On Two Unconventional Summation-Derivative Identities

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Abstract

This paper presents two mathematical formulas involving differentiation and summation. While the derivations do not strictly follow conventional mathematical rigor and certain foundational rules are bypassed, the formulas demonstrate surprisingly consistent results. A full formal proof is currently under construction.

Formula 1

$$\left(\frac{d}{dx}\right)^{k+1} \sum_{n=1}^{x} n^k = k!, \qquad where\ k \in \mathbb{Z}, k \geq 0$$

This identity suggests a deep connection between the discrete sum of powers and higher order derivatives, culminating in the factorial of the power index k. Though nonstandard in derivation, this formula exhibits consistent results even for large values of k, indicating a reliable structural behavior.

Formula 2

$$\sum_{n=1}^{x} n^k = k \int_{0}^{x} \left(C_{k-1} + \sum_{n=1}^{x} n^{k-1} \right) dx, \qquad where\ k \in \mathbb{Z}, k \geq 1$$

This recurrence relation for power sums links the sum of n^k to k multiplied by the integral of the sum of n^{k-1}, plus a constant. It offers a recursive structure that, while not adhering to conventional derivation standards, leads to valid expressions when applied practically.

Conclusion and Status

These formulas offer intriguing shortcuts and recursive structures for computing power sums. However, the mathematical foundation lacks strict formalism, and the derivations currently do not conform to accepted calculus and summation rules. Further refinement and proof construction are ongoing to validate and possibly generalize these expressions.