Six Simultaneous Equations With Integer Solution

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In the below paper we have attempted to find integer solution to six simultaneous equations. In math literature we have not found a similar equation that has been investigated before. The equations consist of three quadratics on the left hand side which sum up to a square on the right side. Even-though the equations are quadratic in nature the degree of difficulty increases with the number of equations considered. The author considered six equations simultaneously. Others can try & solve simultaneous equation which are more than six in the group.

Consider the below equations:

$$(p)x + (q)y + (r)z = a^2 \;------\; (1)$$

$$(p+1)x + (q+1)y + (r+1)z = b^2 \;------\; (2)$$

$$(p+2)x + (q+2)y + (r+2)z = c^2 \;------\; (3)$$

$$(p+5)x + (q+5)y + (r+5)z = d^2 \;------\; (4)$$

$$(p+7)x + (q+7)y + (r+7)z = e^2 \;------\; (5)$$

$$(p+12)x + (q+12)y + (r+12)z = f^2 \;------\; (6)$$

The above 6 equations have 12 unknowns.

Namely, (p,q,r,x,y,z,a,b,c,d,e,f)

We initially consider equation (1) & (2):

$$(p)x + (q)y + (r)z = a^2 \;-------(1)$$

$$(p+1)x + (q+1)y + (r+1)z = b^2 \;------\; (2)$$

We take b=5a & (p,q,r)=(1,2,1) & we get:

$$x + 2y + z = a^2 \;---------(7)$$

$$x + y + z = 24a^2 \ ---------(8)$$

Solving equation (7) & (8) we get:

$$y = -23a^2 \quad \& \quad (x + z) = 47a^2$$

we take, $9x = 10z$ & we get:

$$z = (423a^2)/19 \ ---------(9)$$

Inorder that 'z' be an integer we take, a=19 in equation (9)

& we get, $z = 8037$ & $x = \frac{10z}{9} = 8930$, $y = -23(19)^2 = -8303$

Hence we have:

$$(x, y, z) = [(8930), (-8303), (8037)]$$

& since, b=5a, $b = 5(19) = 95$

Now consider the below equation:

$$(p + n)x + (q + n)y + (r + n)z = w^2 \ ------(10)$$

For (p,q,r)=(1,2,1), equation (10) is equivalent to:

$$(x + 2y + z) + n(x + y + z) = w^2 \ ---------(11)$$

We know from equation (7) & (8), that, (x+2y+z)=a^2 & (x+y+z)=24a^2

Hence equation, (11) becomes:

$$w^2 = a^2(24n + 1) \ -------------(12)$$

Inorder for (LHS) of equation (12) to be a square we make (24n+1) a square.

Now, (24n+1) is a square at, n=(2,5,7 & 12)

& we get: w=[(7a),(11a),(13a),(17a)] for n=(2,5,7 & 12)

Since, a=19 & n=(2,5,7 & 12), we have:

$$(c, d, e, f) = (= [(7a), (11a), (13a), (17a)] = [133{,}209{,}247{,}323]$$

Also, (a,b)=(19,95) & since we have the below equation, we substitute in it

from equation (10) for, n=(1,2,5,7,12) & we get:

$$(p)x + (q)y + (r)z = a^2 \;-----\; (1)$$

$$(p+1)x + (q+1)y + (r+1)z = b^2 \;-----\; (2)$$

$$(p+2)x + (q+2)y + (r+2)z = c^2 \;-----\; (3)$$

$$(p+5)x + (q+5)y + (r+5)z = d^2 \;-----\; (4)$$

$$(p+7)x + (q+7)y + (r+7)z = e^2 \;-----\; (5)$$

$$(p+12)x + (q+12)y + (r+12)z = f^2 \;-----(6)$$

Thus we get after substituting the integer values for (a,b,c,d,e,f):

$$(p)x + (q)y + (r)z = (19)^2 \;-----\; (1)$$

$$(p+1)x + (q+1)y + (r+1)z = (95)^2 \;-----\; (2)$$

$$(p+2)x + (q+2)y + (r+2)z = (133)^2 \;-----\; (3)$$

$$(p+5)x + (q+5)y + (r+5)z = (209)^2 \;-----\; (4)$$

$$(p+7)x + (q+7)y + (r+7)z = (247)^2 \;-----\; (5)$$

$$(p+12)x + (q+12)y + (r+12)z = (323)^2 -----\; (6)$$

In the above we have:

$$(p,q,r) = (1,2,1) \;\&$$

$$(x,y,z) = [(8930),(-8303),(8037)]$$

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