Irrefutable proof of the invalidity of Galois theory

Date: Monday, May 19, 2025

Authors: Ahcene Ait Saadi

E mail: ait_saaadi@yahoo.fr

In this article, I demonstrate the resolution of the 5th degree equation, with algebraic radicals. I contradict Galois theory.

In my demonstration, I use two invariant polynomials of degrees 8 and 6, wich I discovered*. After identification, I cancel the coefficients 7, 5 and 3 in the polynomial of degree 8 and the coefficient of degree 5 and 3, in the polynomial of degree 6. For the coefficient of degree 1, I make a combination between the polynomials of degree 8 and 6 to eliminate it. Finally I find an equation of degree 8 bisquare that I can solve.

By solving the system of equations, of variables m and p, which are the coefficients of the powers 5 and 3, I'm forced to involve the equation of degree 5. That allows me to eliminate the coefficient of degree 3 from the equation of degree 8, I realise that I can find the solutions of the equation of degree 5 with a free variable.

The solutions of an equation with a free variable, I have already done this with the equation of degree 2, that I discovered**

Key words:

Five, six and eight degree equations, Galois theory, free variable

Invariant polynomials of degree six and eight

System of equations of degree 2 with two unknowns

References:

* viXra:2409.0024 **Algebra zéro** Ahcene Ait saadi

** viXra:2407.0140 **Invariant polynomial** Ahcene Ait saadi

Solving the 5th degree equation

Introduction:

In this article I demonstrate that the equation of degree 5, admits solutions with algebraic radicals, therefor the non- validity of Galois theory, using the invariant polynomials of degrees 6 and 8, That I discovered*.

Let be the invariant polynomials of the following 6th and 8th degrees :

$$P(x)=x^6+\lambda_2x^5+\lambda_3x^4+(\frac{2\lambda_2\lambda_3}{3}-\frac{5\lambda_2^3}{27})x^3+\lambda_3x^2+(\frac{\lambda_2\lambda_5}{3}-\frac{\lambda_2^3\lambda_3}{27}+\frac{\lambda_2^5}{81})x+\lambda_7...(\mathrm{I}_A)$$

$$x^8+k_2x^7+k_3x^6+(\frac{3k_3k_2}{4}-\frac{14k_2^3}{4^3})x^5+\lambda_5x^4+(\frac{k_5k_2}{2}-\frac{5k_3k_2^3}{4^3}+\frac{28k_2^5}{4^5})x^3+$$

$$k_7x^2+(\frac{k_7k_2}{4}-\frac{k_5k_2^3}{4^3}+\frac{3k_3k_2^5}{4^5}-\frac{17k_2^7}{4^7})x+k_9........(\mathrm{II}_A)$$

We use these two invariant polynomials to solve with the algebraic radicals the equation of degree 5.

For these two polynomials if we make a change of variable:

$x\to x-\frac{\lambda_2}{6}$ for the invariant polynomial of degree 6

$x\to x-\frac{k_2}{8}$ for the invariant polynomial of degree 8

With: $\frac{\lambda_2}{6}=\frac{k_2}{8}$ and $\lambda_2=6\lambda;k_2=8\lambda$

I) Invariant polynomial of degree 6:

$$P(x+\lambda)=(x+\lambda)^6-6\lambda(x+\lambda)^5+\lambda_3(x+\lambda)^4+(-4\lambda_3\lambda+40\lambda^3)(x+\lambda)^3+\lambda_5x^2+$$
$$(-2\lambda_5\lambda+8\lambda_3\lambda^3-96\lambda^5)(x+\lambda)+\lambda_7$$
$$P(x+\lambda)=x^6+(-15\lambda^2+\lambda_3)x^4+(75\lambda^4+6\lambda_3\lambda^2+\lambda_5)x^2-61\lambda^6+5\lambda_3\lambda^4-h\lambda^2+\lambda_7$$

In reality I can only eliminate the coefficients in power 5 and 3 in the polynomial of degree 5. We write the polynomial in this form:

$$P(x)=x^6-6\lambda x^5+\lambda_3x^4+(-4\lambda\lambda_3+40\lambda^3)x^3+\lambda_5x^2+\lambda_6x+\lambda_7$$

$$P(x+\lambda)=x^6+(\lambda_3-15\lambda^2)x^4+(\lambda_5-6\lambda_3\lambda^2+75\lambda^4)x^2+(96\lambda^5-8\lambda_3\lambda^3+2\lambda_5\lambda+\lambda_6)x$$
$$+35\lambda^6-3\lambda_3\lambda^4+\lambda_5\lambda^2+\lambda_6\lambda+\lambda_7........(\mathrm{I}_B)$$

II) Let the following equation of degree 5 be

$x^5+ax^4+bx^3+cx^2+dx+e=0$, we multiply it by $(x-t)$

$$(x^5+ax^4+bx^3+cx^2+dx+e)(x-t)=$$
$$x^6+(a-t)x^5+(b-at)x^4+(c-bt)x^3+(d-ct)x^2+(e-dt)x-et...........(\mathrm{I}_c)$$

We make the identification of the equation of degree 5 with the invariant polynomial of degree 5 :

$$t=6\lambda+a$$
$$b-at=\lambda_3\Rightarrow\lambda_3=-6a\lambda-a^2+b$$
$$c-bt=-4\lambda_3\lambda+40\lambda^3=-4\lambda(-6a\lambda-a^2+b)+40\lambda^3=40\lambda^3+24a\lambda^2+4(a^2-b)\lambda$$

With: $40\lambda^3+24a\lambda^2+(4a^2+2b)\lambda+ab-c=0.......(\Delta)$

III) Invariant polynomial of degree 8:

$$Q(x)=x^8-8\lambda x^7+k_3x^6+(-6k_3\lambda+112\lambda^3)x^5+lx^4+\sigma x^3+\theta x^2+\omega x+\varphi=0...(\mathrm{II}_B)$$
$$x^8-8\lambda x^7+gx^6+hx^5+lx^4+\sigma x^3+\theta x^2+\omega x+\psi=0$$
$$-m^2-p^2-mp+\beta m+\beta p-a\beta+b=g$$
$$m^2p+mp^2-(\beta+a)mp-am^2-ap^2+a\beta m+a\beta p-b\beta+c=h$$
$$am^2p+amp^2-(a\beta+b)mp-bm^2-bp^2+b\beta m+b\beta p-c\beta+d=l$$
$$bm^2p+bmp^2-(b\beta+c)mp-cm^2-cp^2+c\beta m+c\beta p-d\beta+e=\sigma$$
$$cm^2p+cmp^2-(c\beta+d)mp-dm^2-dp^2+d\beta m+d\beta p-e\beta=\theta$$
$$dm^2p+dmp^2-(d\beta+e)mp-em^2-ep^2+e\beta m+e\beta p=\omega$$
$$em^2p+emp^2-e\beta mp=\psi$$

$$Q(x+\lambda)=x^8+(k_3-28\lambda^2)x^6+(-15k_3\lambda^2+l+350\lambda^4)x^4+(896\lambda^5+20k_3\lambda^3-60k_3\lambda^2+4l\lambda+\sigma)x^3$$
$$(980\lambda^6-45k_3\lambda^4+6l\lambda^2+3\sigma\lambda+3\theta\lambda)x^2+(512\lambda^7-24k_3\lambda^5+4l\lambda^3+3\sigma\lambda^2+3\theta\lambda^2+\omega)x$$
$$+105\lambda^8-5k_3\lambda^6+l\lambda^4+\sigma\lambda^3+\theta\lambda^3+\omega\lambda+\varphi...........(\mathrm{II}_c)$$

IV) Let the following equation of degree 8 be. We multiply it by:

$(x-m)(x-p)(x-q)$

$$(x^5+ax^4+bx^3+cx^2+dx+e)(x-m)(x-p)(x-q)=$$
$$x^8-(m+p+q-a)x^7+(mp+mq+pq-am-ap-aq+b)x^6+$$
$$(-mpq+apq+amp+amq-bm-bp-bq+c)x^5+$$
$$(-ampq+bpq+bmp+bmq-cm-cp-cq+d)x^4+$$
$$(-bmpq+cpq+cmp+cmq-dm-dp-dq+e)x^3+$$
$$(-cmpq+dpq+dmp+dmq-em-ep-eq)x^2+$$
$$(-dmpq+epq+emp+emq)x-empq\text{..........................}(\mathrm{II}_c)$$

We put $a-t=-6\lambda \Rightarrow t=a+6\lambda$ and $\frac{m+p+q-a}{8}=\frac{6\lambda}{6}\Rightarrow q=\beta-m-p$ such as $\beta=8\lambda+a$

We replace q with its value, we obtain:

$$x^8-8\lambda x^7+(-m^2-p^2-mp+\beta m+\beta p-a\beta+b)x^6+$$
$$(m^2p+mp^2-(\beta+a)mp-am^2-ap^2+a\beta m+a\beta p-b\beta+c)x^5+$$
$$(am^2p+amp^2-(a\beta+b)mp-bm^2-bp^2+b\beta m+b\beta p-c\beta+d)x^4+$$
$$(bm^2p+bmp^2-(b\beta+c)mp-cm^2-cp^2+c\beta m+cp\beta-d\beta+e)x^3+$$
$$(cm^2p+cmp^2-(c\beta+d)mp-dm^2-dp^2+d\beta m+d\beta p-e\beta)x^2$$
$$(dm^2p+dmp^2-(d\beta+e)mp-em^2-ep^2+e\beta m+e\beta p)x+em^2p+emp^2-e\beta mp=0\text{.......}(\mathrm{II}_D)$$

We make the identification of the equation of degree 8 with the invariant polynomial of degree 8 :

$$V):Q(x)=x^8-8\lambda x^7+k_3x^6+(-6k_3\lambda+112\lambda^3)x^5+lx^4+\sigma x^3+\theta x^2+\omega x+\varphi=0\text{...}(\mathrm{II}_E)$$

We do the same work with the equation of the 8th degree, to eliminate the coefficients of x^7 and x^5 we obtain:

$$\begin{cases} k_3=-m^2-p^2-mp+\beta m+\beta p-a\beta+p \\ -6k_3+112=m^2p+mp^2-(\beta+a)mp-a(m^2+p^2)+a\beta(m+p)-b\beta+c)\text{ .} \end{cases}$$

Coefficient of degree 5 ⟹

$$-m^2p-mp^2+(\beta+a+6)mp+(a+6)(m^2+p^2)-\beta(a+6)(m+p)+6a\beta+b\beta-6b-c+112=0..(A)$$

The coefficient of x^3 is: $896\lambda^5+20k_3\lambda^3-60k_3\lambda^2+4l\lambda+\sigma=0\text{.......}(B)$

Note 1: the coefficient of the power 3, is eliminated by solving a system of equations of variables m and p, this system is composed of the coefficients of the power 3 and 5 (see (A) and (B)).

But the coefficient of the power 1 will be eliminated by combining the two equations of degrees 8 and 6.

Note 2: how to solve an equation of degree 5 by combining two invariant polynomials of degree 6 and 8.

Note 3: the coefficient of the power 3, is eliminated by solving a system of equations of variables m and p.

Note 4: the coefficient of the power 1 will be eliminated by combining the two equations of degrees 8 and 6.

$VI)$ By replacing the coefficients: $k_5;k_3;l;\sigma$ by their values, we find the value of the equation of the coefficient of power 3 (see(B)) .

By making the change of variable: $x \to x+\lambda$, the coefficients of the power 7 and 5 cancel each other out. There remains the coefficient of the power 3.

The coefficient of power 3 is:

$$(4a+b)(m^2p+mp^2)+(-4a\beta\lambda-4b\lambda-b\beta-c+60\lambda^2-20)mp-(4\lambda b+c+60\lambda^2-20)(m^2+p^2)+$$
$$\beta(4b\lambda+c-60\lambda^2+20)(m+p)+60a\beta\lambda^2-20a\beta-4c\beta\lambda-60b\lambda^2+4d\lambda-d\beta+e+896=0........(B)$$

We have to solve the system consisting of the coefficients of the power

5 and 3.

$$\begin{cases} -m^2p-mp^2+(\beta+a+6)mp+(a+6)(m^2+p^2)-\beta(a+6)(m+p)+6a\beta+b\beta-6b-c+112=0..(A) \\ (4a+b)(m^2p+mp^2)+(-4a\beta\lambda-4b\lambda-b\beta-c+60\lambda^2-20)mp-(4\lambda b+c+60\lambda^2-20)(m^2+p^2)+ \\ \beta(4b\lambda+c-60\lambda^2+20)(m+p)+60a\beta\lambda^2-20a\beta-4c\beta\lambda-60b\lambda^2+4d\lambda-d\beta+e+896=0........(B) \end{cases}$$

Note 5 : This system does not admit solutions. Because, after replacing the value of λ , calculated earlier (see (Δ)

$$(4a\lambda+b)(m^2p+mp^2)+(-4a\beta\lambda-4b\lambda-b\beta-c+60\lambda^2-20)mp-(4\lambda b+c+60\lambda^2-20)(m^2+p^2)+$$
$$\beta(4b\lambda+c-60\lambda^2+20)(m+p)=$$

$$-(4a\lambda+b)\left[(-m^2p-mp^2+(\beta+a+6)mp+(a+6)(m^2+p^2)-\beta(a+6)(m+p)+6a\beta+b\beta-6b-c+112\right]$$

And

$$60a\beta\lambda^2-20a\beta-4c\beta\lambda-60b\lambda^2+4d\lambda-d\beta+e+896\neq$$
$$-(4a+b+10)(6a\beta+b\beta-6b-c+112)$$

Therefore, the degree 5 polynomial of must be introduced by multiplying it by a real (K). So, the two constants of the two equations of variables m and p, must be equal.

$$\left[(x+\lambda)^5+a(x+\lambda)^4+b(x+\lambda)^3+c(x+\lambda)^2+d(x+\lambda)+e\right]=$$
$$kx^5+k(a+5\lambda)x^4+k(b+4a\lambda+10\lambda^2)x^3+k(c+3b\lambda+6a\lambda^2+10\lambda^3)x^2+$$
$$k(d+2c\lambda+3b\lambda^2+4a\lambda^3+5\lambda^4)x+k\left(e+d\lambda+c\lambda^2+b\lambda^3+a\lambda^4+\lambda^5\right)$$

$$-(4a\lambda+b)(6a\beta+b\beta-6b-c+112+k)=$$

$$(60a\beta\lambda^2-20a\beta-4c\beta\lambda-60b\lambda^2+4d\lambda-d\beta+e+896)+(4a\lambda+b+10\lambda^2)k$$

$$10\lambda^2k=-(4a\lambda+b)(6a\beta+b\beta-6b-c+112)-60a\beta\lambda^2+20a\beta+4c\beta\lambda+60b\lambda^2-4d\lambda+d\beta-e-896$$
$$k=\frac{-(4a\lambda+b)(6a\beta+b\beta-6b-c+112)-60a\beta\lambda^2+20a\beta+4c\beta\lambda+60b\lambda^2-4d\lambda+d\beta-e-896}{10\lambda^2}$$

We'll have to resolve the equation:

$$(4a\lambda+b+10k\lambda^2)(m^2p+mp^2)+(-4a\beta\lambda-4b\lambda-b\beta-c+60\lambda^2-20)mp-(4\lambda b+c+60\lambda^2-20)(m^2+p^2)+$$
$$\beta(4b\lambda+c-60\lambda^2+20)(m+p)=0$$

This equation admits infinity of solutions. We give a value for p we find m. or else calculate p as function of m. we obtain solutions with a free variable.

Once we have the value of K , We find that the coefficients of the powers 5 and 3 are equal. It suffices to cancel a single coefficient.

6) Very important note:

The annulet coefficient has two variables. To solve it, there are two possibilities. Either we set a value for p and we find m. or the solution consists in calculating (p) as a function of (m). We will have to solve an equation of degree 2. In this case the solutions of the equation are written with the free variable (m).

$VI)$ Here are some simple examples, where I have chosen the value of $\lambda = 1$, to facilitate the calculations.

Example1: solve the equation: $x^5 - 6x^4 + 9x^3 + 4x^2 - 9x - 6$

We take $\lambda = 1$ we make a change of variable $x \to x+1$ we obtain

$x^5 - x^4 - 5x^3 + 5x^2 + 7x - 7 = 0$ the equation of degree 8 is:

$x^8 - 6x^6 + 12x^4 - 7x^2 = 0$

$x^8 - 6x^6 + 12x^4 - 7x^2 = 0$ $x^5 - 6x^4 + 7x^3 + 12x^2 - 15x - 10 = 0$

Example 2: solve the equation: $x^5 - 6x^4 + 7x^3 + 4712x^2 - 15x - 10 = 0$

We take $\lambda = 1$ we make a change of variable $x \to x+1$ we obtain

$x^5 - x^4 - 7x^3 + 7x^2 + 11x - 11 = 0$; the equation of degree 8 is:

$x^8 - 8x^6 + 18x^4 - 11x^2 = 0$

Example3: solve the equation: $x^5 - 6x^4 + 12x^3 - 8x^2 + 5x - 10 = 0; \lambda = 1$
$x \to x+1 \to x^5 - x^4 - 2x^3 + 2x^2 + 6x - 6 = 0$

the equation of degree 8 is:

$x^8 - 7x^6 + 20x^4 - 38x^2 + 24 = 0$

Summary:

Steps followed for solving the equation of degree 5

Step1: $x^5 + ax^4 + bx^3 + cx^2 + dx + e = 0$

$(x^5 + ax^4 + bx^3 + cx^2 + dx + e)(x - t) = 0$

$$(x \to x+\lambda) \to x^6 + (\lambda_3 - 15\lambda^2)x^4 + (\lambda_5 - 6\lambda_3\lambda^2 + 75\lambda^4)x^2 + (96\lambda^5 - 8\lambda_3\lambda^3 + 2\lambda_5\lambda + \lambda_6)x$$
$$+35\lambda^6 - 3\lambda_3\lambda^4 + \lambda_5\lambda^2 + \lambda_6\lambda + \lambda_7 \text{.............}(III)$$

Step 2: $(x^5 + ax^4 + bx^3 + cx^2 + dx + e)(x - m)(x - p)(x - q) = 0$

$$x \to x+\lambda \to x^8+(k_3-28\lambda^2)x^6+(-15k_3\lambda^2+l+350\lambda^4)x^4+(896\lambda^5+20k_3\lambda^3-60k_3\lambda^2+4l\lambda+\sigma)x^3$$
$$(980\lambda^6-45k_3\lambda^4+6l\lambda^2+3\sigma\lambda+3\theta\lambda)x^2+(512\lambda^7-24k_3\lambda^5+4l\lambda^3+3\sigma\lambda^2+3\theta\lambda^2+\omega)x$$
$$+105\lambda^8-5k_3\lambda^6+l\lambda^4+\sigma\lambda^3+\theta\lambda^3+\omega\lambda+\varphi$$

To eliminate the coefficient of the power 3, we must introduce the condition:

$$\begin{cases} k_3=-m^2-p^2-mp+\beta m+\beta p-a\beta+p \\ -6k_3+112=m^2p+mp^2-(\beta+a)mp-a(m^2+p^2)+a\beta(m+p)-b\beta+c)......(A) \end{cases}.$$

$$(4a+b)(m^2p+mp^2)+(-4a\beta\lambda-4b\lambda-b\beta-c+60\lambda^2-20)mp-(4\lambda b+c+60\lambda^2-20)(m^2+p^2)+$$
$$\beta(4b\lambda+c-60\lambda^2+20)(m+p)+60a\beta\lambda^2-20a\beta-4c\beta\lambda-60b\lambda^2+4d\lambda-d\beta+e+896=0........(B)$$

and introduce the five degree equation:

$$kx^5+k(a+5\lambda)x^4+k(b+4a\lambda+10\lambda^2)x^3+k(c+3b\lambda+6a\lambda^2+10\lambda^3)x^2+$$
$$k(d+2c\lambda+3b\lambda^2+4a\lambda^3+5\lambda^4)x+k\left(e+d\lambda+c\lambda^2+b\lambda^3+a\lambda^4+\lambda^5\right)$$

To involve the term: $k(b+4a\lambda+10\lambda^2)x^3$ hanks to $10\lambda^2k$

So we have to solve a system of two equations. This system does not allow solutions. We must involve the equation of degree 3.

Step 3: Last I find the two equations:

$$x^8+a_6x^6+a_4x^4+a_2x^2+ax+a_0=0$$
$$x^6+b_4x^4+b_2x^2+bx+b_0=0$$

By making a combination between the two equations we cancel the coefficient of the power 1. Which gives an equation of the form:

$$x^8+u_6x^6+u_4x^4+u_2x^2+u_0=0 \ .$$