Infinite Decimals Don't Exist

by
Armando M. Evangelista Jr.
arman781973@gmail.com
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ABSTRACT: This short paper will show that the so-called infinite or non-terminating decimals do *not* exist.

Finite Sum of a Geometric Sequence

Let's consider the sum of a finite geometric sequence below

$$S_n = \sum_{k=1}^{n} a_1 r^{k-1} = a_1 + a_1 r + a_1 r^2 + \cdots + a_1 r^{n-1}$$

$$S_n = a_1 \left(\frac{1 - r^n}{1 - r} \right)$$

If n is approaching infinity and $|r| < 1$, then the sum converges to a definite value or limit

$$S = \lim_{n \to \infty} S_n = \lim_{n \to \infty} a_1 \left(\frac{1 - r^n}{1 - r} \right) = \frac{a_1}{1-r} \qquad |r|<1$$

For example:

$$S = \lim_{n \to \infty} \sum_{k=1}^{n} \frac{3}{10^k} = \frac{3}{10} + \frac{3}{10^2} + \cdots + \frac{3}{10^n} = \frac{0.3}{1-0.1}$$

$$S = \frac{1}{3} = \lim_{n \to \infty} 333\ldots3 \times 10^{-n} = 0.333\ldots3$$

The sum S has a definite value and is a *terminating* decimal.

In contrast, the *geometric* series is indeterminate

$$S_\infty = \sum_{k=1}^{\infty} \frac{3}{10^k} = \frac{3}{10} + \frac{3}{10^2} + \frac{3}{10^3} + \cdots$$

$$S_\infty = \frac{3}{10} \left(\frac{1 - 10^{-\infty}}{1 \;-\; 0.1} \right) = \frac{1}{3}(0.999\ldots) = 0.333\ldots$$

$$= 333\ldots \times 10^{-\infty} = \frac{333\ldots}{10^{\infty}} = \frac{\infty}{\infty} = \text{indeterminate}$$

S_∞ is called a *non-terminating* decimal because it has *no* definite value.

Other examples:

1) $\sum_{k=0}^{\infty} \frac{1}{k!} = 1 + \frac{1}{1!} + \frac{1}{2!} + \cdots = 2.718\ldots = \text{indeterminate} \neq e$

2) $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots = 0.7853\ldots = \text{indeterminate} \neq \frac{\pi}{4}$

3) $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots = 0.693\ldots = \text{indeterminate} \neq \ln(2)$

Conclusions:

- Since non-terminating decimals are indeterminate, they therefore do not exist.
- If infinite decimals do not exist, then the Taylor series must be expressed as a limit:

$$f(x) = \lim_{n \to \infty} \sum_{k=0}^{n} \frac{f^k(a)(x-a)^k}{k!}$$

- If infinite decimals don't exist, then there are *no* such things as *real* numbers. Therefore, all numbers are *rational*.

REFERENCES:

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