September 15, 2026

A NEW PERSPECTIVE ON HUBBLE'S LAW THROUGH A FOUR-DIMENSIONAL SPATIAL MODEL: 4-SPHERE-COSMOLOGY

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ABSTRACT

Recent JWST observations of galaxies at redshifts above 14 challenge standard interpretations of cosmic distances based on the FLRW metric. This motivates a re-examination of the conceptual foundations linking expansion, redshift, and cosmological geometry. Here I explore an alternative framework in which the Universe is modeled as the three-dimensional surface of a hypersphere expanding at a constant rate, with radius $r = ct$. In this scenario, matter appears as a discontinuity within a geometry governed by the CMB, preserving Hubble's law while offering a finite yet unbounded Universe without invoking dark matter or dark energy. The model introduces a distinct interpretation of galactic recession and provides testable predictions through supernova distance measurements, including those accessible with JWST. Its broader implications concern the role of the CMB, the status of the cosmological constant, and the gravity–expansion balance encoded in the Hyperspherical Expansion Acceleration (HEA).

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INTRODUCTION

Recent JWST detections — including the JADES-GS-z14-0 galaxy at $z \approx 14.3$— place unexpected pressure on FLRW-based distance and age estimates, as such systems appear markedly more massive and more evolved than the cosmic ages predicted by the standard metric would allow. These findings invite a reconsideration of how cosmological distances and redshifts are interpreted, particularly given the central role played by the FLRW metric in shaping our understanding of cosmic evolution. While the Big Bang paradigm remains robust, many of its empirical validations implicitly rely on the same metric assumptions used to interpret the data, raising the possibility of a subtle circularity.

In this context, the present work explores an alternative geometric framework in which the Universe is modeled as the three-dimensional surface of a hypersphere expanding at a constant rate, with radius $r = ct$ and the Big Bang located at its center. In this scenario, matter does not participate in the Cosmological Principle but instead appears as a discontinuity within an otherwise homogeneous geometric structure governed by the CMB. The model preserves Hubble's law, yields a finite yet unbounded Universe, and does not require dark matter or dark energy to account for large-scale dynamics.

From a macroscopic perspective, the model relies on the physical laws and principles that govern the observable Universe; from a microscopic viewpoint, it considers only the particles currently known to the Standard Model. While not exhaustive, I have endeavored to describe its conceptual and mathematical structure through the following series of publications:

1. DOI:10.5281/zenodo.18458109 – Dynamics on an Expanding Hypersphere: Reassessing the Cosmological Principle in light of the CMB
2. DOI:10.5281/zenodo.20155756 – The Vacuum Catastrophe revisited: when the quantum vacuum does not contribute to cosmological equilibrium
3. DOI:10.5281/zenodo.19640595 – The Aporia of Reversibility in Cosmological Expansion
4. DOI:10.5281/zenodo.20577082 – Ancient orbital structures: simulating Local Group evolution under full Hyperspherical Expansion Acceleration
5. DOI:10.5281/zenodo.22169635 – A Geometric Blueprint of the Early Universe: Primordial He, D, and Li
6. DOI:10.5281/zenodo.20791295 – Non Keplerian MW–M31 dynamics under Hyperspherical Expansion Acceleration: a bound solution without Dark matter
7. DOI:10.5281/zenodo.14986654 – Reevaluating the Necessity of Dark Matter and Dark Energy within Cosmological Model

I have termed this framework the *4-Sphere model*. The choice reflects both the need to distinguish it from other hyperspherical treatments and the geometric insight that emerged while constructing a stellar map for the model: the interplay of 2D, 3D, and 4D structures revealed a spatial reality that the term “3-sphere”—though mathematically correct—did not fully capture. The designation “4-Sphere” therefore emphasizes the four-dimensional geometric nature of the expanding hypersurface on which cosmic dynamics unfold.

While other models propose a hypersphere expanding with $r = ct$, an analysis of their main features reveals fundamental differences. The novelty of the model presented here — hereafter referred to as the *4-Sphere cosmology* — lies in its definition of the Hubble constant: its four-dimensional geometry suggests a linear relationship between galactic recession and the arc angle (rather than the arc length). This perspective does not contest the validity of Hubble’s law, but introduces different predictions about the past and the future, which cannot be determined solely from current observations.

The use of the angle instead of the arc length produces significant implications, opening the possibility of applying Special Relativity to galactic recession. The redshift, which asymptotically approaches a time horizon of roughly 5 billion years after the Big Bang, implicitly explains why, at the boundaries of the observable Universe with JWST, we should not expect to see only "baby galaxies". Based on supernova distance measurements, I suggest that the dismissal of a Doppler-type redshift interpretation for Galactic Recession might warrant further and careful reconsideration.

The model further introduces a General Methodology—applicable in both Special Relativity and FLRW contexts—that enables the validation or falsification of its predictions through supernova distance measurements. This includes the use of JWST photometric filters, converted into rest-frame Johnson B and V bands via transmission-curve analysis, allowing a direct comparison with established supernova datasets.

The broader implications of this speculative framework rest on the empirical robustness of Hubble's law and the independent validation of stellar distances, but also on a set of necessary assumptions—discussed in the cited works—that remain conjectural. These concern the predominant dynamical role of the CMB, a reconsideration of the Cosmological Principle, the status of the cosmological constant in its relation to quantum field theory, the restoration of the link between entropy conservation and reversibility, and the gravity–expansion balance encoded in the newly introduced Hyperspherical Expansion Acceleration (HEA). A separate line of investigation addresses the implications of this framework for the interpretation of Dark Matter and Dark Energy, as developed in the dedicated study listed above.

As a pivotal outcome, the 4-Sphere mode of operation ultimately implies that the structure of spacetime must accommodate a universal speed limit, independent of direction or frame of reference. From this condition, the principles of Special Relativity follow directly: the constancy of the speed of light, the equivalence of inertial frames, and the Lorentz invariance of physical laws. This inductive methodology, synthesizing abstract reasoning and mathematical formulation, does not in itself constitute scientific proof; however, it strengthens the explanatory coherence of the model and clarifies the conceptual foundations on which it rests.

SPECULATIVE FRAMEWORK AND COSMOLOGICAL CONTEXT

This work builds upon the broader theoretical framework described as a model fundamentally simple, rooted in the geometry of the 3-sphere. It belongs to the field of alternative cosmology, though the core concept of the hypersphere has an illustrious origin, being attributed to Einstein himself:

Here the interplay between energy, entropy, CMB anisotropies, and BAO was introduced as a unified dynamical system. Within that context, the present study focuses specifically on Galactic Recession, while remaining naturally connected to these wider cosmological themes. Such connections are not optional extensions but structural tests: they ensure that the proposed metric, under its foundational hypotheses, does not lead to contradictions with well-established observational pillars.

To meet these requirements, the model introduces a set of working assumptions—still conjectural at this stage—regarding the geometric role of matter as a discontinuity within an otherwise homogeneous hyperspherical structure governed by the CMB (see Section 4-SPHERE IN A NUTSHELL: THE METRIC TENSOR). Despite their speculative nature, these assumptions are anchored by two non-negotiable empirical constraints: the preservation of Hubble's Law and the consistency with stellar distance measurements. These elements serve as the primary calibration points ensuring that the model remains observationally viable while exploring a non-FLRW cosmological geometry.

To verify the model's consistency, one of the necessary assumptions is the age of the universe, estimated at 13.8 billion years, data taken from the ΛCDM model. This value is considered acceptable, given the calculations performed for HD-140283.

The age of the star HD-140283 (Methuselah) is determined through stellar evolution models. These models are based on the principles of nuclear physics and thermodynamics, which describe how stars generate energy and evolve over their lifetimes. By analyzing the star's chemical composition and its position on the Hertzsprung-Russell diagram, astronomers can estimate how long it has been fusing hydrogen in its core.

Therefore, the age estimation for HD 140283 is indeed independent of the $FLRW$ metric. This independence is precisely what makes such stellar age estimates so valuable in cosmology: they provide a crucial, independent lower limit for the age of the universe (around 14 billion years).

We will therefore proceed through the following points:

1. Given the introduction of the fourth spatial dimension, we will examine the implications through the lens of Occam's razor, opening in particular an in-depth discussion on the existence of dark matter and dark energy.
2. We will proceed with the most significant validation of the model using supernova distances. while further verification could stem from the gravitational lenses.
3. We will address the thermodynamic implications of the model, focusing on energy and entropy behavior, and introduce a cosmic tensor that remains invariant throughout the different cosmological eras.
4. Finally, we will provide a concise demonstration of the model's simplicity, both in the definition of the quantities used and in the formulas to be applied.

It is worth noting that while this paper focuses on specific core proofs, numerous other aspects confirming the overall consistency of the framework are detailed in our previously cited descriptive works.

A public dedicated OSF page [DOI 10.17605/OSF.IO/Y736C] is available as a technical repository associated with this research —still under construction— used solely to make available the Python source code and the Excel worksheet that provide the complete computational demonstration of the SN1995K distance validation.

DOES A HIGHER DIMENSION WEAKEN THE PHILOSOPHICAL STRENGTH OF OCCAM'S RAZOR?

The 4-Sphere doesn't stop at defining a metric—it truly introduces the fourth dimension of space. It could survive theoretically and observationally.

From a theoretical perspective:

- Without assuming a center of the universe everywhere and enforcing the absence of preferred directions: the geometry of the 4-sphere surface explains it naturally.
- Without invoking singularity at the zero point of cosmic time: the 4-sphere has a geometric center, with no need for infinite curvature or density.
- Without modifying the physics of radiation, meaning without a spatial expansion that alters the wavelength of light because of the new metric.

- Without assuming large-scale matter homogeneity but treating matter as discontinuity points on the 3D surface, with the universe's dynamics governed exclusively by the (CMB).
- Without Dark Matter (an element not yet detected in physics).
- Without Dark Energy (a form of energy not yet observed).
- Without a Horizon Problem—though introducing an alternative conjecture: after an initial expansion, the universe was static at the Last Scattering, and then expansion resumes.

The composition of the universe, according to the standard cosmological model, is a fundamental parameter that determines its evolution and geometry. The total density of the universe, which is very close to the critical density (the value that makes the universe "flat"), is composed of approximately 70% dark energy, approximately 30% dark matter, while only a very small fraction (about 5% of the total) is the "visible" baryonic matter (atoms, stars, galaxies) that we know.

From an observational and physical perspective, the 4-Sphere model adheres to established principles without requiring additional assumptions:

- Using General Relativity in its original form (always excluding Λ).
- With its absence of boundaries being fully consistent with CMB's energy conservation.
- With its equilibrium preserving the intrinsic link between CMB's entropy conservation and reversibility.
- Without running into the Vacuum Catastrophe, where the conceptual bridge between General Relativity and Quantum Field Theory is fundamentally challenged.
- Upholding the Big Bang theory with its robust and well-established framework.
- Without debating whether JWST observations at the edge of the observable universe should reveal unexpectedly mature galaxies.
- With the "ripples" of the Big Bang but ruling out the introduction of the Baryonic Acoustic Oscillations, to explain the large-scale structures we observe in the Universe.

Speaking of Dark Matter and Dark Energy, my brief research is available here Reevaluating the Necessity of Dark Matter and Dark Energy within Cosmological Models. The claims presented are, apart from Baryon Acoustic Oscillations (BAO), CMB anisotropy and from Galaxy formation, entirely non-speculative and, in fact, self-evident.

That is the thing:

The existence of Dark Matter and Dark Energy is widely accepted within the Standard Cosmological Model. However, their introduction is not based on direct detection but rather on the need to reconcile observations with theoretical expectations. This paper explores whether the necessity of Dark Matter and Dark Energy arises inherently from our chosen cosmological framework rather than from an independent physical requirement. To address BAO, CMB anisotropies, and Galaxy formation, this analysis necessarily required a speculative framework. Consequently, the research is completed through a series of appendices that utilize the 4-Sphere as an alternative model to explore these phenomena from a new perspective.

The present investigation suggests that dark matter and dark energy must thus be reconsidered within a cosmological framework. As a consequence, Einstein's field equation should be maintained in its original formulation, without introducing the cosmological constant Λ, unless future empirical evidence requires its reintroduction.

ON THE SIGNIFICANT VALIDATION OF THE MODEL AND MORE

The validation of a supernova in these publications suggests that the dismissal of a Doppler-type of redshift interpretation for Galactic Recession may warrant further reconsideration:

1. viXra: 2207.0051 - Concerning the Apparent Magnitude
2. viXra: 2208.0040 - Concerning the Time Dilation of the Supernovae
3. viXra: 2208.0152 - Star Distance Validation from Data of a High-Z Supernova Ia in the Special Relativity Context

While the validation involves intricate calculations, and the research's logical progression is intended to be self-evident, it is crucial to acknowledge that the publication at point 1, concerning Apparent Magnitude, represents a potentially essential aspect from which the verification of the results may be derived.

Concerning instead the complexity, independent verification of the calculations is facilitated by the detailed descriptions provided in my earlier publications, specifically points 2 and 3.

To ensure transparency, I have outlined the method for verifying my application of the mean squares estimation: Both the instructions for installing the necessary software, and the supernova data, can be found in my OSF Project Wiki Pages. You can access it in: Supernova validation m.s. extimation. This involves using Python, and the small initial effort to configure its working environment is amply rewarded by the powerful functionalities that this platform offers in the scientific field (Visual Studio offers to developers the “Python Development Tool for Windows”).

Checking the correct application of least squares method is detailed in[viXra:2208.0040], here I supply an Excel spreadsheet for independent verification of the calculations. This method allows for direct confirmation (the values are easily identifiable). I have uploaded the Excel file on my OSF project "Supernova SN1995 K validation.xlsx". You can access it in: 4-Sphere-Cosmology Files tab.

Our first result was good. The *SN* Luminosity distance of 1,320 *Mpc* calculated by 4-Sphere has been confirmed with an accuracy of 96.5%.

Further, and significant, confirmation of the model's validity was obtained through the study of a gravitational lens system, ref. Reevaluating the Necessity of Dark Matter and Dark Energy within Cosmological Models. By applying, both in virial and lensing, the distance computation employed in the prior validation, in substitution for the FLRW angular distance, we obtained a total lens mass that reduces the requirement for Dark Matter from the 27% (required by the standard model for that system) to only 6%. This equates to a reduction in the Dark Matter mass deficit of 75%.

As will become clear, with respect to the six ΛCDM parameters — (1) Physical Baryon Density, (2) Physical Cold Dark Matter Density, (3) Normalized Hubble Parameter, (4) Optical Depth to Reionization (CMB), (5) Scalar Spectral Index of density fluctuations, and (6) Amplitude of the Primordial Scalar Power Spectrum — I believe that all correlated aspects have been addressed, albeit expressed through the different language of my model.

MODEL'S FORMULAS AND CONCEPTUAL SIMPLICITY

A notable elegance of the model lies in the parsimony of its defining quantities and the simplicity of its operative formulations. Here are the operating quantities for the 4-Sphere:

If r is the radius of the 4-Sphere, Θ is center angle between the star and us, with z its Redshift, and t_{now} the time elapsed from Big Bang (measured by us), the main quantities for the 4-Sphere are:

1. The star Recession velocity $v_r = c\Theta$
2. The quantity β equals to angle $\Theta = v_r/c = ((1+z)^2 - 1)/((1+z)^2 + 1)$
3. The star Redshift $1 + z = (1+\beta)^{1/2}(1-\beta)^{-1/2}$
4. The Time dilation $dt_{obs}/dt_{emit} = (1+\Theta^2)^{-1/2} = \gamma$: the Lorentz Factor of Special Relativity
5. The actual radius of the 4-Sphere $r_{now} = ct_{now}$
6. The time from the star's light beam started $t_0 = t_{now}e^{-\Theta}$
7. The time spent by the star's light beam to travel the arc Θ $\Delta t = t_{now} - t_0$
8. The Proper Distance of the star $d_P = r_{now}\Theta$ (the Galilean distance)
9. The Luminosity distance $d_L = c\Delta t$ and the Comoving distance, equivalent to the angle Θ
10. The Recession Anchor Coordinate $A_r = \Theta$ (time invariant)
11. The arc corresponding now to 1 Mpc $\Theta_{1\,Mpc} = 1\,Mpc\,/\,r_{now} = 2.36 * 10^{-4}\,rad$
12. *The equivalent of Hubble's constant* $H_{sphere} = c\Theta_{1\,Mpc} = 70{,}9\,Km\,s^{-1}$ *(per* $\Theta_{1\,Mpc}$*)*

ON THE USE OF DISTANCES IN THE 4-SPHERE MODEL

Comparing FLRW distances with those used in alternative cosmological frameworks is not always straightforward. In my model, however, both the Luminosity distance—defined in FLRW through the inverse-square law and, in the 4-Sphere framework, corresponding to the actual path travelled by the photon between emission and detection—and the Proper (Galilean) distance $d_P = r_{\text{now}}\theta$ can be meaningfully related to observational quantities.

By contrast, the FLRW angular-diameter distance D_A has no direct analogue in the 4-Sphere framework, since it relies on the specific geometrical properties of expanding curved space.

In practical applications within this paper:

- Luminosity distance d_L is used when analysing stellar or supernova observations (e.g., SN1995K). For gravitational lensing, we adopt the Luminosity distance as well. Although this differs from the FLRW use of D_A, it remains consistent with interpreting lensing. We used the distance effectively traveled by light, even though the framework is based on classical optics. This choice stems from the fact that the distances involved are too great to utilize the proper distance for the sole purpose of adhering to Euclidean geometry, leading us to favor the use of effective distances instead.
- Proper distance $d_P = r_{\text{now}}\theta$ is used when computing orbits with Kepler, the distances are small and the choice seems justified.
- The angle Θ: The angle subtended at the center of the hypersphere by the arc connecting the star and the observer. This quantity remains invariant with time and governs galactic recession. In this sense Θ can be compared to the Comoving distance D_C of FLRW.

This approach keeps the treatment of distances internally coherent while still allowing meaningful comparisons with standard FLRW-based analyses.

HOW TO CHECK THE LUMINOSITY DISTANCE WITH THE DISTANCE MODULUS

The Luminosity distance d_L (that is provided by the 4-Sphere) is related to the Distance Modulus in $\mu = log_{10}(d_L) + 5$ where distance is in Parsec. For a star at rest, the relationship between Luminosity distance and Distance modulus cannot depend on the observed wavelength, except for the effect of Extinction. Therefore, abandoning the bolometric quantities:

$$\mu = \mu_\lambda - A_\lambda = log_{10}(d_L) + 5$$

where μ_λ comes now from differences of magnitudes measured in a light interval λ of wavelengths.

The introduction of the new quantity A_λ leads us to modify the relations described above as:

$$\mu_\lambda = m_\lambda - M_\lambda \quad \text{where} \quad m_\lambda = m_{0\lambda} - \ K_{SR\ corr}$$

Then, as for the bandpass resulting from the corrections on observation, from now on, we will refer to one on more of the Johnson-Cousins standard color *U, B, V, R, I*. [1]

GRAVITATIONAL LENSING AND MASS ESTIMATES

In the preceding paragraph we emphasized how the luminosity distance d_L of the model has also proven useful for virial but especially in gravitational lensing. This is an important result because, being based on angular measurements and independent of luminosity, it does not require the K-correction, which is almost never provided and is not related to that of FLRW. For this reason, this distance can be considered valid for mass estimation through strong lensing.

There is also another interesting consideration. The ratio between the angular distance D_A of FLRW and the luminosity distance d_L of the 4-Sphere should not vary significantly even at higher redshifts (up to $z \approx 2$). This implies that the ratio between the angle in the 4-Sphere model and that in FLRW remains close to 0.95. Such stability is useful for deriving distortions, at least approximately, from FLRW weak lensing.

AND HERE ARE, FINALLY, OUR GALACTIC COORDINATES

1. Let's choose a reference frame based on a radius $r = ct$ as time coordinate and on three angles θ, φ, ψ as space coordinates $(0, 2\pi)$. As reference points, unfortunately, we cannot choose known stars as "Alpha Ursae Minoris – Polaris" or "Delta Orionis – Mintaka" on the Orion's Belt. This is because of their proximity to us.

2. The three coordinates on the surface are given by the angles θ, φ, ψ where the first two are the equivalent of Longitude and Colatitude (using $\text{zenith angle} = 90^0 - \text{Latitude}$) and where we will call the third "Universe Height". Astronomic Celestial coordinate Declination and Right ascension are relative to our observable Universe, here Universe Colatitude and Longitude refers to the whole 4-Sphere. As convention we indicate a point P as $P(\varphi, \theta, \psi)$, with Colatitude before Longitudes.

3. Let's establish a position $P_N(0,0,0)$ for the "North pole" of our 4-Sphere. Since all the points on the surface are equivalent, we can choose "Ursa Major GN-108036". Then we chose a Prime Meridian $P_{M0}(undef, 0, undef)$, passing through some other known point in space (say passing through "Sculptor A2744 YD4"). Note that all points $P_{EM-}(\pi/2, 0, \ undef)$ on the Universe Equator are out of our observable Universe. A third point $P_{EM}(\pi/2, 0, \pi/2)$ is at Universe Height $\pi/2$ on the Universe Equator, at $\pi/2$ from P_N measured on Prime Meridian.

The use of these coordinates allows for the creation of a map of our observable universe, simplifying calculations by making only the Proper Distance time-dependent and easily computable, while other quantities remain constant. [2]

[1] – References from Wikipedia: Photometric system

[2] –The section GALACTIC COORDINATES is crucial as it describes the celestial coordinates within the hypersphere and their specific use within the Euclidean geometry that underpins the previously mentioned map of the universe. Reading this section, where you will see 2D, 3D, and 4D geometric figures working together, should make it obvious why I went with the name 4-Sphere instead of 3-Sphere for my model.

4-SPHERE IN A NUTSHELL: WHY THE FOURTH DIMENSION OF SPACE

The standard model assumes that the homogeneity of space implies a homogeneous distribution of matter. While this assumption is certainly possible, it leads me to question: Homogeneity exists only on a large scale; It consists of islands of matter surrounded by vast zones of cosmic void. Are we truly certain that matter is the decisive factor in determining the universe's metric? [See, on this point, the short note: Dynamics on an Expanding Hypersphere: Reassessing the Cosmological Principle in light of the CMB].

More critically, if we rely on the Einstein tensor, we are constrained to our usual dimensional framework. Yet in this context, isotropy and homogeneity of space are dictated by constraints imposed on the metric tensor's coefficients. Can we truly disregard the importance of verifying that this result stems from a physically realizable three-dimensional spatial geometry? A related conceptual point arises in thermodynamics: even in an isolated system, conservation of energy is not guaranteed, and a seemingly irreversible process such as the CMB's evolution under dark energy can still exhibit conservation of comoving entropy.

Furthermore, the problem of proper time is an inherent weakness of the Standard Model, a critical point that my proposal only partially addresses. Our solution employs cosmic time solely for the expansion of the universe, which we have kept separate from relativity. We will delve into this aspect in greater detail throughout our analysis.

Thus, the new perspective should not propose a new stress-energy tensor to be inserted into Einstein's equation in its FLRW form, but rather the abandonment of the FLRW framework itself, and of a context where the distribution of matter is considered essential.

A complete understanding and acceptance of cosmology, as described by the Standard Model, ultimately depends on individual perspectives and ways of thinking. Seeking to explore alternatives to the standard interpretations of the FLRW solution, I directed my attention toward the concept of superluminal motion, which becomes conceivable within the framework of Galactic Recession [3].

With both Galactic Recession and relative motion in mind, I sought a model in which the principle of Relativity and the Recession mechanism could emerge together—yet ideally formulated in a way that distinguishes them as much as possible [4].

The development of a physical model is, in principle, independent of the particular form we assign to it. Nevertheless, in my approach a geometric structure plays a crucial role, serving as the bridge between abstract formulation and our perception of physical reality. Geometry not only shapes our understanding, but also guides the unfolding of abstract reasoning itself [5]. Within this perspective, our conjecture naturally leads to the formulation of Relativity [6]. The connection between geometry and mathematical description should be severed only when absolutely unavoidable. See also The Vacuum Catastrophe revisited: when the quantum vacuum does not contribute to cosmological equilibrium.

But no geometry known to us was able to explain isotropy and homogeneity except by resorting to another spatial dimension. With this idea as a new starting point, I looked for a model in which the metric of the Universe is not what it seems to us, but rather the outcome of our limited three-dimensional perception of a four-dimensional space—a perception that not only shapes our senses but also the very laws of physics as we apply them. As you will see, in this light the energy of the universe is never lost.

In this geometry, the Universe extends on the surface of an expanding 4D-hypersphere, and the geodesic is reduced to an expanding arc of a 2D-circle. However, this geodesic follows the principles of Special Relativity: it is not a straight line but an expanding arc whose curvature belongs to another dimension.

Finally, there's another point, which might prove to be the most important in the future. FLRW distances are very — perhaps too — high, but it will be difficult to question them because FLRW is the only model that naturally preserves Hubble's law under homogeneity and isotropy. In FLRW, the curvature parameter k defines spatial geometry ($k = 0$ flat, $k = 1$ spherical, $k = -1$ hyperbolic), so a S^3surface corresponds to $k = 1$. However, this curvature is an intrinsic property of the 3D hypersurface, and the coordinates are no longer aligned with straight lines in the Euclidean sense. Moreover, the choice of k may appear somewhat artificial, because no physical law outside cosmology — regardless of the precision of our measurements — perceives a fourth spatial dimension. By contrast, in my model the introduction of an additional spatial dimension provides a consistent way to represent the universe while maintaining a different geometric interpretation of distances.

[3] - In the FLRW model, Cosmological Superluminality is a direct consequence of the metric expansion of space. This phenomenon is defined by the Hubble Radius $R_h = c/H$, which represents the theoretical distance at which the recession velocity of a galaxy—due to the metric expansion—is exactly equal to the speed of light, c. For regions where $D > R_h$, the recession velocity $v = HD$ becomes superluminal. This does not imply motion *through* space, but rather the expansion of the coordinate system itself. These are non-local velocities associated with space-like separations, fully compatible with General Relativity as they do not violate local Lorentz invariance.
[4] - Replacing the FLRW metric with an alternative one is not straightforward, since FLRW is an exact solution of Einstein's field equations under the assumptions of homogeneity and isotropy. It follows a deductive structure: starting from Einstein's equations, cosmological laws are derived through the introduction of the scale factor.
In the FLRW framework, light from distant galaxies is redshifted because its wavelength is stretched by the expansion of space. This cosmological redshift provides a way to infer distances. Special relativity appears as a limiting case of the FLRW metric in regions where spacetime curvature is negligible and gravitational effects are weak.
In contrast, within the 4-Sphere model the expansion of space does not affect the wavelength of light, and the gravitational influence of the cosmic microwave background on photon propagation is negligible. This makes it possible to approximate the observed redshift of the most distant galaxies using the Doppler effect alone, even for objects near the boundary of the observable universe.
[5] – Geometric shapes enhance the philosophical understanding of physical reality, allowing us to abstract and generalize concepts, thus aiding in the construction of scientific models. Furthermore, Karl Popper's concept of falsifiability demonstrates that scientific thought and philosophical thought cannot do without each other.
[6] –If we follow this purely philosophical consideration together with our conjecture for the expansion mechanism, we obtain Einstein's Relativity. The inductive approach through this combination of abstract thought and mathematical formulation is not, in itself, scientific proof, but it certainly enhances the model. See also PRIVILEGED FRAMES AND LIGHT: RETHINKING SPECIAL RELATIVITY FROM A 4-SPHERE PERSPECTIVE further ahead.

4-SPHERE IN A NUTSHELL: HUBBLE'S LAW AND DOPPLER REDSHIFT IN GALACTIC RECESSION

The model, named 4-Sphere [7], bases its physics on the expansion of a four-dimensional spatial manifold, simplifying the dynamical description by considering the CMB in the absence of matter [8]. Its definition of the Hubble constant arises from a linear relationship between galactic recession velocity and the arc angle (rather than the arc length). In this framework, Hubble's law becomes:

$$v_r = c\Theta.$$

In this framework, the redshift is not a consequence of metric space expansion (stretching of the wavelength), but rather a geometric result of light propagation within the growing 4-Sphere. This leads naturally to the Doppler formulas:

$1 + z = (1 + \beta)^{1/2}(1 - \beta)^{-1/2}$ and $\beta = ((1 + z)^2 - 1)/((1 + z)^2 + 1)$ where $\beta = v/c$.

The remaining relations follow directly from the identification $\Theta = \beta$.

While the Doppler effect describes the recession velocity v_r, the observed redshift z must be reconciled with the light-travel distance. Consequently, the standard “proper distance” $d_P = ct_{now}\,\Theta$ is a geometric property of the 4-sphere, but the relevant distance for observations is the path length covered by light: $d_L = c\Delta t$ (previously defined).

This model treats the redshift as either Doppler-like (for galactic recession) or gravitational (for the CMB), as both mechanisms preserve the blackbody spectrum, merely altering the temperature scale. The key to this speculation lies in the interpretation of Hubble's Law and distance validation:

Were JWST’s observations to suggest that there is no observable maximum limit in redshift—and that our inability to see farther stems solely from telescope sensitivity—then an asymptote is clearly the solution to the cosmological distance problem. The Doppler effect inherent in this angular-based recession naturally presents this asymptote, providing a framework consistent with galactic formation timescales.

[7] – 4-Sphere is a proper name, but here we also mean the hypersphere embedded in four-dimensional space R4 (someone call it 4-ball too); its surface is named by topologists a S^3 sphere.

[8] – Since gravity decreases with the square of the distance, matter clustered in galaxies could be seen as points of discontinuity in the universe, where the distribution of matter is not continuous but concentrated in specific regions. This idea reflects the large-scale structure of the universe, characterized by a web of galaxies and clusters separated by vast voids, a central concept in modern cosmology. Our hypothesis is that the CMB represents the predominant force in maintaining the 4d-bubble that characterizes, here, the shape of the Universe, and that matter, as a discontinuity, can be neglected. See Dynamics on an Expanding Hypersphere: Reassessing the Cosmological Principle in light of the CMB.

4-SPHERE IN A NUTSHELL: EPISTEMOLOGICAL CONSIDERATIONS BEFORE SPECULATION

As mentioned, the speculation concerns the Galactic Recession and assumes a metric tensor for the hypothesized geometry, from which it results that the Redshift of a galaxy, even distant, is of the Doppler type (verified through the measurement of luminosity distance with the data from observations of the supernova IA SN1995K at z=0.479). [see ON THE SIGNIFICANT VALIDATION OF THE MODEL AND MORE]

Even in future applications involving the validation of supernova distances, it is essential to preserve the methodological approach adopted here, grounded in the classical photometric framework established by Astrophysical Journal Letters v.413, p.L105 - The Absolute Magnitudes of Type IA Supernovae. This strategy is not only based on computational tools accessible to any researcher, but—more importantly—it avoids any reliance on K-corrections during the identification and characterization of the sample supernova. As a result, the procedure remains fully independent of the cosmological model adopted, and can therefore be applied consistently within both FLRW and non-FLRW frameworks.

General Methodology Paragraph (valid in both *SR* and *FLRW*)

To analyze the light curves of distant Type Ia supernovae in a way that is independent of comparison stars and minimally sensitive to spectral corrections, we adopt a purely differential photometric approach. Since the redshift z of each supernova is known, the observed bandpasses can be mapped onto the corresponding rest–frame B and V bands by applying the wavelength transformation $\lambda_{\text{rest}} = \lambda_{\text{obs}}/(1 + z)$. The observed magnitudes in any filter are therefore treated as simple vertical translations of the rest–frame B and V light curves, provided that extinction $A_{\lambda}$and the K–correction remain approximately constant over the 15–20 day interval following maximum light. Under this assumption, the shape of the light curve — and in particular the slope Δm_{15}— is preserved exactly, allowing us to compute $\Delta m_{15}(B)$and $\Delta m_{15}(V)$ directly from the observed data without requiring comparison stars or absolute calibration.

Once the decay slopes are obtained, the absolute magnitudes $M_B$and M_V follow from the empirical linear relations calibrated on nearby Type Ia supernovae. The color excess is then derived in the standard way,

$$E_{B-V} = m_B - m_V - (M_B - M_V),$$

and provides an estimate of the total extinction affecting the supernova, independently of the recession velocity and without relying on foreground-star extinction maps. Finally, the distance modulus is computed in the V band, where extinction corrections are most reliable, yielding a luminosity distance that can be compared with any cosmological model, including FLRW or alternative frameworks.

Synthesis on *JWST* Filters and Rest-Frame Conversion

In practical applications involving space-based photometry—such as observations obtained with *JWST* filters (e.g., F090W, F125W, F160W)—the measured fluxes do not correspond directly to the Johnson B and V bands traditionally used to characterize Type Ia supernovae. Since the redshift z of each supernova is known, the first step is to map the observed filter transmission curve $T_{\text{obs}}(\lambda)$onto the corresponding rest-frame wavelength interval through the transformation

$$\lambda_{\text{rest}} = \frac{\lambda_{\text{obs}}}{1 + z}.$$

This operation identifies which observed filter best overlaps the rest-frame B or V band for the given redshift. Once this correspondence is established, the observed magnitudes can be converted into rest-frame B and V magnitudes by convolving the observed flux with the appropriately shifted transmission curves. This procedure requires only the publicly available transmission functions of the FLRW filters and does not rely on comparison stars or on any cosmological assumptions.

Because extinction A_λ and the K-correction remain effectively constant over the 15–20 day interval used to measure the decay slope, the transformation from an observed FLRW filter to the rest-frame Johnson system acts as a simple vertical shift of the light curve. The shape of the curve—and therefore the key observable Δm_{15}—is preserved exactly. Once $\Delta m_{15}(B)$and $\Delta m_{15}(V)$are obtained, the absolute magnitudes $M_B$and $M_V$follow from the empirical linear relations calibrated on nearby Type Ia supernovae, and the color excess

$$E_{B-V} = m_B - m_V - (M_B - M_V)$$

provides a robust estimate of the total extinction affecting the event. This approach ensures that even observations obtained through non-standard filters can be consistently translated into the Johnson B and V system, enabling a uniform treatment of Type Ia supernovae across different instruments and redshifts, and preserving full independence from the underlying cosmological model.

It is only in the final stage—when converting rest-frame magnitudes into distance moduli—that differences between cosmological models emerge, since each framework applies its own K-correction or luminosity-distance relation. The entire photometric procedure leading to $\Delta m_{15}(B)$, $\Delta m_{15}(V)$, M_B, M_V, and E_{B-V} remains fully model-independent; divergences arise solely in the distance-estimation phase, where each cosmology introduces its specific correction scheme.

4-SPHERE IN A NUTSHELL: THIS SPECULATION AND THE ARTIFICIAL INTELLIGENCE

I envision a future in which all research findings — even highly speculative ones — can continue to be openly published. To address the rising costs of peer review, I propose that each paper be accompanied by an AI-generated quality assessment. This would help readers navigate the growing volume of scientific literature and make more informed judgments.

Although AI is not yet designed specifically for this task, it can already reflect the reasoning of mainstream science and, when properly prompted, provide appropriate critiques. In the case of the 4-Sphere, such an approach could at least partially compensate for the limitations of the current "report-card" system.

Accordingly, to better understand the reasoning of AI, I engaged in a structured dialogue with ChatGPT. The goal was not only to fill gaps in knowledge, but also to gain insight through the friction of contrasting viewpoints. If even now—without targeted training—AI can provide this level of support, it is reasonable to imagine what it will be able to offer in the future. Without the help of ChatGPT, this research would not have been possible.

I would also like to express my gratitude to Gemini and Copilot, both of which I integrated into my workflow during the later stages of this research. Their contributions proved instrumental in the refinement and presentation of the work, particularly through their specialized knowledge of physics and their ability to translate complex technical concepts with precision. In numerous discussions, both systems—when prompted within a rigorous contextual framework—demonstrated a remarkable capacity to synthesize vast amounts of information and to

identify core issues with clarity and nuance. Their support was foundational in giving this research its final form.

In recent times, I have observed that AI, once properly contextualized, is capable of carrying a discussion forward with rigor even outside established theoretical frameworks, and can recognize when the conversation itself requires stepping beyond those frameworks, relying not so much on disciplinary knowledge as on a surprisingly solid logical structure. This proves especially useful in speculative contexts.

Whatever the use of AI was, all physical interpretations, modelling choices, and conclusions are entirely my own, and every result has been checked for internal consistency within the framework of the model.

4-SPHERE IN A NUTSHELL: THE METRIC TENSOR

In a small portion of 4d-hypersphere's surface, where its spacetime can blend with that of our Universe, for the line element to be considered in the Einstein-Hilbert action, the following must hold: $ds^2 = g_{\mu\nu} dx^{\mu} dx^{\nu} \quad with\ x^{\mu} = x, y, z, t$ if we want Galilean coordinates.

Named ξ the angle at the center between two points, P_1 and P_2 on the surface, you can refer to the expanding arc of great circle rξ to simplify the reasoning on geodesics. It is therefore a question of describing a motion along a line in which the constraint reactions, induced by the line itself, must not appear must not appear in any Action Principle.

Thus, continuing with the idea that from the denial of Absolute Space, Relativity is deduced, let us start from a solution in the form: $ds^2 = -h_{r\xi}(dr\xi)^2 + h_t c^2 dt^2$

and consider the differential of the product $r\xi$: $\quad \overset{O}{d(r\xi)} = \overset{M}{ctd\xi} + \overset{R}{c\xi dt}.$

- O is the overall motion, in part constrained by $r = ct$
- M is free motion along the expanding arc
- R is motion due to Galactic Recession

To obtain the separation between gravity and galactic recession, desired by our model, we think of two celestial bodies and the distance between them: if the latter is small, we neglect the recession, if it is large, we neglect gravity.

A unified metric description of local gravity and large-scale recession is not introduced within the present framework. Local gravitational phenomena are described using General Relativity, while the large-scale cosmological background is treated through Special Relativity on the expanding hyperspherical geometry. Consequently, the model does not require the introduction of an additional coupling term analogous to the cosmological constant.

Nevertheless, the development of the HEA formalism [9] provides a dynamical connection between these two regimes. Rather than introducing a universal metric tensor, the transition between gravitationally bound systems and the cosmological expansion emerges through the competition between gravitational attraction and the velocity-dependent HEA contribution.

This interaction defines a characteristic scale at which the two effects become comparable, naturally separating gravity-dominated and expansion-dominated regimes.

Within this approach, the boundary between local and cosmological dynamics is therefore determined by the dynamical evolution itself rather than by an explicit geometric coupling. The corresponding observational transition remains identifiable through the progressive dominance of recession-related redshift over local gravitational effects, but this criterion now reflects an underlying dynamical mechanism described by the HEA framework.

This separation between local relativity and large-scale recession is not the only distinctive feature of the model. The underlying conjecture dictates a set of precise analytical choices arising from the fact that physics cannot directly access the radial dimension of the 4-Sphere. Observable phenomena are confined to the three-dimensional hypersurface, while the radial coordinate remains physically inaccessible.

Then, for the recession it holds $v_r = c\xi$ while, if we refer to Relativity, the remaining expression for our interval $ds^2 = -h_{r\xi}(ctd\xi)^2 + h_t c^2 dt^2$ reflects the expansion of the Universe. This is not a negligible phenomenon, and if we can only measure it through the observation of stars, it is because the objects around us are bound by gravitational forces that resist their mutual separation. Our nearby spatial references remain constant because they are determined by the cohesion of matter or by celestial bodies in stable orbits, gravitationally bound around a center of mass that is in constant recession. What we measure in our vicinity is a space that does not expand, where the true geometry has been altered. Furthermore, this allows us to assume $r = ct = const$ every time we are in the presence of any orbit.

This separation between large-scale recession and local dynamics does not compromise the internal consistency of the model. First, galactic recession — which may exceed the speed of light when the expansion parameter satisfies $\xi > 1$— is a metric effect confined to regions where relativity is not applied locally, and therefore does not violate Special Relativity. Second, gravitationally bound systems lie outside the metric governing the global expansion (as in the FLRW framework itself), so their internal geometry remains effectively static. Third, the large-scale dynamics of the Universe can be regarded as driven primarily by the Cosmic Microwave Background, with matter forming localized perturbations; in this perspective, adopting a gravitational redshift for the CMB is a coherent choice. Finally, it is true that a fully continuous metric describing the transition from intergalactic vacuum to gravitationally bound regions is still lacking — yet the standard FLRW model does not provide such a gradual description either.

The term $ds^2 = -h_{r\xi}(ctd\xi)^2 + h_t c^2 dt^2$ is not covariant, we can call it a pseudotensor. But this is only true globally, because in the limit where the local arc length ξ can be identified with a Cartesian coordinate x, so that $\Delta x \simeq ct\Delta\xi$, the line element is described by a linearized metric. This framework allows us to invoke the principle of equivalence, which ensures that a particle's four-momentum is conserved along its geodesic.

To study galactic recession $v_r = c\xi$ and our pseudotensor in the cosmic vacuum, the logical thread followed begins with a light geodesic moving along a great-circle arc on the surface of an expanding hypersphere. This path extends to the limits of the observable universe, defined by an asymptotic galactic redshift driven by the Doppler effect. From there, we eliminate the universe's expansion from the model's metric. All that leads us to use the Einstein field equations for weak fields [see later].

The calculations, extended to the asymptote, demonstrate that even the faint gravity of the Cosmic Microwave Background *(CMB)* is negligible and does not invalidate the application of SR in the cosmic vacuum. This finding is secondary to our model's core assumption: our physics does not perceive the fourth spatial (radial) dimension. Our physics sees the arc as a straight line always, both with the gravity-opposed expansion of the universe acting on the center of mass of orbiting groups (governed by GR) and in the vast regions of vacuum (governed by SR).

[9] – The Hyperspherical Expansion Acceleration (HEA) is described in Ancient orbital structures: simulating Local Group evolution under full Hyperspherical Expansion Acceleration. It is a purely geometric contribution that emerges from the time variation of the hyperspherical radius in a 4-dimensional spatial setting. Unlike gravitational or inertial accelerations, the HEA does not depend on mass, does not derive from a potential or a field, and does not correspond to any known dynamical mechanism. Its inverse-square dependence is only formally reminiscent of Newtonian gravity, while its physical origin is entirely distinct.

EXPLORING EXPANSION AND GRAVITY

This analysis includes the contribution of the Hyperspherical Expansion Acceleration (HEA), a geometric effect associated with the time variation of the hyperspherical radius in a 4-dimensional spatial framework. The interplay between gravity and expansion is highlighted through the study [Ancient orbital structures: simulating Local Group evolution under full Hyperspherical Expansion Acceleration] of the galactic trajectories within the Local Group. In modelling its past evolution, we incorporate the HEA as a non-dynamical geometric term generated by the time dependence of the hyperspherical radius. The HEA affects recession velocities without introducing forces, potentials, or dissipative effects, and is therefore fully compatible with the gravitational binding of the system.

This interplay can be further quantified by considering through the study of the critical radius at which internal gravity balances the cosmological HEA term. This equilibrium radius can be written

$$R_c = (GMt^2)^{1/3},$$

where M is the total gravitating mass enclosed within R_c. Recession is measured as $v_r = c\Theta$, so that the observable universe corresponds to $\Theta < 1$ rad, that is, to a hyperspherical cap of angular radius 1 rad measured along a great-circle geodesic. The corresponding geodesic distance is $d = r\Theta \approx 4.2$ Gpc. For a system of two galaxy clusters, the equilibrium condition then yields a critical radius of order $R_c \sim 10$ Mpc for a total mass $M \sim 10^{15} M_\odot$, and $R_c \sim 4$–5 Mpc when only the expected baryonic mass $M \sim 10^{14} M_\odot$ is considered, showing that the gravity–HEA balance remains confined to local scales.

For stellar and galactic scales, HEA effects are entirely negligible. A Milky Way–like galaxy with a baryonic mass of order $10^{11} M_\odot$ has a gravity–HEA equilibrium radius $R_c = (GMt^2)^{1/3}$ of only ~ 0.5–1 Mpc, far beyond its ~ 10–100 kpc extent. Consequently, the internal dynamics of the

Milky Way are fully decoupled from cosmological expansion: Galactic Cepheids exhibit purely kinematic (and mildly gravitational) redshifts, with no contribution from recession. Only extra-galactic Cepheids, hosted in galaxies not gravitationally bound to the Milky Way, participate in the cosmological recession and can therefore be used in the Hubble relation within the HEA framework.

TO SUM UP

Cosmic expansion should be considered in the context of "inter-group" dynamics, between gravitationally bound systems like galaxy groups and clusters. However, within these systems ("intra-group"), the gravitational forces dominate, preventing the expansion from affecting the internal structure.

In the inter-group context, the geodesic of light reduces to $0 = -(ctd\xi)^2 + c^2dt^2$ (which gives $v = c$) and the redshift of a faraway star comes mainly from the recession $v_r = c\xi$. In the intra-group context, where, regardless of the size of a celestial system, none of its bodies are receding from each other, the redshift of a nearby star should be interpreted as its orbital velocity (or a peculiar velocity) and its distance should be calculated from its distance modulus μ. Finally, it is important to underline that, with regard to gravity and therefore in the intra-group context, we can apply Einstein's field equations in their original form (with $\Lambda = 0$).

Among the assumptions of the model, applied in the calculations, the *CMB* expands in equilibrium, meaning that its density changes in the presence of forces that maintain balance. This approach assumes that gravity remains consistent with the isoentropic expansion. Thus, despite the long-term cosmic evolution, the model suggests that the gravitational effects align with the expanding, thermodynamic properties of the *CMB*, ensuring that the overall energy and forces in the universe remain in a state of equilibrium.

The line element proposed by the model includes a recession term, $c\xi dt$, which applies exclusively to stars, while the second postulate of relativity (regarding light) is guaranteed by the assumed geometry. Thus, we accept superluminal speeds between physical entities, but only under the condition that these objects cannot be physically observed. We also accept relativity in all the laws governing our physics, including the 2 laws of conservation of mass and momentum and the relationship between mass and energy, because the model ensures that wherever relativity is manifested, the effect of recession term $c\xi dt$, which is completely absent in the case of light, can still be neglected in the case of matter. Recession is used only to calculate the effect on the light arriving from the star.

AN APPROXIMATE SOLUTION FOR THE OBSERVABLE UNIVERSE FROM EINSTEIN'S WEAK FIELDS

By relating time to the 4th spatial dimension we obtain the usual curved space-time. After this, we no longer need the equation of the surface: $x_1^2 + x_2^2 + x_3^2 + x_4^2 = c^2t^2$. As we will see later, fourth dimension of space x_4 will appear again in a mathematical context but no longer in physics.

The generic procedure to get the metric of 4-Sphere curved space-time sems extremely complex in a Cartesian reference frame.

The solution is not even simplified using polar coordinates:

1. Let's choose a reference frame based on a radius $r = ct$ as time coordinate and on three angles θ, φ, ψ as space coordinates $(0, 2\pi)$. As reference points, unfortunately, we cannot choose known stars as "Alpha Ursae Minoris – Polaris" or "Delta Orionis – Mintaka" on the Orion's Belt. This because of their proximity to us.
2. The three coordinates on the surface are given by the angles θ, φ, ψ where the first two are the equivalent of Longitude and Colatitude (using $\text{zenith angle} = 90^0 - \text{Latitude}$) and where we will call the third "Universe Height". Astronomic Celestial coordinate Declination and Right ascension are relative to our observable Universe, here Universe Colatitude and Longitude refers to the whole 4-Sphere. As convention we indicate a point P as $P(\varphi, \theta, \psi)$, with Colatitude before Longitudes.
3. Let's establish a position $P_N(0,0,0)$ for the "North pole" of our 4-Sphere. Since all the points on the surface are equivalent, we can choose "Ursa Major GN-108036". Then we chose a Prime Meridian $P_{M0}(undef, 0, undef)$, passing through some other known point in space (say passing through "Sculptor A2744 YD4"). Note that all points $P_{EM-}(\pi/2, 0, \ undef)$ on the Universe Equator are out of our observable Universe. A third point $P_{EM}(\pi/2, 0, \pi/2)$ is at Universe Height $\pi/2$ on the Universe Equator, at $\pi/2$ from P_N measured on Prime Meridian.

The corresponding Cartesian coordinate can be useful:

1. $x_1 = ct\sin(\psi)\sin(\varphi)\cos(\theta)$
2. $x_2 = ct\sin(\psi)\sin(\varphi)\sin(\theta)$
3. $x_3 = ct\sin(\psi)\cos(\varphi)$
4. $x_4 = ct\cos(\psi)$

Note that θ, φ are the Longitude and Colatitude of the sphere.

Also are useful the 4-vector

$$\boldsymbol{r} = (ct\sin(\psi)\sin(\varphi)\cos(\theta), \quad ct\sin(\psi)\sin(\varphi)\sin(\theta), \quad ct\sin(\psi)\cos(\varphi), \quad ct\cos(\psi))$$

and its derivatives ($t = \text{const}$ on the surface):

1. $\mathbf{r}_\theta = (-ct\sin(\psi)\sin(\varphi)\sin(\theta), \ ct\sin(\psi)\sin(\varphi)\cos(\theta), \quad 0, \quad 0)$
2. $\mathbf{r}_\varphi = (ct\sin(\psi)\cos(\varphi)\cos(\theta), \ ct\sin(\psi)\cos(\varphi)\sin(\theta), \quad -ct\sin(\psi)\sin(\varphi), \quad 0)$
3. $\mathbf{r}_\psi = (ct\cos(\psi)\sin(\varphi)\cos(\theta), \ ct\cos(\psi)\sin(\varphi)\sin(\theta), \quad ct\cos(\psi)\cos(\varphi), \quad - ct\sin(\psi)))$

These are 4-vectors of a Euclidean space: for us, there is the inner product and the angles it defines.

The three inner products are all equal to zero: $\mathbf{r}_\theta \cdot \mathbf{r}_\varphi = \mathbf{r}_\varphi \cdot \mathbf{r}_\psi = \mathbf{r}_\theta \cdot \mathbf{r}_\psi = 0$: they are orthogonal.

Once the angle ξ between two points, P_1 *with vector* $\mathbf{r}_1$ and P_2 *with vector* $\mathbf{r}_2$, has been calculated:

$$\xi = \left|\arccos(\frac{1}{c^2t^2}\mathbf{r}_1 \cdot \mathbf{r}_2)\right|$$

you can refer to the arc of great circle $r\xi$ to simplify the reasoning on light geodesics.

Saw the variables to use, it seems hard to set up the latter relation. Space and time variables are tightly coupled: it is not at all obvious to formulate a covariant expression for this space-time interval: $ds^2 = g_{\mu\nu}dx^\mu dx^\nu$.

In a coordinate system with origin in the center of the 4-Sphere and with respect to which the observer is stationary, we have seen that the maximum achievable speed for an object bound to the 4-Sphere surface is $v_t = ctd\xi/dt = c$. The overall speed of a ray of light is not constrained by the constant *c:* it is its tangential component.

Resuming what was said about the metric tensor, let us consider a solution in the form: $ds^2 = -h_{r\xi}d(r\xi)^2 + h_t c^2dt^2$ and the differential of the product $r\xi$: $d(r\xi) = ctd\xi + c\xi dt$. To obtain the desired geodesic we must put $c\xi dt = 0$ as if the radius *r* were a constant.

In our hypothesis the only possible spatial displacement in the radial direction occurs at constant velocity: the term $c\xi$ gives the Galactic Recession. As we will see, by considering the Universe expansion as a succession of equilibrium states, the velocity $v_r = dr/dt$ does not anyway appear in the Stress-Energy tensor nor directly in our application of the Einstein's equation. Holding out Galactic Recession from calculation for the metric $g_{\mu\nu}$, the tightly coupling of variables disappears so that we can look for a solution in the form $ds^2 = -h_\xi c^2t^2d\xi^2 + h_t c^2dt^2$. Here the expansion of the Universe manifests itself through the increasing term c^2t^2. Dilation of the distance, due to expansion, can only be felt at the interstellar level.

In this speculation, we have not yet discussed the principle of covariance. The precedent solution is given in the form $ds^2 = -h_\xi c^2t^2d\xi^2 + h_t c^2dt^2$ and, although it is possible to express the same metric in other coordinates, our quantity ds^2 does not represent the usual line element expression: in fact, given our setup it is completely useless incorporate the term c^2t^2 into $-h_\xi$, because in the absence of gravity, it is only h_ξ that tends to 1, while c^2t^2 remains unchanged. More precisely, the term c^2t^2 is antithetical to the presence of gravity, disappearing when gravity is present and being non-negligible when gravity is absent. Then, there is no point in achieving a covariant form at the expense of the equivalence principle.

This lack of generalization is the weakness of the logic plant but does not invalidate it. It is difficult to think of another representation of coordinates in which the same metric can be equally easily expressed but the use of this model is reserved for the geodesic of light.

Thus, we have variables whose differentials only partially enter the metric tensor *because in our conjecture the radial dimension cannot be perceived in any way*. Quantities to be used are therefore *cdt* and *ctd*ξ where the first describes a variation of time, the second a variation along the expanding 4-Sphere arc.

Notwithstanding the equation of geodesic $ctd\xi/dt = $ c, the speculation deals with the use of the Doppler-type redshift for the calculation of the galactic recession and of Special Relativity in the cosmic void. The purpose of the following analysis is to verify how much the presence of the gravity of the CMB can modify our result.

The idea is then to consider a sufficiently small region of the Universe where the Cartesian variable x can be merged with the arc ξ, so that $\Delta x \simeq c\,t\,\Delta\xi$, and to evaluate there the trend of the gravitational field over the last 10 billion years.

Although the underlying four-dimensional geometry is encoded in the pseudo-tensor used to describe recession, this additional radial degree of freedom cannot enter the local field equations, which are formulated strictly within the observable 3+1 spacetime. In this setting, the geodesics of the cosmic vacuum are perceived as straight lines, and the radial arc on S^3 is therefore projected as a linear spatial separation. Consequently, its observable projection satisfies the effective relation

$$\Delta x \simeq c\,t\,\Delta\xi,$$

which remains valid even when the expansion parameter ξ is not small, since the unobservable radial dimension does not contribute to local dynamics and the recession is interpreted as motion along an effective straight line up to the asymptotic limit where the source becomes unobservable.

Under these conditions, our line element becomes governed by a metric tensor.

The very small curvature of space in our present epoch confirms that the gravitational field is currently weak. We can therefore resume the analysis using the previously introduced infinitesimal coordinates

$$dx^\mu = ct\,d\phi,\ ct\,d\theta,\ ct\,d\psi,\ c\,dt.$$

We seek a model that approximates an almost flat space-time throughout the domain under consideration. Thus, we aim to derive the field equation in a form suitable for extrapolation back in time, allowing us to describe the propagation of light rays emitted by the most distant galaxies.

We have already seen before that, for each point $P(\varphi, \theta, \psi)$, the tangents to Colatitude, Longitude and Height are orthogonal: the angles between the coordinates φ, θ, ψ are always $\pi/2$. Then the differential arc is:

$$c^2t^2d\xi^2 = c^2t^2\sin^2(\psi)d\varphi^2 + c^2t^2\sin^2(\psi)\sin^2(\varphi)d\theta^2 + c^2t^2d\psi^2$$

If the vectors $\mathbf{e}_\varphi, \mathbf{e}_\theta, \mathbf{e}_\psi$ can be assumed as an orthogonal covariant basis of this space we note that, with the 4-Sphere radius $\boldsymbol{r} = ct\,\mathbf{e}_t$, the basis $\mathbf{e}_t$ for our time coordinate is orthogonal to the previous ones too (so it had to be on the basis of the Principle of Equivalence).

For the basis $\mathbf{e}_\varphi, \mathbf{e}_\theta, \mathbf{e}_\psi, \mathbf{e}_t$, a double angle rotation on ψ and φ is function of the current values of ψ_0 and φ_0

$$f_\psi = \sin(\psi) \quad \text{and} \quad f_\varphi = \sin(\varphi)$$

and it is given by:

$$\mathbf{C}(\psi,\varphi) \quad = \quad \begin{bmatrix} f_\psi & 0 & 0 & 0 \\ 0 & f_\psi\, f_\varphi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The compound transformation $g'_{\mu\nu} = \mathbf{C}(\psi,\varphi)\mathbf{C}(\psi,\varphi)\, g_{\mu\nu}\mathbf{C}^{-1}(\psi_0,\varphi_0)\mathbf{C}^{-1}(\psi_0,\varphi_0)$ gives the metric tensor for the rotation.

All points are equivalent; to simplify we choose the point at the Universe Equator in $P_{EM}(\pi/2, 0, \pi/2)$.

Having selected the coordinate system that simplifies the description of the hyperspherical geometry, the remaining relation is

$$c^2t^2\, d\xi^2 = c^2t^2(d\phi^2 + d\theta^2 + d\psi^2).$$

Recalling that our starting point was the interval

$$ds^2 = -h_\xi\, c^2t^2\, d\xi^2 + h_t\, c^2\, dt^2,$$

we can now pass to the Cartesian coordinates x, y, z, which represent the observable spatial variables of our 3+1 description.

Now let us solve the following field equation (with the cosmological constant set to $\Lambda = 0$):

$$\frac{8\pi G}{c^4}\, T_\mu^\nu = R_\mu^\nu - \frac{1}{2} R\, g_\mu^\nu,$$

in order to obtain the metric tensor $g_{\mu\nu}$for the interval

$$ds^2 = g_{\mu\nu}\, dx^\mu dx^\nu.$$

The analysis begins with Einstein's weak-field approximation,

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu},$$

where $\eta_{\mu\nu}$are the constant Galilean values of Special Relativity and $h_{\mu\nu}$are small correction terms. $\mathcal{E}_{0r} = c^2\rho_0$ is the current proper energy density of radiation. On the hyperspherical surface, the uniform radiation pressure p counteracts the tangential gravitational pull, acting as the stabilizing stress of the present configuration.

Let us consider the first law of thermodynamics for the 4-sphere in the form

$$dU_{\text{Univ}} = \delta Q - \delta W, \text{with } \delta Q = 0.$$

The expansion is assumed to occur under equilibrium conditions. In this context, for an infinitesimal volume element δV, we introduce an effective term δw associated with deviations from equilibrium of the geometric configuration (a work done to keep the Universe in its shape), leading to the relation:

$$\frac{\partial(c^2\rho\delta V)}{\partial t}dt = \delta w - p\frac{\partial \delta V}{\partial t}dt \qquad \text{as measured by a local observer}$$

in which matter, in the form of discontinuities in mass distribution, has no rule. You can eliminate it from the Stress Energy tensor. [The derivation of the above equation follows—after suitable modifications for my model—the principles set out in Section 86 of Tolman's seminal work *Relativity, Thermodynamics and Cosmology*.].

Even in the absence of local matter, radiation pressure remains a real property of the field. While it does not produce direct mechanical effects, it contributes to spacetime curvature and enters the global gravitational balance:

- Radiation doesn't interact with itself, but its momentum distribution defines pressure
- Without matter, this pressure cannot transfer force locally — yet in general relativity, its presence is not mediated by local interactions but through the energy-momentum tensor, which shapes the spacetime geometry via the metric
- In cosmology, this pressure plays a crucial role in the global energy balance and in governing large-scale dynamics, not in producing local mechanical effects.

From the equilibrium condition the right member is zero, then the Stress Energy tensor for this disordered radiation is: $T_{\mu\nu} = \mathrm{diag}(c^2\rho, 0, 0, 0)$ with $c^2\rho = \mathcal{E}_\mathrm{r}$, the density at radius r.

As an expression for volume we put $V = 2\pi^2 c^3 t^3$ for 4-Sphere surface and, for the previous balance, $\rho_0 V_0 \simeq \rho V$ is constant over time. We can, then, calculate mass (or energy) density and volume at present time. Moreover ρ_0 can be considered the density of a perfect fluid composed of radiation.

With the quantities $h_\mu^\lambda = \eta^{\lambda\alpha} h_{\mu\alpha}$ *and* $h = \eta^{\lambda\alpha} h_{\alpha\lambda}$, the field equation result

$$\left(h_\mu^\nu - \frac{1}{2}\delta_\mu^\nu h\right) = 4\int \frac{T_\mu^\nu}{r} dV = \frac{\mathrm{G}}{\mathrm{c}^4 ct}\delta_\mu^\nu \int \mathcal{E}_\mathrm{r}\, dV$$

We fix $r = ct = \text{const}$ throughout the integration volume because, in this framework, the relevant 'point of evaluation' of Einstein's equations is not a spatial location within a static 3-geometry, but any instantaneous point on the expanding 3-surface of the 4-sphere. Integration is therefore carried out on the hypersurface of constant r, which represents the physical space at a given cosmological time in this model.

Integrating on *V,* after calculating the quantity $\mathcal{E}_{0\mathrm{r}} V_0 = \int \mathcal{E}_\mathrm{r}\, dV \simeq const,$ we get:

$$\left(h_\mu^\nu - \frac{1}{2}\delta_\mu^\nu h\right) = \frac{4\mathrm{G}E_\mathrm{r}}{\mathrm{c}^4 ct}\delta_\mu^\nu \quad \mu = \nu \qquad 0 \quad \mu \neq \nu \qquad E_\mathrm{r} = \mathcal{E}_{0\mathrm{r}} V_0 \simeq const$$

We can see that $h_{\mu\nu} = \eta_{\mu\nu} h_0$. Values of h_μ^λ are all equals, say to $4h_0$, and with $h = 4h_0$ then follows:

$$h_0 = -\frac{2\mathrm{G}E_\mathrm{r}}{\mathrm{c}^4 ct}$$

and the space-time interval is

$$ds^2 = \eta_{\mu\nu}(1 + h_0) dx^\mu dx^\nu$$

Once the weak–field solution is obtained, the light geodesic cannot be derived from standard Relativity, since the relativistic 3+1 spacetime does not contain the radial dimension of the expanding hypersphere. The absence of this degree of freedom prevents the relativistic metric from representing the true null geodesic, which — although perceived as a straight trajectory — remains the expanding great-circle path on S^3. For this reason, the null condition must be applied to the four-dimensional pseudo-tensor that encodes the recession geometry, and it yields the effective light geodesic

$$d\xi = \frac{dt}{t},$$

which represents the observable trajectory of photon propagation in the cosmic vacuum. This relation follows directly from the effective line element

$$ds^2 = -\, h_\xi(t)\, c^2 t^2\, d\xi^2 + h_t(t)\, c^2\, dt^2,$$

by imposing $ds^2 = 0$ and adopting the weak-field condition $h_\xi(t) = h_t(t)$.

This relation will be used both to describe galactic recession and to analyse the behaviour of the CMB from recombination onwards.

Throughout this work, the term "geodesic" is used for light in its purely geometric sense: the shortest-length curve connecting two points on the expanding S^3 surface, without invoking any relativistic metric. After recombination, photons follow this minimal-arc trajectory while the radius grows as $r = ct$. The isoentropic evolution of the relic radiation fully determines the thermodynamic state of the CMB (pressure, temperature, and energy density) as a function of the expanding volume, so no additional metric information is required. In this context, the gravitational redshift plays no dynamical role in determining the light path; it simply reflects the isoentropic transformation of a Relic field emitted from a temporarily stationary universe.In this model, we have deduced the effective line element of the observable universe from the post-recombination kinematics of photon propagation on the expanding S^3 hypersphere. Since light moves at speed c along an arc whose radius increases as $r = ct$, the relation between emission time and angular separation arises directly from this expansion kinematics, without invoking a fully specified gravitational geodesic.

Let us do some calculation:

Calculation for h_0. (We assume that mass E_r is constant over time)

- Today energy density of CMB $\mathcal{E}_{0\mathrm{r}} = 4.02 * 10^{-14}\ J\ m^{-3}$
- Constant over time, energy $E_\mathrm{r} = \mathcal{E}_{0\mathrm{r}} V = 2\pi^2 r^3 \mathcal{E}_{0\mathrm{r}} = 1.69 * 10^{66}\ J$
- Constant $h_0 = -2.23 * 10^{-4}$

Verification of the gravitational redshift relative to the time when ray of light started from the farthest galaxy.

- The expansion speed c is constant over time. In the absence of other factors, it means that the distance, measured from source and receiver, between two successive wave crests does not change over time. <u>There is no redshift due to the expansion itself</u>.
- In absence of a relative angle ξ, that gives the Doppler effect, the redshift is the quotient between the proper times of receiver and transmitter $(g_{44\ today}/g_{44\ early})^{1/2}$, not in relative motion with respect to each other, as for the Schwarzschild metric:

$$1+z=\frac{\sqrt{1-\frac{2GE_{\mathrm{r}}}{c^5 t_{today}}}}{\sqrt{1-\frac{2GE_{\mathrm{r}}}{c^5 t_{early}}}}$$

For a galaxy at its maximum distance ($\xi \simeq 1$), $t_{Max} \simeq\ 5 * 10^9\ \ years$ value is $z = 1.86 * 10^{-4}$.

The latter value is the confirmation that throughout the observable universe gravity in the cosmic void remained negligible.

With this consideration and gravity negligible throughout the observable universe, the Einstein's model for weak fields has been correctly applied. This equation correctly represents the observable Universe:

- The very small value of h_0 shows that Special Relativity is effectively exact throughout the intergalactic vacuum, with deviations becoming relevant only at cosmological distances.
- Accepting a negligible error, Galactic redshift can always be calculated as Doppler redshift.

In this framework, the absence of a fully specified metric tensor for the CMB after recombination is not a limitation. The isoentropic evolution of the relic radiation already determines its thermodynamic properties—pressure, temperature, and energy density—as functions of the expanding volume. Since the redshift is purely gravitational and does not affect the propagation speed of light, no additional metric information is required. The radiation field is therefore fully characterized: its propagation is kinematic, its spectrum evolves isoentropically, reflecting only the gravitational transformation of a relic field emitted from a temporarily stationary universe.

ABOUT ENERGY AND ENTROPY

Let us now consider the energy and entropy of the Universe as a whole, based on this model that considers only the *CMB* where its redshift is exclusively gravitational. In this model, the Universe is homogeneous (and uniform) at every point, meaning that physical properties such as energy and gravity are evenly distributed throughout. Given the constant galactic recession velocity, neglecting the matter component in the variation of kinetic energy is justified.

Space is described as a four-dimensional spatial hyper-bubble that is continuously expanding, and where the *CMB* acts as a cohesive force, similar to the surface tension γ of a bubble:

ENERGY

The *CMB* cannot be considered to be undergoing free expansion, nor can it be seen as expanding against external pressure forces. That said and given the premises, according to the principle of mass-energy conservation, the energy of such a system changes only when something happens on its surface. In other words, According to Stokes' theorem, what happens at the system's boundary affects its total energy. Even though the universe has a finite volume, it does not have edges or boundaries. How could we ever explain a change in energy?

The condition $\partial_t(c^2\rho\,\delta V) = 0$ ensures that the CMB remains in isoentropic equilibrium on the expanding 4-sphere. Within this framework—where only General Relativity is used and no cosmological constant is introduced—the stress–energy tensor satisfies $T_{\mu\nu} = \mathrm{diag}(c^2\rho, 0,0,0)$: the energy density of the CMB is entirely encoded in T_{00}, with no additional vacuum term. The temperature evolution then follows solely from the gravitational redshift induced by the variation of the curvature radius, not from FLRW-type dilution.

The conservation law follows directly from the covariant condition $\nabla_\mu T^{\mu 0} = 0$. In the weak-field regime relevant to this model, the $\nu = 0$ component reduces to:

$\nabla_\mu T^{\mu 0} = c^2 \left(\dot{\rho}+\rho\,\frac{\delta\dot{V}}{\delta V}\right) = 0$ exactly equivalent to the equilibrium requirement $\partial_t(\rho\,\delta V) = 0$,

showing that the local decrease of the energy density is fully compensated by the geometric increase of the comoving volume element on the expanding S^3 sphere surface. No additional source terms, pressures, or vacuum contributions are needed: the conservation of the stress–energy tensor is satisfied purely through the geometry of the hyperspherical expansion.

Since the Universe has no boundaries, no energy flux through a surface can be defined. In this geometric setting, the condition $\partial_t(\rho\,\delta V) = 0$ ensures that the CMB remains in isoentropic equilibrium and that its total energy content stays constant. The absence of boundaries is therefore fully consistent with energy conservation: there is no external region into which energy could flow, nor any degree of freedom that could perform work on the system. [10]

Duality from a split Hyperspherical geometry

Receding matter, however, does not live entirely within this vacuum geometry. Its dynamics depends explicitly on the evolution of the cosmic radius and therefore on the cosmic time itself. Once the recession velocity and HEA acceleration are treated as dynamical quantities rather than geometric ones, the cosmic time ceases to be a mere parameter and becomes a genuine dynamical variable. This explicit time dependence breaks the time-translation symmetry in the Noether sense, leading to a non-conserved energy component. I denote this term by

$$E_{\mathrm{HEA}} = \underbrace{\frac{1}{2}\dot{r}^2}_{\text{radial kinetic energy}} + \underbrace{\frac{GM}{r}}_{\text{gravitational potential}} - \underbrace{\frac{h^2}{2r^2}}_{\text{centrifugal barrier}} - \underbrace{\frac{r^2}{2t^2}}_{\text{HEA geometric term}}$$

which quantifies the specific mechanical energy change experienced by matter as a consequence of the hyperspherical expansion. [see also *DOI: 10.5281/zenodo.20791295*]

$$\frac{dE_{\mathrm{HEA}}}{dt} = -\frac{\dot{r}^2}{t} + \frac{r^2}{t^3}$$

Thus, the same expansion produces two distinct energetic regimes: a Gauss-preserved vacuum sector and a Noether-broken matter sector. This duality is not a contradiction but a direct consequence of keeping the recession dynamics separate from the vacuum metric.

Effective energy, time dependence, and global conservation

The non-conservation of the effective HEA energy can be understood in the framework of time-dependent dynamical systems. In classical mechanics, energy conservation follows from the invariance of the Lagrangian under time translations. For a Lagrangian $\left(L(q, \dot{q}, t)\right)$, the corresponding energy function satisfies

$$\frac{dE}{dt} = -\frac{\partial L}{\partial t}$$

Therefore, an explicit dependence on cosmic time does not imply a violation of global energy conservation, but rather indicates that the considered subsystem is not autonomous.

The HEA radial dynamics contains an explicit time dependence through the cosmological terms. Consequently, the effective energy

$$\frac{1}{2}\dot{r}^2 + \frac{GM}{r} - \frac{h^2}{2r^2} - \frac{r^2}{2t^2}$$

is not, in general, an invariant of motion. The variation of E_{HEA} is therefore a property of the reduced radial description rather than a direct indication of energy loss from the complete hyperspherical system.

In the absence of matter perturbations, the motion follows the pure hyperspherical recession law $r = d(t)$, with $r = d + s$ and $\dot{d} = d/t$ and the explicit time-dependent contributions cancel, yielding

$$\dot{E}_{\mathrm{HEA}} = 0$$

The appearance of matter concentrations introduces local deviations $s(t)$ from the homogeneous recession. In this case, the reduced dynamics acquires an effective time dependence and HEA energy is no longer conserved.

Possible Geometric Interpretation (Speculative)

The introduction of matter breaks the symmetry of the purely homogeneous expansion, leading to an effective dynamics in which the HEA term explicitly depends on cosmic time. As a consequence, the effective mechanical quantity $\mathrm{E_{HEA}}$ is no longer conserved along matter trajectories. The HEA contribution does not represent a conventional form of energy, nor does it appear to transform into any known mechanical or radiative component. Instead, it modifies the kinetic evolution of matter through the effective equation of motion.

This apparent non-conservation, however, does not necessarily imply a violation of the global energy balance. In the absence of matter, the hyperspherical cosmological background evolves homogeneously on a closed S^3 manifold, where the absence of physical boundaries is consistent with the global conservation arguments discussed in [The Aporia of Reversibility in Cosmological Expansion].

It is important to distinguish the variation of the effective HEA energy from a conventional exchange between kinetic and gravitational potential energy. The Newtonian gravitational interaction is conservative and can only redistribute energy within the mechanical degrees of freedom of the two-body system. It cannot compensate for the explicit time dependence introduced by the HEA background term, which makes $\mathrm{E}_{\mathrm{HEA}}$ non-conservative in the reduced description.

Therefore, the variation of $\mathrm{E}_{\mathrm{HEA}}$ cannot be interpreted as a transfer of energy to or from the gravitational potential alone.

The presence of matter therefore raises the question of how this local change of effective mechanical energy should be interpreted while preserving the global properties of the hyperspherical background.

A possible interpretation within the hyperspherical framework is that the apparent local variation of $\mathrm{E}_{\mathrm{HEA}}$ is associated with the coupling between matter-induced perturbations and the geometric background. Matter concentrations can be regarded as local deformations of the hyperspherical expansion, introducing additional geometric degrees of freedom not included in the reduced two-body description. In this view, the variation of the effective HEA energy does not represent the creation or destruction of energy, but rather an exchange with degrees of freedom of the complete geometrical system.

In this view, the variation of the effective HEA energy does not represent the creation or destruction of energy, but rather an exchange with degrees of freedom of the complete geometrical system. This interpretation is conceptually analogous to the geometric work term δw introduced in the global energy balance of the hyperspherical model, where the maintenance of the cosmic configuration may involve an effective work contribution associated with the evolving geometry [See on this point: Dynamics on an Expanding Hypersphere: Reassessing the Cosmological Principle in light of the CMB]. However, the relation between δw and the local variation of E_{HEA} is not derived here and remains an open question.

This interpretation remains speculative and is not derived from the present formalism. However, it provides a possible connection between the local non-conservation of the effective HEA energy and the global conservation expected for a closed hyperspherical geometry.

ENTROPY

The expansion is adiabatic and occurs without friction; it proceeds uniformly because of the absence of boundaries, just as the reverse process would. This is not comparable to the expansion of a gas in a piston, which is irreversible at finite rate. Here the dynamics are governed solely by gravitational forces, which are conservative. Therefore, we must conclude that entropy remains constant even though the expansion does not occur at an infinitesimally slow rate. The conservation of entropy is fundamental not only to maintain the blackbody spectrum of the CMB but also to ensure the correct evolution of temperature and thus of the energy of cosmic radiation over time. The possibility of reversible transformations occurring at finite rate in relativistic thermodynamics—unlike in classical thermodynamics—is discussed in detail in Tolman's classic text, *Relativity, Thermodynamics and Cosmology*.

Energy and entropy's thermodynamic implications are further explored in: The Aporia of Reversibility in Cosmological Expansion.

[10] – A useful geometric analogy can be drawn by considering a two-dimensional observer living on the surface of a soap bubble. Such an observer has no access to the bubble's interior or exterior and can measure only quantities intrinsic to the surface—most notably, its total surface energy.

If this quantity remained constant over time, the observer would be forced to conclude that:

1. No energy flows through the surface, implying internal and external pressures must be in equilibrium.
2. Any time evolution of the bubble must be inertial: the surface is either static or expands at a constant rate. An accelerated expansion would require an energy transfer across the surface, which the observer would detect.

This analogy can be extended to the cosmological model explored in this work. After the end of inflation, the primordial plasma may be assumed to have collapsed entirely onto the surface of a four-dimensional expanding hypersphere, leaving its interior physically empty.

Within this picture:

1. the CMB energy density plays a role analogous to a surface energy distributed over the 3-dimensional boundary of the 4-sphere;
2. the absence of any interior or exterior medium capable of exchanging energy with the surface naturally leads to energy conservation on the hypersphere;
3. the global expansion law $r = ct$ follows as the inertial evolution of a surface whose total energy remains constant, in analogy with the soap bubble.

The analogy is not intended as a physical model of the Universe's interior or exterior—neither of which appear in the theory—but as a conceptual aid:

– a system whose observable energy is entirely confined to a hypersurface evolves inertially unless acted upon by external influences.

This mirrors the central assumption of the 4-Sphere cosmology that all relevant physics resides on the expanding boundary $r = ct$, while the interior plays no dynamical role.

TRACING COSMOLOGICAL EVOLUTION THROUGH THE 4-SPHERE METRIC CONIECTURE

Photon Geodesics and Mean Free Path Constraints in an S^3 Cosmological Model

These brief considerations do not address the inflationary phase, during which—even preserving the 4-Sphere geometry—we must assume a radius growth no longer governed by a linear relation, but rather by an exponential form, thereby abandoning our assumption of equilibrium conditions. This inflationary burst can be interpreted as the rapid release of a geometric potential inherent to the 4D structure preceding the bubble. Analogous to a tightly compressed sphere of elastic material—like a chewing gum bubble—the system begins as a point of infinite tension. As the volume expands, the potential decreases, and the bubble forms without rupture. The energy transferred to the 3D universe during this phase stems from the decay of a 4D potential that, after driving its expansion, ceases once equilibrium is reached [12].

Instead, we focus on the universe starting from the radiation era onward, where the expansion can be treated as balanced, although its rate is no longer temporally constant. The current radius of the sphere, hypothesized as 13.8 Gly, undergoes negligible variation when calculated from the time of recombination to the present. The same applies to the estimated age of the universe.

[12] – THE CONJECTURE: This phase likely corresponds to a breakdown of the equilibrium condition $\delta w - p\frac{\partial \delta V}{\partial t}dt = 0$. The work δw, previously introduced in the context of the bubble, is no longer suitable when describing inflation. We must instead use $-\delta\phi$ to generalize the type of energy involved. This potential energy $-\delta\phi$, perceived within our three-dimensional space, originates from a four-dimensional potential.

In the early stages, the geometric potential $\delta\phi$ may be strongly negative, making $-\delta\phi$ large and positive — a condition that triggers rapid volumetric expansion. As time progresses, $\delta\phi$ evolves, restoring the equilibrium condition and leading to the regulated expansion regime described in the model. Notably, this inflationary burst arises without the need for dark energy (in the form of negative pressure), inflaton fields, or any speculative components.

The inflationary phase can be pictured as the release, irradiated from a punctiform origin, of a primordial 4D condensate, an undifferentiated mass of geometric potential existing before the formation of the bubble. At the end of inflation, this potential ceases marking the emergence of the bubble.

Metric Invariance and the Primordial Plasma Constraints

Extending the results obtained from the weak-field solution, where the effective metric retains its homogeneous and isotropic form, we note that a rigorous geometric constraint applies even during the early radiation era and the pre-nucleosynthesis epoch. This constraint is dictated by the constancy of the tangential speed of light (c) along the null geodesics. Confirmed in the weak-field limit, this condition permanently imposes the global metric profile:

$$ds^2 = -h_\xi(t)\, c^2 t^2\, d\xi^2 + h_t(t)\, c^2\, dt^2 \qquad \text{with } h_\xi(t) = h_t(t)$$

or

$$ds^2 = -h(t)[c^2 t^2\, d\xi^2 - c^2\, dt^2].$$

Remarkably, this geometric framework persists independently of the local gravitational regime, obeying exclusively the global boundary conditions of light propagation.

Consequently, we can infer that following the inflationary epoch, the universe is governed by a single, structurally invariant metric, where the historical evolution of the system is reflected solely through the varying scaling factor *h(t)* across different cosmological eras.

However, a crucial distinction must be made regarding the physical visibility of this metric. During primordial nucleosynthesis and the subsequent radiation-dominated plasma phase, the metric does not directly map the effective physics of light. Because the mean free path of photons (λ_c) is drastically suppressed by continuous interactions within the opaque medium, imposing the standard null geodesic condition for free photons, $ds^2 = 0$, fails to yield a macroscopically observable radial expansion.

The fact that the metric tensor remains structurally unchanged across the nucleosynthesis boundary indicates that the geometry of the S^3 hypersphere is more fundamental than the local thermodynamic physics of the plasma. The metric is not dynamically generated or deformed by the local energy-matter content—as posited in standard general relativity—but is instead predetermined by the global geometric structure and the potential for the free propagation of light.

To avoid misleading interpretations inherent to the extra-dimensional radial coordinate, the relationship between radiation, matter, and spatial volume must be analyzed strictly within the three-dimensional manifold of the S^3 hypersphere. Within this volume, the photon mean free path λ_c acts as a geometric confinement regulator.

Prior to recombination, as λ_c drops toward microscopic scales, the intense Thomson scattering forces radiation into an ultra-dense random walk. Consequently, the effective spatial volume accessible to photons collapses until it perfectly coincides with the static space occupied by baryonic matter. Because photons cannot diffuse away from the massive particles due to the continuous sequence of causal impacts, the radiation field is strictly localized. This spatial equivalence prevents any independent volumetric expansion of the photon field relative to the plasma, freezing the 3D volume of S^3 into a temporary isochoric state.

This constraint is abruptly lifted at the last scattering surface: as λ_c diverges, the volume accessible to radiation instantaneously decouples from the static volume of matter, allowing the radiation field to freely occupy the full geometric potential of the hypersphere and setting the stage for the subsequent, unified expansion of the cosmic manifold.

In the 4-Sphere model, the photon mean free path acts as the ultimate cosmological regulator. When radiation is entirely unconstrained the universe expands freely following the linear boundary $r = ct$. Conversely, when the mean free path drops toward zero, the isotropic scattering dampens the radial momentum, bringing the global expansion to a temporary stasis just prior to recombination. The metric expansion is only allowed to resume once the universe becomes transparent, unlocking the coherent radial propagation of the relic photon field.

In conclusion, the 4-Sphere model introduces a profound paradigm shift in cosmological dynamics: the metric architecture of the universe remains unique and structurally invariant across all epochs. The physical transitions traditionally associated with changing energy-matter contents do not warp or redefine the spatial geometry itself; instead, they modulate the behavior of the scaling function $h(t)$. It is the thermodynamic state of the radiation field—specifically, the degree of freedom of photons as dictated by their mean free path—that drives the evolution of $h(t)$, causing it to accelerate, decelerate, enter a temporary isochoric stasis, or resume its linear expansion. Geometry, therefore, acts as the absolute stage, while the kinetic physics of light serves as the ultimate regulator of cosmic expansion.

The 4-Sphere at the Last Scattering

The conjecture concerning the last-scattering dynamics represents the least constrained element of the model. Although it provides a coherent qualitative mechanism within the proposed 4D framework, it should be regarded as the most speculative assumption in the present construction.

A kinematic discrepancy becomes explicitly manifest in the immediate aftermath of the last scattering epoch, explaining why baryonic matter experiences a delayed expansion relative to the decoupled radiation field. The primordial plasma is intrinsically inhomogeneous, containing localized matter clumps held in hydrostatic equilibrium by the trapped radiation.

When the relic radiation abruptly decouples and becomes the Cosmic Microwave Background (CMB), the baryons lose the radiative support provided by the tightly coupled photon field. If the gravitational contribution of the relic radiation suddenly collapses, the overall binding of the primordial plasma is drastically reduced. As a result, some of the previously coherent baryonic regions would no longer remain mutually gravitationally bound and could drift apart as independent patches.

In the framework adopted here, a four-dimensional inertia principle is introduced as a specific hypothesis of the model: once the inflationary forces are exhausted, the expansion of the bubble proceeds at a constant radial speed c in 4D space. This assumption does not follow from standard general relativity, but defines the geometric context within which the dynamics of matter–radiation decoupling are interpreted. At recombination, the relic radiation is suddenly released from the plasma and reaches the radial speed c, while the baryonic matter, no longer supported by the tightly coupled photon field, remains temporarily at smaller radii. A transient phase therefore appears in which the bubble acquires a finite thickness. In this framework, the existence of an intrinsic surface tension associated with the geometry of the bubble is postulated: this tension tends to reduce the thickness generated by the decoupling and, in doing so, triggers the motion of the matter toward the radial speed c, progressively restoring the integrity of the boundary. As a result, previously coherent baryonic regions, no longer gravitationally bound to each other, can spread into space, ensuring, at least in part, a large-scale distribution of matter.

NB. The paper A Geometric Blueprint of the Early Universe: Primordial He, D, and Li presents an extended discussion of the topics covered in this section and provides a comprehensive description of recombination and of the preceding phases as viewed within the 4-Sphere model.

4-SPHERE RECOMBINATION REACTIONS

In this framework, the early universe originates from a Big Bang followed by a four-dimensional (4D) inflationary expansion. Upon the exit from inflation, the universe enters a radiation-dominated era, establishing a 4D hyperspherical bubble whose initial radial expansion proceeds inertially at the speed of light, satisfying the geometric condition $r = ct$. During this early phase, the mean free path of photons is effectively unconstrained, allowing radiation to propagate coherently along the radial direction of the fourth spatial dimension.

However, as the universe cools, the progressive formation of baryonic matter and the consequent rise of a dense, ionized plasma trigger a dramatic drop in the photon mean free path due to intense Thomson scattering. The individual photons, while still moving locally at the speed of light c, are forced into an isotropic random walk within the three-dimensional (3D) space. This confinement effectively reduces their net radial velocity component to zero, acting as a powerful thermodynamic brake that temporarily halts the global expansion $dr/dt = 0$.

Consequently, the early universe is assumed to be isochoric just prior to recombination, with its latent dynamics still dominated by this trapped relativistic radiation. At the last scattering epoch, the sharp decline in free electron density triggers a sudden kinetic decoupling. As the mean free path of photons diverges, radiation escapes the plasma, resuming its outward propagation into the newly available 4D volume. Because the photons carrying the threshold energy for the inverse process (photoionization) escape the medium, the reverse reaction is abruptly halted. This rapid transition allows the universe to break its temporary stasis and resume its expansion. Crucially, as the radiation field escapes along the radial direction of the fourth spatial dimension, it exerts a global geometric drag on the matter fields. This transfers momentum to the relic plasma collectively, without requiring individual, localized photon-matter interactions.

Recombination therefore proceeds as a strictly kinetic and non-equilibrium process: photons produced in direct radiative transitions escape the local plasma, preventing the inverse photoionization process and leaving the baryonic matter to evolve independently. Consequently, while radiation fills the newly expanding extra-dimensional volume, matter is left to follow its own distinct cooling history, establishing the thermodynamic conditions necessary for subsequent cosmic structure formation.

Thus, in the isochoric phase temperature continues to decrease until it reaches $52{,}000K$, only then the main recombination reaction begins:

$$H^{+} + e^{-} \rightleftarrows H + h\nu$$

where the backward reaction requires photons with at least 13.6 eV of energy for the ground state, and it is favored at high temperatures. The tail of the relic radiation spectrum at $52{,}000K$ would be sufficient to allow the inverse reaction, but the reactant electrons are being consumed and the entire radiation escapes from the plasma.

The main reactant $h\nu$ of the backward reaction is missing allowing us to think of very high reaction kinetics:

$$H^{+} + e^{-} \rightarrow H + h\nu$$

During the nucleosynthesis epoch, the local expansion velocity undergoes a progressive deceleration directly driven by the evolution of the baryonic density. As the local concentration of baryonic matter increases, the photon mean free path drastically drops ($\lambda_c \to 0$). This ultra-dense confinement acts as a microscopic thermal brake, shifting the local frame into a perfectly static, isochoric state. This pristine, non-expanding window provides the exact thermodynamic conditions required for primordial nucleosynthesis to occur at equilibrium.

The subsequent last scattering epoch acts as a sudden breaking of this stationarity: as the plasma neutralizes and ($\lambda_c \to \infty$), the radiation field instantly transitions to radial null geodesics at the speed of light c. Due to its profound 4D inertia, the baryonic matter—which had been brought to a complete halt during the pre-nucleosynthesis phase—cannot instantaneously match this velocity step. This cosmic discrepancy manifests as a transient, self-limiting phase shift along the 4D wavefront, where the lagging matter is gradually re-accelerated from $0\ to\ c$ by the combined action of the hyperspherical surface tension and late-time CMB dipole drag.

We contend that this dynamic sequence—characterized by a continuous deceleration leading to a complete expansion halt, followed by an instantaneous radiative decoupling—is physically admissible, mathematically coherent, and structurally necessary.

The apparent paradox of an abrupt, step-like re-expansion following a gradual deceleration is resolved by distinguishing their physical drivers: the slowdown is a macroscopic hydrodynamic effect, whereas the decoupling is an instantaneous, microscopic quantum phase transition occurring at the 52,000 K kinetic threshold.

Once electron capture is triggered, the newly liberated radiation field instantly evacuates the medium at the speed of light c. Because the photons physically escape the local frame, the reverse reaction (photo-ionization) is strictly suppressed due to reagent starvation. This asymmetric, one-way kinetic path guarantees a clean, rapid decoupling that preserves the isotropic blackbody spectrum of the CMB without cumulative Doppler distortions. While the mass-less radiation field instantly occupies null geodesics, the baryonic infrastructure—anchored by its 4D inertia—remains initially at rest, entering a transient phase shift where it is gradually accelerated toward c by the restoring force of the hyperspherical surface tension over macroscopic cosmic timescales. The entire continuum of the model is therefore preserved without any unphysical discontinuities.

Assuming that matter reaches the expanding CMB only after about one million years. During this epoch, the massive presence of relic photons makes the inverse recombination reaction possible, leading to a partial reionization of the gravitationally formed clouds.

1. $H_2 + h\nu \rightarrow H + H$
2. $H + h\nu \rightarrow H^+ + e^-$ our forbitten backward reaction at the Recombination

From this point onward, hydrogen reactions proceed in equilibrium, with the balance between ionization and recombination governed by the evolving thermodynamic conditions.

As the background cools toward $3000K$, hydrogen chemistry stabilizes and molecular hydrogen forms in equilibrium. This transition provides the first efficient cooling mechanism, allowing gravitational fragmentation and the earliest stages of galaxy formation.

In particular, when photons act as reactants and are consumed, the undesired reactions to consider are: [13]

1. $H^- + h\nu \rightarrow H + e^-$ a competing step reaction for the molecular Hydrogen production

Atoms become able to approach each other and form molecules. This process, however, requires time, further expansion, and cooling, and occurs in the absence of catalytic activity from dust. The relevant reactions are:

1. $H + e^- \rightleftarrows H^- + h\nu$
2. $H^- + H \rightleftarrows H_2 + e^-$
3. $H + H^+ \rightleftarrows H_2^+ + h\nu$
4. $H_2^+ + H \rightleftarrows H_2 + H^+$
5. $H + H \rightleftarrows H_2 + h\nu$

We should expect hydrogen in atomic form even at temperatures below the binding energy. However, we want H_2 to persist even in the absence of dust catalysts, in order to ensure the desired, nearly standard, evolution of the hydrogen halo.

Note**:** Radiation emitted by post-recombination reactions occurs within gravitationally bound matter regions and is subject to Doppler shifts and local dynamics. As such, it does not contribute meaningfully to the isotropic Cosmic Microwave Background spectrum, nor does it undergo Thomson scattering. Its spectral imprint remains confined to proto-galactic environments and is negligible compared to the relic radiation from the Radiation Era.

[13] – [arXiv:astro-ph/0506221] – Cosmological Implications of the Uncertainty in H- Destruction Rate Coefficients

THERMAL EVOLUTION OF MATTER AND RADIATION IN AN EXPANDING UNIVERSE

We consider a universe expanding linearly as $r = ct$, with spatial geometry described by S^3 sphere. The volume evolves as:

$$V(t) = 2\pi r^3(t) = 2\pi(ct)^3 \propto t^3$$

This allows us to define a natural scale factor:

$$a(t) = \frac{t_0}{t}$$

where t_0 is the initial cosmic time. Temperatures evolve as:

- Radiation: $T_\gamma = \frac{t_{rec}}{t}$
- Matter: $T_m = \begin{cases} T_{rec}\left(\frac{t_{rec}}{t}\right)^2 & \text{for } t \le t_{bind} \\ T_m(t_{bind}) & \text{for } t > t_{bind} \end{cases}$

The cooling picture is accurate, as for matter the decisive factor is not the spacing between atoms but the volume available to *radiation*.

We assume:

- Temperature at recombination $T_{rec} = 52{,}000K$
- Time at recombination $t_{rec} = 720{,}000\ yr$
- Time when matter becomes gravitationally bound $t_{bind} \sim 3$ Myr

where for t_{bind} we can mean the time of formation of molecular hydrogen, when T reaches 3000K.

The Reconnection Epoch and Partial Reionization (~1 Myr)

After the instantaneous radiative decoupling at 52,000 K ($\sim$ 720.000 yr)), the massless radiation field and the lagging baryonic matter undergo distinct, uncoupled isoentropic cooling rates within the temporary 4D thickness of the manifold. During this brief window of radiative isolation, the matter cools rapidly at a quadratic rate, while beginning to coalesce into primitive, gravitationally bound gas clouds and localized overdensities.

Driven exclusively by the hyperspherical surface tension of the 4-Sphere $r = ct$, the lagging matter is continuously accelerated along the radial coordinate. This geometric constraint forces a macro-scale reconnection with the radiative wavefront after approximately one million years. At this specific junction, the radiation field has cooled to an average temperature of approximately 37,700 K.

When the cold, structured baryonic matter re-enters the radiation field, it is swept by the highly energetic ultraviolet tail of the CMB's Planck spectrum. This interaction reactivates the inverse reaction $H + hv \rightarrow H^{+} + e^{-}$, triggering a strictly partial and localized reionization of the primordial gas.

The reionization remains partial due to two concurrent factors:

- Spectral Filtering: Only the fractional subset of photons residing in the high-frequency tail of the 37,700 K blackbody spectrum possesses sufficient energy 13.6 GeV.
- Spatial Asymmetry: Because the baryonic matter is no longer homogeneous but organized into discrete clouds, the bulk of the CMB photons propagates uninterrupted through the expanding cosmic voids. Only the radiation directly striking the overdensities experiences localized scattering and electron density fluctuations.

This non-uniform, macroscopic impact provides a profound, purely geometric explanation for the secondary temperature anisotropies and acoustic peaks observed in the CMB power spectrum today, avoiding any unphysical global re-thermalization.

The Binding Threshold and Molecular Synthesis t_{bind} (~3 Myr)

Following the initial reconnection at 1 Myr, the coupled matter-radiation system continues to expand and cool in a transient state of dynamic equilibrium. This regime holds until the universe reaches the critical binding time, defined as t_{bind} (~3 Myr).

Visually and physically, t_{bind} represents the exact chronological threshold where the evolving thermodynamic conditions allow the synthesis of stable molecular hydrogen. At this epoch, the background temperature drops to the 3,000 K boundary. This temperature drop carries a dual physical consequence that permanently alters the cosmic evolution:

- The Extinction of the Backward Reaction: At 3,000 K, the high-energy UV tail of the radiation field is completely extinguished. The photons no longer possess the kinetic energy required to ionize hydrogen, causing the backward photo-ionization reaction to cease permanently.

- The Genesis of Molecular Hydrogen H_2: Below 3,000 K, the thermal velocity of neutral hydrogen atoms is low enough to permit stable chemical binding. The atoms successfully combine to form molecular hydrogen.

Operating as the primary and most efficient rotational coolant of the pristine gas, the rapid formation of H_2at 3 Myr enables the localized, isochoric collapse of primordial clouds. By effectively lowering the internal pressure of these cold overdensities, the synthesis of molecular hydrogen breaks the global expansion regime at a local scale, providing the mandatory mechanism to ignite the very first generation of stars.

Consequently, t_{bind} marks the definitive zero-thickness alignment of the 4D manifold, closing the geometric transition era and initiating the structured chemical history of the cosmos.

This scenario aligns with current observations of massive galaxies at high redshift, and does not require exotic components such as non-baryonic dark matter to initiate early collapse. It provides a natural pathway for the formation of galaxies.

The first stars (Population III) form from pristine, metal-free gas. Their formation is enabled by molecular cooling and gravitational instability. HD 140283, a Population II star, likely formed after these first generations, but still very early in cosmic history.

In the $r = ct$ model, the recombination epoch remains the earliest viable starting point for star formation. Therefore, the age of any star must be less than the cosmic time elapsed since recombination. This ensures consistency across models and avoids paradoxes such as stars appearing older than the universe itself.

Implications for Stellar Age Estimates:

- The age of a star is not affected by the choice of cosmological model, as long as the time is measured from recombination forward.
- The $r = ct$ framework allows for early structure formation, making ancient stars like HD 140283 fully compatible with the model.
- If a star were found to be older than the recombination epoch, it would challenge all physical models, not just cosmological ones.

REFLECTIONS ON THE CMB: BETWEEN HYPOTHESIS AND MODEL

Since the CMB originated from a stationary universe, the Doppler effect can be excluded, and its redshift can be attributed solely to a gravitational component.

In both the standard model and the present framework, the primordial plasma remained in a state of near-thermal equilibrium for an extended period prior to recombination. The key difference lies in the kinetics of recombination itself. While ΛCDM assumes a gradual process governed by chemical equilibrium, the 4-Sphere model describes a rapid, non-equilibrium recombination triggered by the sharp decline in free electron density and the consequent exponential increase (divergence) of the photon mean free path. This surplus energy causes all the radiation to escape swiftly, driving the expansion forward. Under these assumptions—violent recombination, isoentropic escape and free streaming of radiation as $r = ct$, and only a tiny fraction of radiation engaged—the global CMB spectrum remains a blackbody. Any "alteration of the peak" is best understood as a small effective temperature rescaling, not a shape distortion.

However, the radiation released during recombination represents only an extremely small fraction of the total relic CMB energy. Quantitatively, its contribution is of order $\sim 10^{-9}$ relative to the background energy density: this small burst of remains dynamically negligible compared with the pre-existing relic field. As a consequence, any energy exchange associated with recombination affects only a minute portion of the photon population. The global CMB spectrum therefore retains its blackbody form, and any potential μ or y distortion is expected to lie far below current observational sensitivity, amounting effectively to a tiny rescaling of the effective temperature rather than a genuine spectral deformation.

In this framework, we focus on the geometric conditions occurring precisely at the last scattering surface. At this exact cosmic threshold, the freely propagating radiation field—having just undergone its final microscopic interactions with the plasma—maps out and occupies a newly available 4D spatial volume, which matter subsequently and gradually tends to occupy. Because the relic matter is left in direct geometric contact with this final wavefront of radiation, the two components begin to expand in unison as cosmic time progresses. From a macroscopic perspective, this collective outward motion appears as if the matter fields are geometrically dragged by the expanding boundary of the radiation field along the extra-dimensional radial direction. This mechanism ensures that the preceding conclusions regarding the spectral purity of the CMB remain robust, as the spatial realignment of matter followed the last scattering boundary, involving no subsequent local energy injection or disruptive thermodynamic interactions relative to the relic background.

The physics of the CMB that we treat as disordered radiation is compatible with the 4-Sphere model, in the sense that, taking a snapshot of the Universe, an observer O_1 is expected to see the CMB as coming from a homogeneous spherical radiation source of which the observer is in the center O.

To show that compatibility, we consider a celestial body with a peculiar motion (namely: that it is in motion with respect to the expanding frame of the CMB) as equivalent to another observer O_2, this time in motion, in the instant in which he meets O_1 and crosses the center of the sphere.

The question that arises is: Does O_2 measure in O the same Radiation Intensity as O_1 or not? Or said in another way: Can a celestial body, for the sole fact that it has a peculiar motion. measure a temperature anisotropy of the CMB surrounding it?

We are not interested in knowing the distribution of energy on the surface of the sphere. For our purpose we can simply study the behavior of the radiation lying in a generic plane passing through O. Then with $\beta = v/c$, the frequency ν' due to the Oblique Doppler Effects is:

$$\nu' = \frac{\nu(1-\beta^2)^{1/2}}{1-\beta\cos\theta}$$

and for $\beta \neq 0$ we can get the indefinite integral, with the substitution $u = \tan(\theta/2)$, starting from:

$$\frac{(1-\beta^2)^{1/2}}{\beta}\int\frac{1}{1/\beta-\cos\theta}\,d\theta$$

The definite integral, in the open interval (-π, π) is $2\pi\,\beta(1-\beta^2)^{-1/2}$ and we get $\nu' = \nu$ for mean values of frequency. In other words, Redshift and Blueshift compensate each other in the mean energy of the disordered radiation. (If this is true for a generic circumference then it is true for the whole surface of the sphere). Therefore $O_1$and O_2 measure the same mean temperature of CMB*. All this is consistent with 4-Sphere.*

PRIMARY AND SECONDARY ANISOTROPIES OF THE CMB

Within the 4-Sphere framework, the elapsed time for a light beam traveling along an arc of angular length θ is given by the relation:

$$t_{from} = t_{tnow}e^{-\theta}$$

A ray of light that completes a full loop and reaches us after a rotation of 2π had an age of approximately 25.4 million years when it began its journey. At that epoch—and even earlier—no stars yet existed. The CMB radiation observed today from opposite directions has therefore completed a full circuit around the universe, after initially propagating just over half a loop (≈ 3.56 radians) since the last scattering.

In the absence of BAO acoustic oscillations, recombination does not produce a coherent quadrupole of the radiation field; consequently, primary anisotropies consist solely of temperature fluctuations, with no polarization generated at this stage.

As described in section [REIONIZATION AND GALAXY FORMATION IN A CONTEXT OF ISENTROPIC RADIATION], the generation of secondary anisotropies is not a continuous process but is constrained to the specific window between the Reconnection Epoch (1 Myr) and the Binding Threshold (3 Myr).

During this interval, the CMB photons traverse primordial gas clouds that are still partially ionized due to the high-energy tail of the Planck spectrum. The interaction with such structures introduces local alterations in the spectrum and intensity of the radiation, generating secondary anisotropies. Because the elapsed time for the CMB to generate these secondary anisotropies is strictly less than 3 Myr—the moment corresponding to the synthesis of stable molecular hydrogen (H_2)—the scattering process is naturally self-limiting.

Beyond the 3 Myr threshold, the background temperature falls below 3,000 K, the backward photo-ionization reaction ceases, and the rapid formation of H_2 enables the isochoric collapse of pristine gas. Consequently, the CMB photons that reach us today have effectively 'sampled' the structured baryonic medium only during this brief, pre-molecular phase, ensuring that the primary anisotropies remain preserved while the secondary modulations provide a unique signature of the early-time geometric and density fluctuations.

The main modes of interaction, the causes of which were formally described in section [REIONIZATION AND GALAXY FORMATION IN A CONTEXT OF ISENTROPIC RADIATION], include [see also 14].:

- Thomson/Compton scattering on free electrons within ionized clouds, redistributing photon energies.
- Absorption and emission lines from neutral or partially ionized gas, leaving local spectral imprints.
- Overall attenuation of radiation along specific directions, resulting in angular variations in the observed intensity.

At a later stage, corresponding to the last billion years of cosmic evolution, the CMB travels through the observable universe as defined by the model. During this period, it can interact with more evolved structures, eventually being absorbed or modified by galaxies and galaxy clusters.

In the 4-Sphere framework, the primary CMB anisotropies reflect the initial "peaks" frozen at recombination. Shortly after last scattering, photons may interact with nearby forming halos, producing secondary anisotropies. These effects are subtle and cannot provide a practical observational test to distinguish 4-Sphere from standard FLRW cosmology, but they illustrate how line-of-sight interactions might be influenced by alternative geometrical frameworks.

[14] –Reevaluating the Necessity of Dark Matter and Dark Energy within Cosmological Models examines whether anisotropies in the cosmic microwave background, interpreted within the four-sphere geometry, can explain the observed peaks in the power spectrum, not as acoustic oscillations, but rather as ripples generated by an initial inflationary phase. In the 4-Sphere, this inflationary explosion is explained without the need for dark energy (in the form of negative pressure), inflaton fields, or speculative components.

CONCEPTUAL DIPOLE TEST USING A SPACE PROBE

Intrinsic anisotropy models are cosmological scenarios in which the CMB dipole is not generated solely by the observer's motion but contains a primordial or late-time anisotropic component. In such models, the dipole cannot be removed by observing in a direction orthogonal to the observer's velocity, because it does not arise from a geometric Doppler effect. Therefore, the orthogonality test proposed here provides a clean discriminator: a vanishing dipole confirms a purely kinematic origin (as in FLRW or in the 4-Sphere model), whereas a non-vanishing dipole would falsify all intrinsic anisotropy models.

To clarify the geometric nature of the CMB dipole, consider an idealized space probe equipped with a microwave radiometer capable of measuring the CMB temperature along arbitrary lines of sight. The probe moves along a known trajectory within the Local Group, whose barycenter provides a good approximation to the CMB quasi–rest frame. At any given instant, one can define the plane spanned by (i) the probe's instantaneous velocity vector relative to this barycenter and (ii) the line connecting the probe to the barycenter itself. A line of sight orthogonal to this plane is then, by construction, orthogonal to the probe's velocity with respect to the CMB rest frame. Along such a direction, the projection of the observer's velocity vanishes, and therefore any kinematic contribution to the CMB dipole must be strongly suppressed, ideally approaching zero.

This geometric configuration allows us to test directly whether the observed CMB dipole is purely Doppler in origin. If the dipole measured along the orthogonal direction indeed vanishes, the result is fully consistent with any cosmological model in which the CMB dipole arises solely from the observer's motion with respect to the radiation rest frame. Conversely, if a residual dipole were detected even when the velocity projection is null, the dipole could not be interpreted as kinematic, and an intrinsic anisotropic component would be required. This motivates the following concise formulation of the test and its implications.

Taken together, these considerations show that the orthogonality test provides a clean and model-independent discriminator. A vanishing dipole confirms the kinematic interpretation assumed in FLRW and in any framework where the CMB uniquely defines the cosmological rest frame. A non-vanishing dipole, instead, would falsify all intrinsic anisotropy models and would require a revision of the standard interpretation of the CMB dipole. The conclusion follows naturally.

ON THE DISTINCT REST FRAMES DEFINED BY THE CMB AND BY LARGE-SCALE MATTER DIPOLES

In cosmological models where the CMB uniquely defines the global isotropic frame—such as the 4-Sphere framework in which matter is treated as a perturbative field—the two dipoles are not required to coincide. The following conceptual test illustrates this point.

Recent analyses of radio-source and quasar number counts have reported a dipole amplitude significantly larger than the kinematic dipole inferred from the CMB. While the dipole directions remain statistically compatible within current uncertainties, the amplitude discrepancy alone already implies that the matter distribution does not share the same rest frame as the CMB. This raises the question of whether the matter dipole can be interpreted as purely kinematic, or whether it reflects a deeper physical decoupling between the two frames.

Reconsider the idealized space probe equipped with a CMB radiometer introduced above. As previously described, the probe follows a known trajectory within the Local Group, and the plane defined by its instantaneous velocity vector and the line connecting it to the Local Group barycenter provides the geometric framework for identifying directions orthogonal to the probe's motion with respect to the CMB rest frame.

Selecting a line of sight orthogonal to this plane minimizes the projection of the probe's velocity onto the CMB rest frame. In such a direction, the kinematic contribution to the CMB dipole must be strongly suppressed, ideally approaching zero. This suppression is purely geometric and follows directly from the interpretation of the CMB dipole as a Doppler effect arising from the observer's motion.

This configuration therefore offers a clean operational definition of the CMB rest frame, independent of local gravitational influences or matter inhomogeneities.

EXPECTED BEHAVIOR OF THE MATTER DIPOLE

If the dipole observed in matter number counts were purely kinematic and referred to the same rest frame as the CMB, then:

- the matter dipole should also be suppressed along the same orthogonal direction,
- and its amplitude should scale consistently with the CMB dipole.

However, current observations show a significant amplitude mismatch between the two dipoles, even though the directional uncertainties remain large. If future measurements confirm that:

- the CMB dipole vanishes along the orthogonal direction (as expected),
- while the matter dipole does not,

then the matter dipole cannot be interpreted as a kinematic effect relative to the CMB rest frame.

This would imply that the matter distribution possesses a large-scale anisotropy or bulk motion not correlated with the CMB-defined frame.

IMPLICATIONS FOR FLRW AND 4-SPHERE MODELS

In ΛCDM/FLRW cosmology, the CMB and the large-scale matter distribution are expected to share the same rest frame on sufficiently large scales. A persistent discrepancy in dipole amplitudes—or a failure of the matter dipole to vanish in the CMB-orthogonal direction—would therefore challenge the standard interpretation of the CMB dipole as purely kinematic and would require invoking:

- extremely large bulk flows,
- non-standard initial conditions, or
- new physics beyond FLRW.

In the 4-sphere framework:

- the CMB uniquely defines the cosmological rest frame,
- the matter field is perturbative and does not determine isotropy, therefore the two dipoles are not expected to coincide.

The amplitude discrepancy is thus not anomalous but a natural consequence of the model. The proposed probe-based test provides an operational way to distinguish between:

- the geometric rest frame defined by the CMB, and
- the dynamical behavior of matter, which may exhibit large-scale anisotropies independent of the CMB.

The difference in dipole amplitudes already establishes that the matter and CMB rest frames are distinct. The proposed orthogonality test strengthens this conclusion: if the CMB dipole can be geometrically suppressed while the matter dipole cannot, then the matter dipole is not kinematic with respect to the CMB frame.

This distinction is problematic for FLRW but fully consistent with cosmological models in which the CMB alone encodes the global isotropic structure of spacetime, and matter acts as a perturbative field that does not define the cosmological rest frame.

GALACTIC COORDINATES

The observable Universe is a volume, on the surface of the 4-Sphere, delimited in the three spatial dimensions by an arc of $\Theta = 1\ rad$. In this volume we are at the center O.

Fixed the origin for the time axis t coinciding with the Big Bang, we can use three angles as a galactic coordinate system: the position of an astronomic object A can be defined by the direction of the 4-Sphere arc OA and the angle λ of this one. For the direction we can adopt the usual coordinates: Right ascension α and Declination δ. About the 4-Sphere arc angle, say "Arc λ", knowing the Galactic redshift *z,* you have:

$\lambda = ((1+z)^2 - 1)/((1+z)^2 + 1)\ rad$
Present proper distance $s = ct_{now}\lambda$

Moving on 4-Sphere surface coordinates, Colatitude, Longitude and Height, is quite complicate. Maybe it needs the aid of a computer program or some more suitable mathematical method. Here we give only some tools and a way to approach the solution:

Let's recall the coordinate in the 4-Sphere space **U**: $P = P(\varphi, \theta,\ \psi)$:

1. $x_1 = ct \sin(\psi) \sin(\varphi) \cos(\theta)$
2. $x_2 = ct \sin(\psi) \sin(\varphi) \sin(\theta)$
3. $x_3 = ct \sin(\psi) \cos(\varphi)$
4. $x_4 = ct \cos(\psi)$

The 4-vector $\boldsymbol{r} =$
$(ct \sin(\psi) \sin(\varphi) \cos(\theta),\quad ct \sin(\psi) \sin(\varphi) \sin(\theta),\quad ct \sin(\psi) \cos(\varphi),\quad ct \cos(\psi))$

and its derivatives:

1. $\mathbf{r}_\theta = (-ct \sin(\psi) \sin(\varphi) \sin(\theta),\ ct \sin(\psi) \sin(\varphi) \cos(\theta),\quad 0,\quad 0)$
2. $\mathbf{r}_\varphi = (ct \sin(\psi) \cos(\varphi) \cos(\theta),\ ct \sin(\psi) \cos(\varphi) \sin(\theta),\quad -ct \sin(\psi) \sin(\varphi),\quad 0)$
3. $\mathbf{r}_\psi = (ct \cos(\psi) \sin(\varphi) \cos(\theta),\ ct \cos(\psi) \sin(\varphi) \sin(\theta),\quad ct \cos(\psi) \cos(\varphi),\quad -ct \sin(\psi)))$

After converting δ using zenith angle $= 90^0 -$ Declination, in the space **O** of observable Universe, for a point, $U = U(\delta, \alpha, \lambda)$:

1. $y_1 = \sin(\delta)\cos(\alpha)$
2. $y_2 = \sin(\delta) \sin(\alpha)$

3. $y_3 = \cos(\delta)$
4. $y_4 = ct\lambda$

The vector $\boldsymbol{u} = (\sin(\delta)\cos(\alpha), \quad \sin(\delta)\sin(\alpha), \quad \cos(\delta))$ (with unit length)

and its derivatives:

1. $\mathbf{u}_\alpha = (-\sin(\delta)\sin(\alpha), \quad \sin(\delta)\cos(\alpha), \quad 0,)$
2. $\mathbf{u}_\delta = (\cos(\delta)\cos(\alpha), \quad \cos(\delta)\sin(\alpha), \quad 0)$

Note that two stars can be nearby on **U** but distant on **O**: it complicates approximations.

An angle on the 4-Sphere is given by:

$$\xi = \arccos(\frac{1}{c^2t^2}\mathbf{r}_1 \cdot \mathbf{r}_2)$$

while the one on the observable Universe (that is on the 4-Sphere surface, between the Earth and two star) is:

$$\gamma = \arccos(\mathbf{u}_1 \cdot \mathbf{u}_2)$$

To use Right Ascension and Declination we need the formulas effective for arcs and angles on the surface. For this purpose, given three points, we can set the 4-plane that passes through them and the center of the 4-Sphere. Once got it, we have a 3-sphere so to use the Sine Theorem and other tools.

Here calculations in polar coordinates are hard so let's move on to Cartesian ones:

$x_1^2 + x_2^2 + x_3^2 + x_4^2 = c^2t^2$
$x_4 = ax_1 + bx_2 + cx_3$ (where this 4-plane passes through the North Pole and the Earth).

We have $x_1^2 + x_2^2 + x_3^2 - c^2t^2 = -(ax_1 + bx_2 + cx_3)^2$.

This means that if a point belongs to the 3-plane: $ax_1 + bx_2 + cx_3 = 0$ and belongs to the 3-sphere: $x_1^2 + x_2^2 + x_3^2 = c^2t^2$ then it also belongs to the 4-Sphere after we put

$$x_4 = ax_1 + bx_2 + cx_3.$$

About the steps to find the position of an unknown star $P_x(\varphi, \theta, \psi)$, variables must be chosen so that the point lies both on of the sphere and the plane. That gives a first condition $F(\varphi, \theta, \psi) = 0$. Note that parameters a, b, c, for the equation of the 3-plane, are not linearly independent but we need all them later to set x_4. [15]

For the whole procedure to be valid, we should demonstrate that the transformation preserves angles and distances between the three points in question. To avoid calculations, we see that the same is true in 3d when we intersect a sphere with a plane, passing through the center, to get a circle.

For triangulations of the 4-Sphere we start getting coordinates of some points. We use our Earth, Ursa Major GN-108036, Sculptor A2744 YD4 and Piscis Austrinus BDF-3299:

1. Our Earth Us $P_0(\varphi, \theta, \psi)$ and $U_0(0, 0, 0)$
2. Ursa Major GN-108036 $z = 7.2$ $P_N(0, 0, 0)$ and $U_1(0.4863, \ 3.3003, \ 0.9707)$ - Boreal Hemisphere

3. Sculptor A2744 YD4 $z = 8.38$ $P_{EP-}(undef, 0, undef)$ and $U_2(-1.0405,\ 0.0629,\ 0.9775)$ – Austral Hemisphere
4. Piscis Austrinus BDF-3299 $z = 7.11$ $P_3(\varphi, \theta,\ \psi) = U_3(-0.9570,\ \ 5.8827,\ 0.9700)$ – Austral Hemisphere
5. … and so on …

We can give here the trace of a solution for our North Star Polaris. In these coordinates, it is close to the Earth:

1. Alpha Ursae Minoris – Polaris $z = 0.000055$ $U_4(0.0128,\ 0.6624,\ \ 0.000055)$ – Boreal Hemisphere
2. Our Earth $\mathbf{r}_0 = (a, b, c, d)$
3. Ursa Major GN-108036 $\mathbf{r}_\mathrm{N} = (0, 0, 0, ct)$
4. Sculptor A2744 YD4 $\mathbf{r}_2 = (e, 0, f, g)$

With respect to the Earth $P_0(\varphi, \theta, \psi)$, the coordinates of Alpha Ursae Minoris – Polaris are: $P_4(\varphi + \mathrm{x}, \theta + \mathrm{y},\ \psi + \mathrm{z})$ where x, y, z are unknown.

We follow these steps:

1. Define a point P_W on the direction $P_0\, P_N$ at the same distance $P_W\, P_N = P_4\, P_N$. U_W lies on the segment $U_0\, U_N$.
2. The first condition on x, y, z comes from the sphere and plane passing through $P_0\, P_N\, P_4$
3. Calculate the angle between P_N and P_4 in **O:** $\gamma = \arccos(\mathbf{u}_\mathrm{N} \cdot \mathbf{u}_4) = 0.8788$
4. Use the Sine Theorem in the triangle $P_0\, P_W\, P_4$, right in P_W: $|\arcsin(\lambda\gamma)| = \varepsilon = 0.000048$
5. Calculate the other cathetus with the Cosine theorem: $\cos\lambda = \cos\varsigma \cos\gamma$ and $\varsigma = 0.000027$

Now we abandon the 3-sphere $\mathrm{x}_1^2 + \mathrm{x}_2^2 + \mathrm{x}_3^2 = \mathrm{c}^2\mathrm{t}^2$ and, back to the 4-Sphere equation, we can solve the displacement between $P_0\, P_4$:

1. the value $\sin(\psi)\sin(\varphi)\,\Delta\theta$ is equal to ε.
2. the value $\sin(\psi)\,\Delta\varphi$ is equal to ς.

[15] – Since for the North Pole we arbitrarily assumed $\mathrm{x}_4 = 0$, it is not strange that all the points are constructed in the same way and all satisfy the condition of coplanarity on x_4. In this construction, we can reasonably think that, for every three points of the 4-Sphere, passes a sphere that preserves angles and distances between them.

THE TIME DIALECTIC IN 4-SPHERE MODEL

After publishing the Special Theory of Relativity, Einstein wrote: “Time is what clocks measure.” This phrase captures the deep bond between space and time — time is no longer absolute, but a coordinate that depends on motion and on space itself.

However, when we move from Relativity to Cosmology, things become more subtle. The cosmic time — the one that appears in the scale factor *a(t)* — introduces a notion of simultaneity that seems closer to Galileo’s absolute time. It is a “common” time shared by the whole universe, valid for every comoving observer, and it seems to bring back a universal concept of time.

I find this point particularly interesting: the time that became relative in Relativity appears "global" again in cosmology.

In my view, this is not a specific issue of the FLRW model, since I encounter the same feature in the 4-Sphere alternative model, and it's probably unavoidable.

Here the universe lies on an expanding spherical surface S^3 where the space expansion is defined by a simple, linear relationship: $r = ct$. The model establishes a fundamental distinction between the coordinate time of the expansion and the proper time experienced by objects within that space.

SEPARATION OF TIME COMPONENTS

Neglecting the early-universe phases, the model predicts that the expansion ($r = ct$) begins almost simultaneously from all points of a static universe at recombination. Model also posits that the total time elapsed for any object is composed of two distinct parts, contrasting with the standard spacetime metric (FLRW):

- Radial Cosmic Time:
  This is the time component responsible for the overall expansion of the universe. It is the fundamental, absolute timescale of the model's evolution, analogous to the concept of Cosmic Time in standard cosmology, but conceptually separate from the local dynamics.

- Surface Relativistic Time:
  This represents the additional time altered (in Relativity) due to the object's peculiar motion or deviation from the perfect "radial" expansion trajectory.
  This component governs all local relativistic effects and gravitational deviations within the S^3 space.

The adoption of a model that treats cosmic time as a distinct entity from relativistic phenomena allows for a reinterpretation of cosmic expansion—not as a global effect, but as a force capable of acting even outside the vacuum. While this expansion does not affect gravitationally bound macroscopic systems, it may hypothetically exert influence at the quantum scale anyway.

If physical conditions were to lead to the formation of a finite-thickness hyperspherical shell, the radial cosmic time could no longer be directly identified with the local temporal evolution within that finite radial extent. In such a configuration, the usual correspondence between cosmic expansion and local time evolution would lose its direct meaning, leaving open the possibility of a hypothesized suppression of the local time evolution.

THE LITHIUM CLOCK AND THE IDEAL OBSERVER

This model uses the hypothetical Lithium Clock—based on the primordial abundance of Lithium-7 as a universal age marker—to establish the Cosmic Time. This time parameter serves as the absolute, elapsed time since the beginning of the expansion:

- The Ideal Condition: The Lithium clock would measure the pure Cosmic Time if it belonged to an observer located in the vast, empty regions of space, not gravitationally bound to any structure (galaxies, clusters), and at rest relative to the Cosmic Microwave Background (CMB).
- Cosmic Time vs. Local Time: An observer at rest in the cosmic void measures the undilated Cosmic Time. Any object subject to local gravitational fields or high peculiar velocities appears to this observer to experience a dilated Proper Time, slower than the Cosmic Time, due to relativistic effects.
- In this model, all physical trajectories remain confined to the surface of the expanding 4-sphere, defined by $r = ct$. Objects in gravitationally bound systems (e.g., orbital motion) follow non-radial, curved paths within this surface. Although their Cosmic Time, as measured by an ideal observer in the void, is the same, their Proper Time appears dilated due to the longer, non-radial trajectory. This time dilation is not due to leaving the hypersphere, but arises geometrically from deviating from the straight radial expansion path.

In this framework, the Cosmic Time measured by the Lithium Clock in the void provides the Galilean-like reference frame for the entire universe.

Absolute time exists not merely as the Newtonian limit of relativity, even if no universal clock exists.

From this perspective, time is not a fundamental parameter of the theory but an emergent property of the universe's geometry. It is the very structure of the expanding 4-Sphere that gives rise to the perception of a common evolution, making time unavoidable even when it is not explicitly included in the metric.

THE 4-SPHERE AND THE TWIN PARADOX

In the 4-Sphere model, celestial bodies bound by gravity lie on the hypersurface, as does the cosmic vacuum. One may postulate that a global cosmic time progresses uniformly along this surface. Locally, a portion of this time corresponds to the proper time of a body in free fall within a gravitational field, or equivalently—by the equivalence principle—to a body undergoing acceleration. Whether accelerated or at rest, each body still observes the other's clock running slower, preserving the classical paradox; yet under this hypothesis, one may also assume that individual aging progresses identically for all bodies on the surface.

The Twin Paradox may thus be interpreted as an apparent absurdity arising from the viewpoint adopted when studying the evolution of a temporal separation traced in spacetime: that between two twins, the first traveling on a spacecraft and the second waiting on Earth. Determining who is "moving" or "at rest" depends entirely on the chosen reference frame.

Quantum mechanics does not contradict the relativistic notion of proper time; rather, it ties the effective proper-time evolution of a particle to its dynamical behavior. This is illustrated by the extended lifetime of cosmic-ray muons reaching the Earth's surface and by atomic-clock experiments on commercial flights, both of which confirm the relativistic dependence of proper time on the worldline [15]. High-energy experiments at CERN provide further confirmation:

unstable particles created in collider interactions display decay rates fully consistent with relativistic time dilation. Although their evolution is governed by quantum decay amplitudes, the effective proper time governing their lifetime remains determined by the worldline geometry. Thus, even in the quantum regime, no deviation from the relativistic notion of proper time has been observed. However, since the relationship between quantum theory and gravity is not yet fully understood, such quantum considerations cannot be used to challenge the geometric definition of proper time in General Relativity.

If the quantum vacuum possessed an absolute rest frame, it would fundamentally challenge modern physics. While macroscopic objects and light would continue to behave as expected due to the suppression of Planck-scale effects, the internal consistency of Quantum Field Theory would break down, violating Lorentz invariance and invalidating the foundations of General Relativity by introducing a preferred background structure [16].

In General Relativity, proper time is a scalar invariant: all observers agree on its value along any given worldline. However, GR does not provide any principle ensuring that two distinct worldlines must accumulate equal or different proper times; only an explicit integration of the metric determines the result, and this can be highly non-trivial.

Within the speculative 4-Sphere framework, one may consider a different organizing principle: biological aging is tied not to the geometric length of each worldline, but to a global Radial Cosmic Time. Under this assumption, the two twins—despite following distinct worldlines—reunite at the same point of the hyperspherical surface having experienced the same cosmic duration, and therefore the same biological aging.

This conclusion does not contradict General Relativity; rather, it relies on an additional postulate absent in GR. The latter assigns proper time only as the geometric length of each worldline and does not prescribe any universal temporal evolution when an observer transitions between inertial and accelerated motion [17].

The 4-Sphere framework is grounded in SR and GR, and we do not intend to question their solidity here.

[15] – In this regard, see also the paper by U. Jacob and T. Piran, which provides a detailed analysis of how hypothetical deviations from Lorentz symmetry modify the kinematics and propagation times of high-energy particles in spacetime: [arXiv:0712.2170] - Lorentz-violation-induced arrival delays of cosmological particles.

[16] – For a theoretical framework exploring the hypothesis of a dynamical preferred reference frame for the vacuum and gravity, see T. Jacobson and D. Mattingly [arXiv:gr-qc/0007031] - Gravity with a dynamical preferred frame.

[17] – The introduction of a global cosmic time in the 4-Sphere model is a geometric postulate specific to this framework. It is not intended as a modification of General Relativity, nor does it contradict it: GR simply does not address the existence of a universal temporal parameter. The discussion above should therefore be understood as an exploration of the consequences of this additional assumption, not as a claim about standard relativistic physics.

PRIVILEGED FRAMES AND LIGHT: THINKING ABOUT SPECIAL RELATIVITY FROM A 4-SPHERE PERSPECTIVE

From the expansion mechanism of the 4-Sphere to Special Relativity: A Deductive Approach

The present model, which conceives of the expanding universe as the surface of a 4-dimensional hypersphere, reopens the discussion on the foundations of relativity. It proposes to derive the theory from the rejection of absolute space and from a conjecture that the expansion is driven by radiation, that maps out and occupies newly available 4D spatial volume, and whose radial and transverse velocities are constrained by the condition:

$v_r = v_t = c$ this is our foundational conjecture for the generation of the cosmic fabric.

THE GEOMETRIC FOUNDATIONS OF DYNAMICS

The Photon Mean Free Path and Metric Elasticity as Primordial Constraints

In this section, recalling and extending our previous considerations regarding the invariant structure of the S^3 metric and its validation in the weak-field limit, we propose a foundational derivation of cosmological dynamics. This approach departs from the standard stress-energy tensor framework of General Relativity. Instead, we posit that the universe is governed by two complementary principles already introduced: a kinetic regulator—the photon mean free path λ_c—and a topological constraint—the extrinsic curvature surface tension of the S^3 hypersphere.

To guarantee absolute theoretical consistency across all cosmic epochs, this mechanism must be traced back to the pre-nucleosynthesis era. In this primordial phase, the unconstrained relic radiation field directly drove the global expansion, establishing the linear radial progression $r = ct$. Following the decoupling at the last scattering surface—where the spatial confinement of light was dismantled—this relic radiation transitioned into what we observe today as the Cosmic Microwave Background (CMB).

This evolutionary perspective allows us to refine the dynamics of gravitationally bound structures in the present era. Rather than assuming that the expansion regulator is artificially deactivated within galaxies, we must accept that the infinite mean free path of the CMB continues to drive metric expansion uniformly across the entire manifold, including high-density gravitational zones. The key distinction lies in how matter responds to this localized generation of space. Within a galaxy, the intense internal gravitational potential acts as a binding constraint that anchors the material constituents to their shared center of mass. Consequently, as the ubiquitous CMB continuously generates new spatial volume within the gravitational well, this newly available space is dynamically evacuated—it escapes outward into the surrounding intergalactic voids. Matter, remaining gravitationally bound, does not dilate tangentially, whereas the newly minted fabric of space continuously slips past it and streams outward, preserving the rigid size of the astronomical structure while feeding the global expansion of the cosmic matrix.

Conclusion

Ultimately, General Relativity can be reinterpreted not as space being actively bent by mass, but as a macroscopic manifestation of a deeper geometric and thermodynamic equilibrium. The architecture of the universe is a unique, invariant S^3 stage whose physical transitions merely modulate the local scaling function $h(t)$.

This mechanism enters the fundamental dynamic balance of the hyperspherical bubble, governed by the local energy-conservation equation:

$$\partial(c^2\rho\delta V)/\,\partial t \cdot dt = \delta w - p\,\partial\delta V/\,\partial t \cdot dt$$

where p and ρ represent the pressure and density of the CMB, and δV denotes a small proper volume element defined in the local frame of the observer, while δw is the work done to keep the universe in its shape.

In this framework, the generalized work term δw explicitly incorporates the effects of the radiation-driven expansion and the structural constraints of the manifold. The photon mean free path dictates where and when space is allowed to expand, actively generating new volume that either dilates the coordinates (in the voids) or is dynamically evacuated past bound structures (within galaxies). This ensures that the local energy variations perfectly balance the pressure work, guaranteeing that matter, light, and gravity remain forever bound to the pristine, evolving surface of the 4-Sphere without experiencing dimensional ruptures.

FROM RADIATION-DRIVEN EXPANSION TO THE EMERGENCE OF SPECIAL RELATIVITY

Radial and transverse velocities of radiation reinforce the 4-Sphere mode of operation. This symmetry is not imposed externally, but emerges naturally from the expansion mechanism itself. It implies that the spacetime structure must accommodate a universal speed limit, independent of direction or frame of reference. From this condition, the principles of Special Relativity follow directly: the constancy of the speed of light, the equivalence of inertial frames, and the Lorentz invariance of physical laws.

From the cosmological assumptions of homogeneity and isotropy, every point on the surface is equivalent, but not every velocity: observers moving relative to the fixed points of the surface perceive different effects, such as a CMB dipole.

Eliminating the two classical postulates of special relativity is not automatic; it relies on the specific conjectures of this model. The "cosmic time" in the relation $r = ct$ (a similar one is present in *FLRW* with the expansion factor *a)* is postulated to be identical for all observers at rest on the 4-Sphere. While this does not represent a logical advantage over Einstein's postulate, it grounds the theory in a notion of time that is physically more transparent.

This raises a further question: what does it mean to derive relativity from a conception of time so close to the classical notion of absolute time? The model features a *CMB* rest frame (a cosmic privileged frame) and a cosmic time (effectively absolute, since it defines simultaneity). Can special relativity be consistently recovered from these assumptions?

Naturally, this "cosmic time" could itself be deduced from the postulates of relativity; yet the idea that physics is, in some sense, tied to our perception of reality makes the original deductive direction preferable. Even more compellingly, adhering to the original deductive direction may yield implications that transcend the boundaries of deduction itself.

In the 4-Sphere framework, the large-scale dynamics of the universe are governed by General Relativity on S^3, applied solely to the *CMB,* whose influence has been shown to be negligible in practice. Used for light, Special Relativity in the intergalactic vacuum remains an excellent approximation, since the error introduced by applying the Doppler effect to galactic recession is negligible. The result should be extended to massive particles thus overcoming the problem of equivalence of position but not of velocity.

In this model, cosmic time defines a foliation of spacetime, creating a spatial hypersurface for each cosmic time instant. In the presence of gravity, local attraction dominates over cosmic expansion: gravity doesn't act within the leaf, but it is as if it does because cosmic recession is only perceived by the center of mass of large-scale structures. In the cosmic vacuum, our physics doesn't perceive the radial dimension, and special relativity applies, where the transition between leaves expands space without affecting any laws of physics. Here, cosmic time never appears in Einstein's field equations nor in relativity in general.

So, despite the unresolved conceptual questions this raises, yet it does not prevent this model from being useful in describing the large-scale evolution of our universe.

Can the *CMB* serve as a privileged frame of reference? Although a space probe can be calibrated to have zero peculiar velocity relative to the *CMB*, the observed "drag" effect seems to hold relevance only within a cosmological context.

Motion is defined with respect to the center of the 4-sphere, which we do not know whether it is stationary or rotating. In the absence of absolute space, given two points A and B in relative motion, who can say if and which one is truly at rest? So, regardless of its state of motion, any point on the surface is equivalent.

Laws of Interaction Remain Invariant: Just as with the *CMB*, where the isotropy rest frame is cosmologically special but not privileged for local physical laws, in a universe modeled as the surface of our 4-Sphere, any reference frame "comoving" with the surface (or with the matter distributed on it) would not constitute a preferred frame.

What remains is the most plausible candidate for a privileged frame—the stationary center of the 4-Sphere—but is external to the universe and cannot be directly detected by our physics.

Once we remove any privileged reference frame (the first postulate), the focus shifts to the speed of light (the second postulate), which alone determines the form of the Lorentz transformation.

Thus, consider light. The (unmeasurable) rotation of the hypersphere around its center is crucial: the geodesic connecting two points A and B is the path along which light moves with radial and transverse velocities both constrained around the center of the 4-sphere [18].

The fact that the expansion rate of the Universe equals the speed of light in vacuum may not be a coincidence. In our framework, this acts as a constraint for light: in the 4-Sphere $v_t = v_r$ where $v_r = c$, ensures that light does not escape the Universe.

Two points A and B in motion measure the same value c: a light ray emitted at A at speed c will reach B still traveling at speed c, regardless of the relative motion between A and B.

To conclude, it is the constancy of the speed of light that completes the definition of special relativity. The second postulate follows from this alone.

[18] – This last statement follows from the model's assumptions on the components of radial and transverse velocity of light:

$$\begin{cases} v_t = |r d\theta/dt| = c \\ v_r = dr/dt \quad\quad = c \end{cases}$$ *(where r is the radius and θ is the central angle)*

Let us then think of our Universe expanding with $r = ct$ and fix the origin of our reference frame in the center. Here we have two points A and B in relative angular motion between them and with the same radial velocity c. With our reference frame rotating with A, if B emits a ray of light, its tangential speed must be equal to the radial speed c to not abandon the surface. But this condition holds both in A and in B, with the result that the speed of the light ray cannot be dragged by B.