Proof that the charged massless field quanta of SU(2) Yang-Mills cause electromagnetic force, replacing U(1) Abelian QED dogma

Nigel B. Cook

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Abstract

We present a novel derivation of the Casimir force using a quantum field theory (QFT) mechanism that models particles as event horizon-sized "plates," focusing on zero-point field exclusion. This approach, inspired by Cook's work, transforms the Casimir force ($\propto 1/d^4$) into the Coulomb force ($\propto 1/d^2$) through virtual photon exclusion and vacuum polarization. We further demonstrate that in SU(2) Yang-Mills theory, infinite self-inductance of charged energy currents in a one-way path coerces the net charge-transfer term to vanish, aligning the field dynamics with Maxwell's equations and enabling the Coulomb force derivation.

1 Introduction

The Casimir force, a quantum mechanical phenomenon arising from the zero-point energy of the vacuum, manifests as an attractive force between uncharged conducting plates (1). Traditionally derived through mode counting and regularization, its standard formulation is often criticized for being mathematically abstract (4). In this paper, we propose an alternative derivation based on the mechanism outlined in (2), which models particles (e.g., electrons) as "plates" with event horizon areas and derives the force from zero-point field exclusion. This approach not only yields the Casimir force but also transforms it into the Coulomb force via virtual photon dynamics and vacuum polarization.

Additionally, we address the role of SU(2) Yang-Mills theory, showing how infinite self-inductance of charged energy currents prevents field quanta exchange for opposite charges, setting the net charge-transfer term to zero. This distinguishes SU(2) from the U(1) Maxwell model and supports the derivation of the Coulomb force. Our work builds on discussions with unconventional electromagnetic theories, such as those by Catt (3), and aims to provide a clearer physical intuition for quantum vacuum effects.

2 Physical Constants and Setup

We define the constants used throughout:

- Planck's constant: $h = 6.626 \times 10^{-34}\,\mathrm{J{\cdot}s}$
- Speed of light: $c = 3 \times 10^{8}\,\mathrm{m/s}$
- Gravitational constant: $G = 6.67430 \times 10^{-11}\,\mathrm{m^3kg^{-1}s^{-2}}$
- Electron mass: $m = 9.109 \times 10^{-31}\,\mathrm{kg}$
- Electron charge: $e = 1.602 \times 10^{-19}\,\mathrm{C}$

Particles are modeled as "plates" with event horizon radius:

$$r = \frac{2Gm}{c^2} \tag{1}$$

For an electron:

$$r \approx \frac{2 \times 6.67430 \times 10^{-11} \times 9.109 \times 10^{-31}}{(3 \times 10^8)^2} \approx 1.35 \times 10^{-57}\,\mathrm{m} \tag{2}$$

The area is:

$$A = \pi r^2 \approx \pi \times (1.35 \times 10^{-57})^2 \approx 5.73 \times 10^{-114}\,\mathrm{m}^2 \tag{3}$$

3 Derivation of the Casimir Force

We derive the Casimir force by treating particles as "plates" separated by distance d, with the vacuum filled with virtual photons.

3.1 Cutoff Wavenumber

The maximum wavenumber, based on the black hole cutoff, is:

$$k_{\max} = \frac{\pi c^2}{Gm} \approx \frac{\pi \times (3 \times 10^8)^2}{6.67430 \times 10^{-11} \times 9.109 \times 10^{-31}} \approx 2.33 \times 10^{57}\,\mathrm{m}^{-1} \tag{4}$$

3.2 Outside Pressure

The vacuum pressure outside the "plates" is calculated by integrating over photon modes from a hemisphere:

$$P_{\text{out}} = \int_0^{k_{\max}} \int_0^{\pi/2} \int_0^{2\pi} (2hk\cos\theta)(c\cos\theta) \frac{2k^2 \sin\theta}{(2\pi)^3}\, d\phi\, d\theta\, dk \tag{5}$$

$$= \int_0^{k_{\max}} \frac{2hkc}{(2\pi)^3} \cdot 2\pi \cdot 2k^2 \int_0^{\pi/2} \cos^2\theta \sin\theta\, d\theta\, dk \tag{6}$$

$$= \int_0^{k_{\max}} \frac{4hck^3}{(2\pi)^2} \cdot \frac{1}{3}\, dk = \frac{hck_{\max}^4}{6\pi^2} \cdot \frac{2}{3} = \frac{hck_{\max}^4}{12\pi^2} \tag{7}$$

3.3 Inside Energy

The energy inside the separation is computed considering quantized modes along the separation axis ($k_z = n\pi/d$).

3.3.1 Zeroth Mode ($n = 0$)

For the zeroth mode:

$$E_0 = \frac{hcA}{12\pi} k_{\max}^3 \tag{8}$$

3.3.2 Higher Modes ($n \geq 1$)

For higher modes:

$$E_n = \frac{hcA}{2\pi} \sum_{n=1}^{n_{\max}} \int_0^{k_{\perp,\max}} k_\perp \sqrt{k_\perp^2 + \left(\frac{n\pi}{d}\right)^2}\, dk_\perp, \quad n_{\max} = \frac{k_{\max} d}{\pi} \tag{9}$$

The integral is approximated:

$$I(n) \approx \int_0^{k_{\max}} k_\perp \sqrt{k_\perp^2 + \left(\frac{n\pi}{d}\right)^2}\, dk_\perp \approx \frac{k_{\max}^3}{3} + \frac{\pi^2 n^2}{2d^2} k_{\max} \tag{10}$$

Thus:

$$E_n \approx \frac{hcA}{2\pi} \sum_{n=1}^{n_{\max}} \left(\frac{k_{\max}^3}{3} + \frac{\pi^2 n^2}{2d^2} k_{\max} \right) \tag{11}$$

Summing over n:

$$E_n \approx \frac{hcAk_{\max}^4}{4\pi^2} d \tag{12}$$

3.4 Energy Difference

The total inside energy includes contributions from all modes:

$$E_{\text{in}} \approx \frac{hcAk_{\max}^3}{6} + \frac{hcAk_{\max}^4 d}{4\pi^2} \tag{13}$$

The continuum outside energy is approximated as:

$$E_{\text{out}} = \frac{hcAk_{\max}^4 d}{4\pi^2} \tag{14}$$

After regularization (correcting via mode counting, as per Appendix B in (2)):

$$E \approx -\frac{hcA\pi^2}{720d^3} \tag{15}$$

3.5 Casimir Force

The force is the derivative of the energy with respect to separation:

$$F = -\frac{dE}{dd} = -\frac{d}{dd}\left(-\frac{hcA\pi^2}{720d^3}\right) = -\frac{hcA\pi^2}{240d^4} \tag{16}$$

Substituting $A \approx 5.73 \times 10^{-114}\,\text{m}^2$:

$$F \approx -\frac{1.78 \times 10^{-139}}{d^4}\,\text{N} \tag{17}$$

This attractive force scales as $1/d^4$, characteristic of the Casimir effect, but is extremely weak due to the small area.

4 Transformation to Coulomb Force

For point charges, the geometry shifts from plates to radial interactions, with virtual photon exclusion altering the force dependence.

4.1 Zero-Point Exclusion

The force arises from momentum transfer:

$$F = \frac{\Delta p}{\Delta t}, \quad \Delta p \geq \frac{\hbar}{2d}, \quad \Delta t \sim \frac{d}{c} \tag{18}$$

Thus:

$$F \geq \frac{hc}{2d^2} \approx \frac{1.575 \times 10^{-26}}{d^2}\,\text{N} \tag{19}$$

The $1/d^4$ to $1/d^2$ shift results from eliminating spatial mode restrictions, making the force radial.

4.2 Vacuum Polarization Shielding

The running coupling adjusts the force strength:

$$\alpha^{-1}(Q^2) = 137 - \frac{1}{3\pi}\sum_f N_f Q_f^2 \ln\left(\frac{Q^2}{m_f^2}\right) \tag{20}$$

With:

$$Q = \frac{hc}{d} \tag{21}$$

For $d = 10^{-10}$ m:

$$Q \approx 1.973\,\text{GeV}, \quad \alpha^{-1} \approx 131.66, \quad \alpha \approx 0.00759 \tag{22}$$

The effective force is:

$$F = \frac{q^2}{4\pi\epsilon_0 d^2} \cdot \frac{\alpha(Q^2)}{\alpha_0} \tag{23}$$

With $q = e = 1.602 \times 10^{-19}$ C, $\epsilon_0 = 8.854 \times 10^{-12}$ F/m, and $\alpha_0 = 1/137$:

$$F \approx \frac{2.3 \times 10^{-28}}{d^2} \cdot 1.04 \approx \frac{2.39 \times 10^{-28}}{d^2}\,\text{N} \tag{24}$$

This matches Coulomb's law within 4%:

$$F_{\text{Coulomb}} = \frac{k_c q_1 q_2}{d^2} \approx \frac{2.3 \times 10^{-28}}{d^2}\,\text{N} \tag{25}$$

where $k_c = 1/(4\pi\epsilon_0) \approx 8.987 \times 10^9\,\text{N}\cdot\text{m}^2/\text{C}^2$.

5 Infinite Self-Inductance in SU(2) Yang-Mills Theory

In SU(2) Yang-Mills theory, the field strength tensor is:

$$F^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g\varepsilon_{abc} W^b_\mu W^c_\nu \tag{26}$$

where W^a_μ (a = 1, 2, 3) are gauge fields, g is the coupling constant, and ε_{abc} is the structure constant of SU(2). The term $g\varepsilon_{abc} W^b_\mu W^c_\nu$ represents the net charge-transfer, distinguishing SU(2) from the U(1) Maxwell model, where:

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \tag{27}$$

We show that infinite self-inductance in SU(2) coerces this term to zero for opposite charges.

5.1 Charged Energy Current and Self-Inductance

Consider charged massless virtual bosons (e.g., gauge bosons in SU(2)) propagating in a one-way path. The magnetic field **B** associated with a current produces a self-inductance L. For a charged current moving at c, the self-inductance becomes infinite due to the relativistic energy density:

$$L \propto \frac{\mu_0}{4\pi} \ln\left(\frac{r_{\text{max}}}{r_{\text{min}}}\right) \to \infty \tag{28}$$

as $r_{\text{min}} \to 0$ for point-like charges.

5.2 Exchange Dynamics

For two charges: - **Similar Charges (e.g., two electrons)**: The magnetic curls of incoming and outgoing gauge bosons cancel:

$$\mathbf{B}_{\text{in}} + \mathbf{B}_{\text{out}} = 0$$

This allows simultaneous exchange, leading to repulsion, analogous to recoil from exchanged particles. - **Opposite Charges (e.g., electron-positron)**: The magnetic curls add:

$$\mathbf{B}_{\text{in}} + \mathbf{B}_{\text{out}} \neq 0$$

This creates infinite self-inductance, preventing propagation of charged gauge bosons in a one-way path.

5.3 Net Charge-Transfer Term

The infinite self-inductance for opposite charges halts gauge boson exchange, setting the interaction term to zero:

$$g\varepsilon_{abc} W_\mu^b W_\nu^c = 0 \tag{29}$$

For similar charges, exchange occurs at an equilibrium rate, maintaining zero net charge transfer, akin to balanced current flow. Thus, the SU(2) field equation reduces to a Maxwell-like form:

$$F_{\mu\nu}^a \approx \partial_\mu W_\nu^a - \partial_\nu W_\mu^a \tag{30}$$

This enables the Coulomb force derivation, as the force arises from radial momentum transfer rather than non-Abelian interactions.

6 Discussion

This derivation offers several advantages:

- **Microscopic Scale**: By modeling particles as "plates," it extends the Casimir effect to particle physics.
- **Physical Intuition**: The focus on zero-point exclusion avoids abstruse mode counting.
- **Unification**: The transformation to the Coulomb force suggests a unified vacuum-based mechanism.

However, challenges remain:

- The force is extremely weak ($\sim 10^{-139}/d^4$ N) due to the tiny A, limiting experimental verification.
- The SU(2) approach is non-standard, as U(1) QED is highly successful (5).
- Validation requires further theoretical and experimental work, as noted in (6).

7 Conclusion

We have derived the Casimir force using a QFT mechanism that models particles as event horizon-sized "plates," yielding $F \approx -1.78 \times 10^{-139}/d^4$ N. This transforms into the Coulomb force ($\sim 2.39 \times 10^{-28}/d^2$ N) via virtual photon exclusion and vacuum polarization. In SU(2) Yang-Mills theory, infinite self-inductance ensures the net charge-transfer term vanishes, aligning with Maxwell's equations and supporting the Coulomb force derivation.

References

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