# Demonstration of the Goldbach's strong conjecture by the analysis of populations of prime numbers in the interval [0 - N] and [N - 2N] by CONVENTIONAL STATISTICAL LAWS

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**ABSTRACT.**

In this article I apply classical statistical laws to analyze prime numbers assimilated to populations. The statistical analysis focuses on prime numbers in the intervals [0 - S/2] and [S/2 – S] with S an even > 4. The results show that the even number S > 4 is enclosed by two populations of prime numbers P in the interval [0 - S/2] and Q in [S/2 - S] which have approximately the same standard deviation relative to their means. Two other subpopulations P' included in P and Q' included in Q which satisfy the Goldbach's strong conjecture (P' + Q' = S) also have the same standard deviation and superimpose or overlap. This result shows that an even number is enclosed by two populations P' and Q' of prime numbers which are symmetric with respect to S/2 and therefore S = P' + Q'. This result also shows that any natural number N > 4 is enclosed by at least two equidistant and symmetric prime numbers. Therefore for every N > 4 there exists a number t < N such that N – t = P' and N + t = Q' are primes and so 2N = P' + Q'.

## INTRODUCTION

I have already reported various works on the Goldbach Strong Conjecture (GSC) according to which an even number denoted here S is the sum of two prime numbers p and q such that p < S/2 and q > S/2 and therefore S = p + q (***references 1 - 8***). In this article, I use a completely different approach based on the conventional laws of statistics. Indeed, the GSC is certainly a function of the distribution of prime numbers, which remains unresolved. Here is my method. I posit an even number S > 4 as resulting from two intervals of numbers [0 – S/2] and [S/2 – S]. I consider the prime numbers as being equivalent to a population in the conventional statistical sense. We therefore have the population P of the interval [0 - S/2] and the population Q of the interval [S/2 – S]. We will therefore compare the populations P and Q of even numbers taken at random and try to understand how the distribution of prime numbers induces the GSC. Note that the populations P and Q correspond to the set of prime numbers < S/2 and > S/2, respectively. While the populations P' and Q' correspond to the prime numbers that satisfy the GSC such that P ' + Q' = S. Therefore P' and Q' are subsets included in P and Q.

## METHODS

So I will compare the populations P and Q of randomly chosen even numbers and try to understand how the distribution of prime numbers induces the GSC. I therefore calculate the mean (M) as well as the standard deviation (SD) of the populations P and Q. I also calculate the same parameters for the prime numbers P' and Q' which are known to satisfy the GSC. The list of prime numbers is obtained from the site ***https://compoasso.free.fr*** and the mean and standard deviation are calculated on the site https://miniwebtool.com/fr/standard-deviation-calculator/**.** Primes satisfying Goldbach's strong conjecture were obtained from the site ***https://www.dcode.fr/conjecture-goldbach***

**RESULTS**

***I- The value of an even number S is linearly correlated to the standard deviation of the populations of prime numbers P < S/2 and Q > S/2***

**Table 1** shows SD values of populations P < S/2 and Q > S/2 of a randomly chosen sample of even numbers S. Note that the SD values have been rounded to integer values.

**Table 1 : SD values of P and Q.**

| S | SD (P) | SD (Q) |
|---|---|---|
| 40 | 5 | 5 |
| 60 | 8 | 8 |
| 100 | 14 | 13 |
| 200 | 28 | 32 |
| 400 | 61 | 55 |
| 600 | 88 | 86 |
| 1000 | 149 | 142 |
| 2000 | 296 | 289 |
| 3330 | 491 | 530 |
| 4000 | 591 | 575 |

**Figure 1A** and **1B** show correlation coefficients between SD (P) or SD (Q) and S from Table 1.

**Figure 1A :**

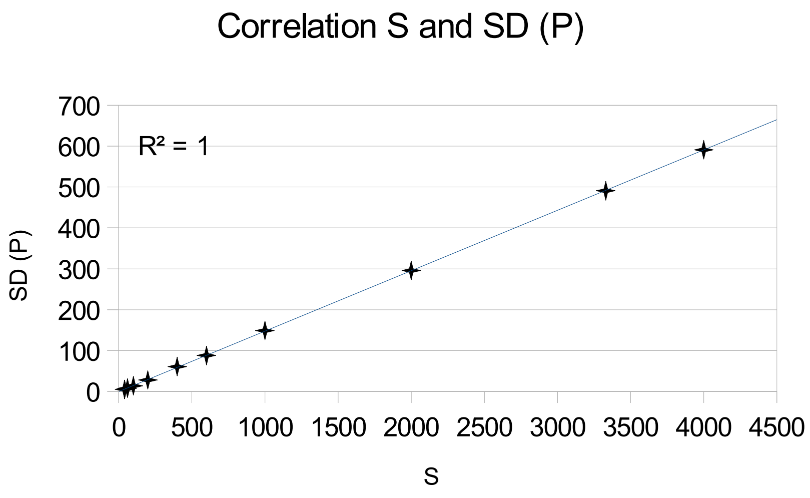
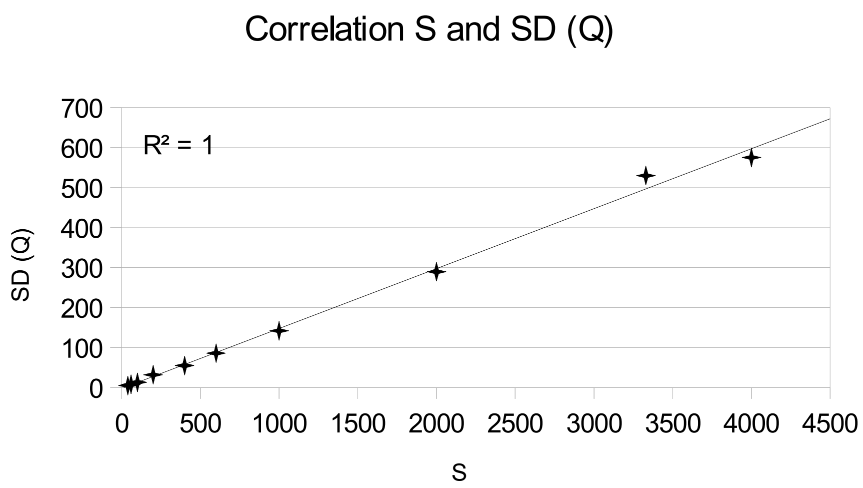


**Figure 1B :**

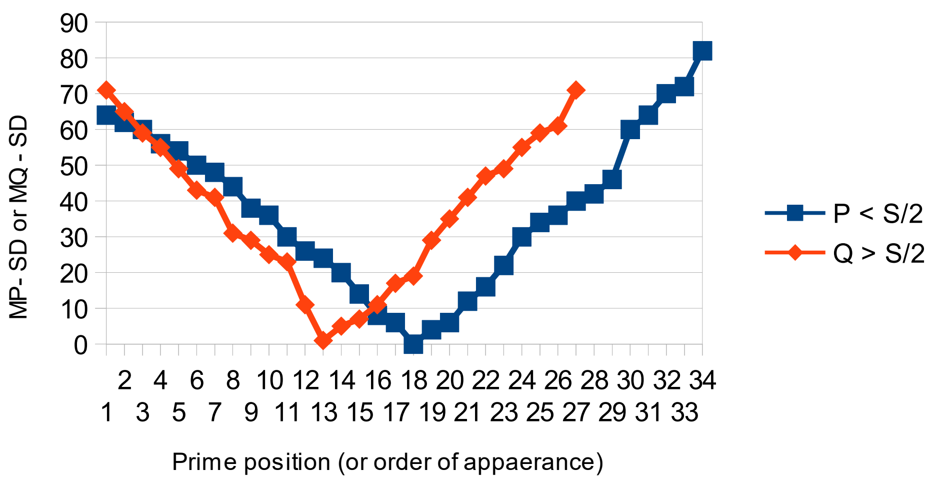



II- ***Analysis of populations of composite and prime numbers whose sum equals an even number.***

Let C be any odd composite number; and P or Q any prime number. An even number denoted S > 4, whatever it may be, is either S = C1 + C2 with C1 < S/2 and C2 > S/2; S = P + C; and S = P + Q with P < S/2 and Q > S/2. I compared the populations P, C, and Q in each case with respect to their averages. Here are the results for the number S = 300 as a example.

First of all, the population of prime numbers P < S/2 is compared to the population Q > S/2. We see that the two populations do not have the same dispersion relative to their average and do not overlap (**Figure 2A**). The mean (M) and standard deviation (SD) of each population is calculated and then M - SD is determined for population P and population Q.

**Figure 2A**



Note that P or Q represents the set of prime numbers < S/2 and > S/2, respectively. But those that satisfy the strong Gldbach conjecture are denoted P' and Q' such that S = P' + Q'. Knowing that E = C + P or E = P' + Q', I compared the populations C and P in the first case and P' and Q' in the second case. Indeed in both cases, the populations C and P on the one hand (**Figure 2B**) and P' and Q' on the other hand (**Figure 2C**) are superimposed at all points which is expected *because C and P; or P' and Q' are symmetric or equidistant with respect to S/2*. In fact if S = C + P then S/2 - C = P - S/2 or S/2 - P = C – S/2 ; and if S = P' + Q' then S/2 - P' = Q' - S/2. This symmetry explains why the number S is formed by the addition of C and P or P' and Q'.

**Figure 2B ( E = C + P)**

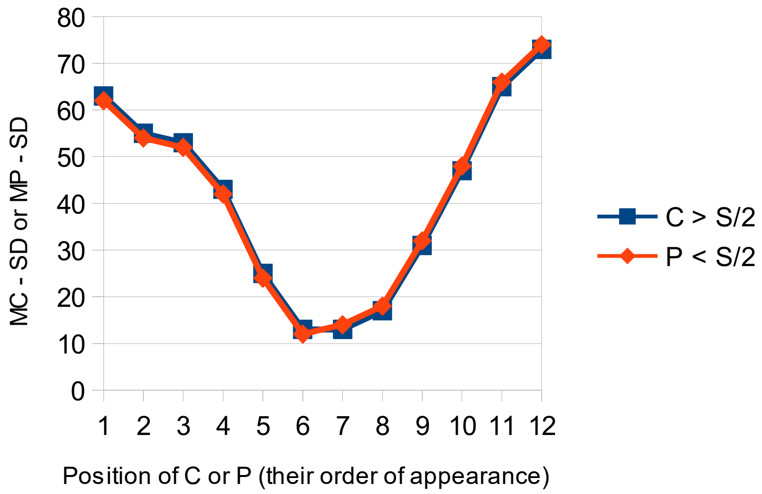
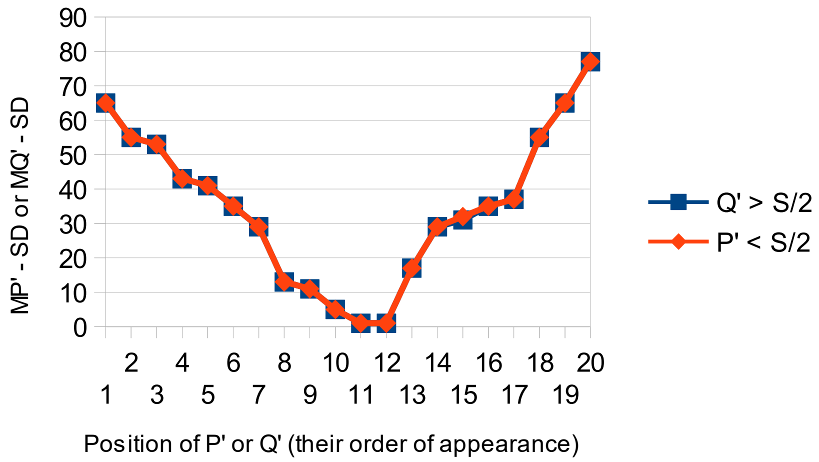


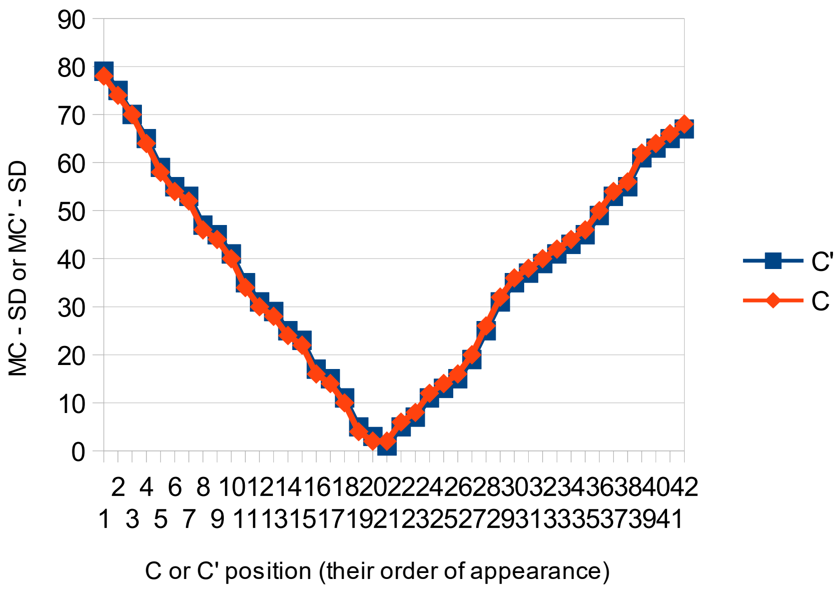

**Figure 2C ( E = P' + Q')**



Let us note a very important point here: *this symmetry is not the result of the addition but it is behind the addition. It is because P′ and Q′ are symmetric populations with the same standard deviation that S can be formed by their addition. This symmetry explains all the partitions of S. This symmetry occurs in [0 – S/2] and [S/2 – S] intervals. A natural number is primarily a point on a line with a position coordinate, and therefore geometric properties precede arithmetic properties. Goldbach's strong conjecture is primarily based on geometry on a line. This is what this article aims to demonstrate by calculating the mean and standard deviation.*

Finally, in the case of S = C + C' with C < S/2 and C > S/2, and as expected, the two populations C and C' overlap at all points (**Figure 2D**).

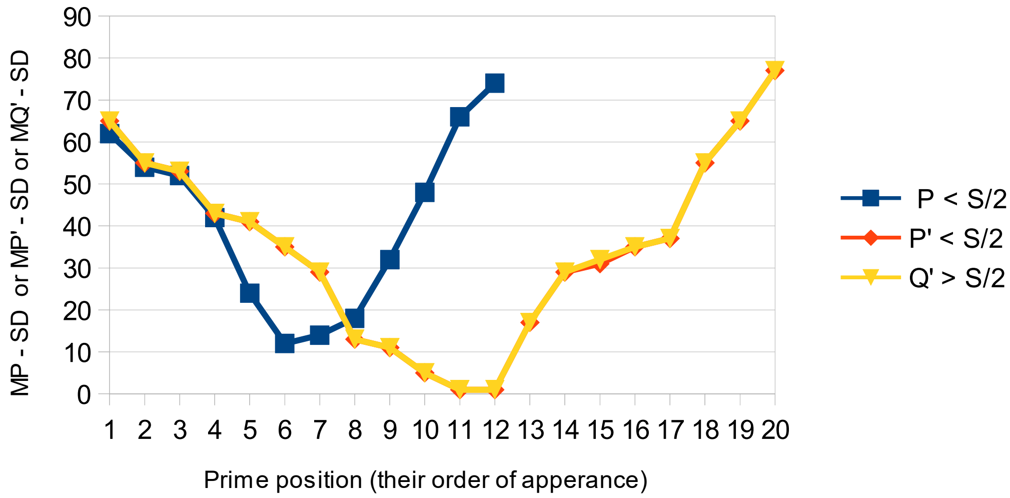
**<u>Figure 2D</u>**



III. *For an even number S > 4 there are two distinct populations of prime numbers such that S = <u>P</u> + C and S = <u>P′</u> + Q′. P and P′ or Q′ are distinct populations of prime numbers which are not symmetric*.

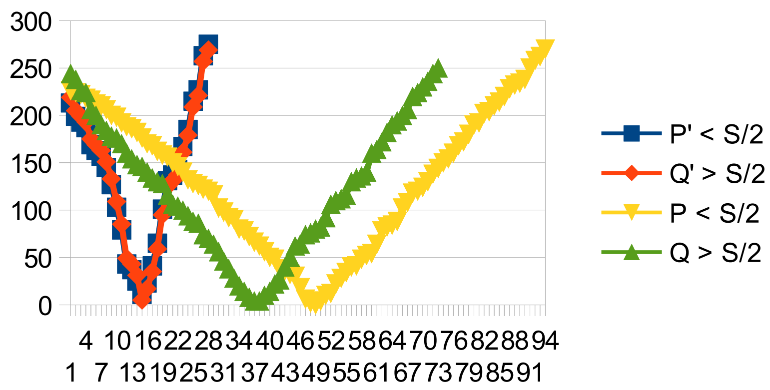
Prime numbers are divided into two distinct populations: those that do not satisfy the strong Goldbach conjecture (P) (**S = P + C**) and those that do (P' and Q' with **S = P' + Q'**). Each even number S in the set N has a specific distribution and standard deviation for these populations (**Figure 3A**).

<u>**Figure 3A**</u>



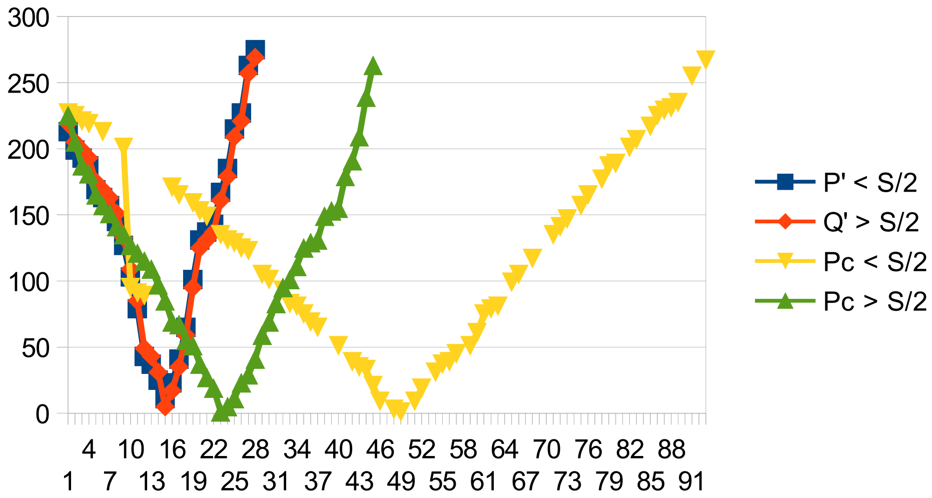

The same results were fud with other even numbers including S = 1000 (**Figure 3B**). Again primes satisfying GSC (P' + Q ' = S) form a population separate from other prime numbers (P and Q) and are the only symmetric prime numbers in the two intervals [0 – S/2] and [S/2 – S] (**Figure 3B**). This again shows that Gldbach's strong conjecture is linked to prime number symetry in the two intervals.

<u>**Figure 3B**</u>



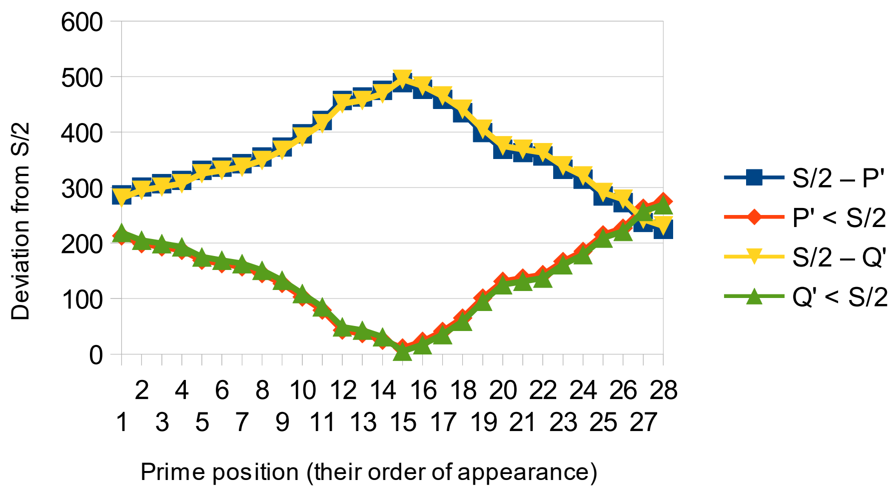
Prime numbers that do not satisfy the strong Goldbach conjecture denoted Pc or Qc such that S = Pc + C or S = Qc + C are not symmetric and therefore this confirms that the only symmetry observed between [0 – S/2] and [S/2 - S] concerns the prime numbers P' and Q' such that S = P' + Q' (Figure 3C).

**Figure 3C**




The deviations between S/2 and P' or Q' that satisfy the strong Goldbach conjecture are also symmetric in the two intervals [] and []. This proves the existence of two subpopulations of prime numbers P' and Q' symmetric with respect to their means and to S/2. This is why S = P' + Q' (Figure 3D).

**Figure 3D**



***V. The arithmetic operations of addition S = C + C′; S = P + C and S = P′ + Q′ can only be understood by modular calculus.***

- S = C + C' → S – C' = C. Let's pose for simplicity C = pq (p and q are prime factors) Then **S = xp + r** and **C' = yp + r** or **S = x'q + r'** and **C' = y'q + r'** → S ≡ C' (p) and S ≡ C' (q) thyerefore C – C' = n x p x q (n > 0). This is also true for S – C = C'.
- S = C + P → S - C = P → **S = xP + r** and **C = ( x – 1)P + r** → S ≡ C (P).
- In the case S – P = C and if C = pq then Then **S = xp + r** and P **= yp + r** or **S = x'q + r'** and P **= y'q + r'** → S ≡ P (p) and S ≡ P (q) thyerefore C – P = n x p x q (n > 0).
- S = P' + Q' → S - Q' = P' → **S = xP' + r** and **Q'= ( x – 1)P' + r** → S ≡ Q' (P'). In a similar way, S ≡ P' (Q'). These rules apply to infinity.
- To verify the Goldbach's Strong conjecture (GSC) to infinity let us pose S = P + X with the known prime P < S/2 and so S – X = P. Then if **S = xP + r** then **X = (x – 1)P + r**. If X is prime GSC is verified ; if X composite GSC is not verified. If at least one X > S/2 is prime then GSC is true for S. To minimize factorisation time, we may start with X numbers close to S/2.

- We deduce from the results here that if S tends to infinity, the standard deviation of primes P and Q would increase to infinity. It is non possible to perform analyses shown here to infinity because of the number of P and Q primes increasing to infinity. However, the paper tell that there will be very likely symmetrical P' and Q' primes in [0 – S/2] and [S/2 – S] intervals leading to S = P' + Q'.

**Remarks**

- The data have been reproduced with other evens (not shown) and are true for any even.
- A quasi-linear correlation exists between the value of the even number S and the standard deviation of the populations of primes P and Q.
- The standard deviations of the populations P and Q increase with the value of the even number but remain close to each other.
- The standard deviations of the populations P' and Q' are very close and the two populations overlap.
- Since P' and Q' have the same standard deviation at their respective means, they are also symmetrical with respect to S/2 and are therefore equidistant from S/2. Note that the sum of the means M1 (P) + M2 (Q) = S (with some variations that can be neglected).
- This result also shows that any natural number N > 4 is enclosed by at least two equidistant and symmetric prime numbers P' and Q'. Therefore for every N > 4 there exists a number t < N such that N – t = P' and N + t = Q' are primes and so 2N = P' + Q'.

## Conclusion

Every even number gives two subpopulations P' and Q' of prime numbers each having its mean, having a standard deviation approximately the same, and equidistant with respect to S/2 and therefore S = P' + Q'. GSC means that every even number has its own two symmetric subpopulations P' and Q'. Note P' is in [0 – S/2] and Q' in [S/2 – S].

**An even number**
**S > 4**

Two prime numbers populations P < S/2 and Q > S/2
Populations P and Q have close SD and each has a Mean

Two symmetrical prime numbers subpopulations

Subpopulation P' of [0 – S/2]

Subpopulation Q' of [S/2 - S]

***P′ and Q′ are equidistant at S/2***
**S = P + Q'**

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