting Mount Everest, I resurrected the old FLT adaptation of Composite numbers, and found some use for it, in this proof attempt. However, not having the final metaphysical math tool required for the FBT proof, I let this proof attempt go back to sleep, and took a big math

break of a few months. While there were clear indicators "wisps" of inspiration, and a feeling of nearness to a proof, I could not identify a clear path thru the atmosphere, being the clouds were too thick to see thru.

So if you did not sleep thru all of the above, you will now understand the foundation for the BC proof being adaptation of FLT to Composite numbers. We will proceed shortly, after our ground crew releases the cords which bond our airship to terra nova.

April 5, 2025
The BC proof never materialized, the Apex proof, was approximately the 15th attempt at summiting Mt. Everest, but with each subsequent attempt the path became easier to navigate and power thru. The Apex proof, is an amalgam of multiple proof attempts, that were made over a 2 year period. The directory it was written in was Equations\FLT, 2025\\The D Proof. Many other directories with many thousands of words and formulas, and 8 or more bound paper notebooks exist, to attest to this travail.