# Falling Apples, Quarks, and Expanding Galaxies

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## Abstract

I will reverse-engineer the equation for the photon sphere radius to derive a Modified Gravity equation applicable to all conditions and energy densities. I will test this equation under conditions present on Earth's surface, in regions describing distant galaxies, and under the extreme conditions that may exist deep inside a quark.

## 1. Introduction

## Photon sphere

„A photon sphere, or photon ring[1] or photon circle,[2]…

The circular photon orbit is said to be the last photon orbit.[3] The radius of the photon sphere, which is also the lower bound for any stable orbit, is, for a Schwarzschild black hole“ [4]

$$r = \frac{3GM}{C^2} = \frac{3r_S}{2} \quad (1)$$

where the Schwarzschild radius is:

$$r_S = \frac{2GM}{C^2} \quad (2)$$

### Derivation of the MOND equation

$$r = \frac{3GM}{C^2} \quad (3)$$

From here, we express the centripetal acceleration of a photon that has a circular orbit around a mass M at a distance r.

$$\frac{C^2}{r} = \frac{3GM}{r^2} \quad (4)$$

We have obtained the modified Newtonian equation for centripetal gravitational acceleration:

$$a_c = \frac{3GM}{r^2} \quad (5)$$

But we immediately notice a problem because this equation gives values that are 3 times larger than expected for the Earth's surface.

Let's try again:

$$r = \frac{3GM}{C^2} \quad (6)$$

$$\frac{C^2}{r} = \frac{3GM}{r^2}$$

$$\frac{C^2}{r} = \frac{2GM}{r^2} + \frac{GM}{r^2}$$

$$\frac{C^2}{r} - \frac{2GM}{r^2} = \frac{GM}{r^2}$$

$$\frac{C^2}{r}\left(1 - \frac{2GM}{rC^2}\right) = \frac{GM}{r^2}$$

$$\frac{C^2}{r} = \frac{GM}{r^2}\frac{1}{\left(1 - \frac{2GM}{rC^2}\right)} \quad (7)$$

Or

$$\frac{C^2}{r} = \frac{GM}{r^2}\frac{1}{\left(1 - \frac{r_S}{r}\right)} \quad (8)$$

We reason that under the conditions m≪M, the equation holds for masses m that can be very different. Now, someone might say that a photon has no mass, but it doesn't have to. Here, the masses m and M can represent the actual rest mass of an object or (and) mass as a measure of the total energy associated with objects. That is, if a photon feels gravitational acceleration (and it does), there is no reason why the same acceleration at the same location wouldn't be felt by another particle or object under the same conditions.

Here, on the Earth's surface, objects of any mass, under the condition m≪M, experience the same acceleration in free fall. The same acceleration of about 9.81 m/s^2 is experienced by all, whether the object is a building, a truck, a person, an atom, an electron, or a photon.

Therefore, although the given equation for acceleration was derived for a photon traveling at the speed C, I will assert that the same acceleration applies to all other objects at the same distance, whether they are moving or just beginning free fall.

Therefore, I will assume that the central acceleration for any mass m≪M at a distance r from M is:

$$a_c = \frac{GM}{r^2}\frac{1}{\left(1-\frac{r_s}{r}\right)} \quad (9)$$

and

$$r_s = \frac{2GM}{C^2} \quad (10)$$

**Testing:**

**1. It holds for the radius of the photon sphere**, r, which we do not need to derive again because the equation is identical.

The only difference is that I assume a hypothetical gravitational force that acts as the centripetal force on the photon, keeping it in a circular orbit by providing it with that centripetal acceleration.

**2. It holds for the surface of planet Earth**

The gravitational acceleration at the surface of Earth is g=9,81 m/s^2:

For planet Earth:

$$r_s = \frac{2\cdot 6.67\cdot 10^{-11}\frac{Nm^2}{kg^2}5.97\cdot 10^{24}kg}{(3\cdot 10^8\frac{m}{s})^2} = 8.85\cdot 10^{-3}m = 0.00885m \quad (11)$$

Thus, using the modified acceleration equation:

$$a_c = \frac{GM}{r^2}\frac{1}{\left(1-\frac{r_s}{r}\right)}$$

$$= \frac{6.67\cdot 10^{-11}\frac{Nm^2}{kg^2}5.97\cdot 10^{24}kg}{(6370000m)^2}\cdot\frac{1}{\left(1-\frac{0.0085m}{6370000m}\right)}$$

$$= 9.81\frac{m}{s^2}\cdot 1.000000001 = 9.81\frac{m}{s^2} \quad (12)$$

According to Newton:

$$g = a_c = \frac{GM}{r^2} = \frac{6.67\cdot 10^{-11}\frac{Nm^2}{kg^2}5.97\cdot 10^{24}kg}{(6370000m)^2} = 9.81\frac{m}{s^2} \quad (13)$$

**3. Dark Energy**

Gravitational acceleration, and consequently gravitational force, is defined by the following equation (9), where (10) applies.

There are solutions where the gravitational force is attractive, i.e., where $r_s < r$, but there also exist regions (or conditions) where the **gravitational force is repulsive**, meaning that the equation changes its sign under the conditions where $r_s > r$.

Let us assume that repulsive gravitational force is what accelerates distant objects away from each other.

Now, we will determine the specific conditions under which this occurs:

If we are to believe some of our colleagues:

„ At 1 Mpsec1 Mpsec it is 10^−18 km/s$^2$ (if my algebra didn't fail me), or 10^−15 m/s$^2$" [5]

„At the edge of the observable universe now (about 47 Gly) it is only about 10^−11 m/s$^2$ (again, with simple multiplies I hope I got right)" [6]

**Example:**

Let us calculate the acceleration of a galaxy at the edge of the observable universe.

The mass of the observable universe:

M=$10^{53}$ kg [7],[8]

Distance to the edge of the observable universe:

r=$4.4\cdot 10^{26}$ m [9],[10]

$$a_c = \frac{GM}{r^2}\frac{1}{\left(1-\frac{2GM}{rC^2}\right)} \quad (14)$$

(Contrary to convention, for no particular reason, attractive acceleration is assigned a positive sign, meaning that repulsive acceleration will manifest as a negative sign.)

$$a_c = \frac{6.67\cdot 10^{-11}\frac{Nm^2}{kg^2}\cdot 10^{53}kg}{(4.40\cdot 10^{26}m)^2}\frac{1}{1-\frac{2\cdot 6.67\cdot 10^{-11}\frac{Nm^2}{kg^2}\cdot 10^{53}kg}{4.40\cdot 10^{26}m\cdot(3\cdot 10^8\frac{m}{s})^2}} \quad (15)$$

$$a_c = 3.445\cdot 10^{-11}\frac{1}{1-\frac{1.48\cdot 10^{26}m}{4.40\cdot 10^{26}m}}\frac{m}{s^2} \quad (16)$$

$$a_c = 3.445\cdot 10^{-11}\frac{1}{1-0.336}\frac{m}{s^2} \quad (17)$$

$$a_c = 5.188\cdot 10^{-11}\frac{m}{s^2} \quad (18)$$

The magnitude of the attractive acceleration, i.e., equation (18), would represent the acceleration with which a galaxy at the edge of the observable universe accelerates toward us. However, this is not what has been detected.

Now, note that the equation also supports real solutions for $r_s$>r.

Let us determine the mass M for which $r_s$ is slightly greater than r. In that case:

$$r = 4.4\cdot 10^{26}m \quad (19)$$

$$r_s = 4.41\cdot 10^{26}m \quad (20)$$

$$4.41\cdot 10^{26}m = \frac{2GM}{C^2} \quad (21)$$

$$M = \frac{4.41\cdot 10^{26}m\cdot(3\cdot 10^8\frac{m}{s})^2}{2\cdot 6.67\cdot 10^{-11}\frac{Nm^2}{kg^2}} \quad (22)$$

$$M = 2.975\cdot 10^{53}kg \quad (23)$$

The central gravitational acceleration in that case is:

$$a_c = \frac{6.67\cdot 10^{-11}\frac{Nm^2}{kg^2}\cdot 2.975\cdot 10^{53}kg}{(4.40\cdot 10^{26}m)^2}\frac{1}{1-\frac{2\cdot 6.67\cdot 10^{-11}\frac{Nm^2}{kg^2}\cdot 2.975\cdot 10^{53}kg}{4.40\cdot 10^{26}m\cdot(3\cdot 10^8\frac{m}{s})^2}}$$

(24)

$$a_c = 1.024\cdot 10^{-10}\frac{1}{1-\frac{4.41\cdot 10^{26}m}{4.40\cdot 10^{26}m}}\frac{m}{s^2} \quad (25)$$

$$a_c = 1.024\cdot 10^{-10}\frac{1}{1-1.002}\frac{m}{s^2} \quad (26)$$

$$a_c = -5.12\cdot 10^{-8}\frac{m}{s^2} \quad (27)$$

This (27) is the acceleration of the galaxy away from us, i.e., under these conditions, a repulsive gravitational force acts on the galaxy, accelerating it away from us. This could effectively be part of the explanation for what we call dark energy.

The average density of the galaxy in the observable universe, under these conditions, is:

$$\rho = \frac{M}{V} = \frac{M}{\frac{4}{3}\cdot r^3\pi} = \frac{2.975\cdot 10^{53}kg}{\frac{4}{3}(4.40\cdot 10^{26}m)^3\cdot 3.14}$$

$$= 8.342\cdot 10^{-28}\frac{kg}{m^3} \quad (28)$$

All in all, under these conditions, in this sense:

$r > r_s$ gravity is an attractive force

$r < r_s$ gravity is a repulsive force

$r = r_s$ gravity is undefined

(For practice, if $r = r_s/2$, then it is $M = 5.397\cdot 10^{53}kg$, $a_c = -2.045\cdot 10^{-9}\frac{m}{s^2}$ etc.)

## 4. The Strongest Force in Nature

First, let me generalize gravitational acceleration and thus the gravitational force for two objects with comparable masses. Specifically, the acceleration given by equation (14):

$$a_c = \frac{GM}{r^2}\frac{1}{\left(1-\frac{2GM}{rC^2}\right)}$$

This (14) holds under the condition m<<M. If we were to use this equation for two objects with comparable masses, it wouldn't comply with Newton's 3rd law, and so on. Therefore, we generalize the equation for the intensity of acceleration, i.e., the gravitational force between two objects of masses m and M:

$$F_{mM} = F_{Mm} = \frac{GmM}{r^2}\frac{1}{\left(1-\frac{2G(m+M)}{rC^2}\right)} \quad (29)$$

$$a_m \cdot m = a_M \cdot M = \frac{GmM}{r^2}\frac{1}{\left(1-\frac{2G(m+M)}{rC^2}\right)} \quad (30)$$

$$a_m = \frac{GM}{r^2}\frac{1}{\left(1-\frac{2G(m+M)}{rC^2}\right)} \quad (31)$$

Acceleration (31) of mass m toward (or away from) mass M.

$$a_M = \frac{Gm}{r^2}\frac{1}{\left(1-\frac{2G(m+M)}{rC^2}\right)} \quad (32)$$

Acceleration (32) of mass M toward (or away from) mass m.

Now, I will calculate the distance between two point-like electrons at which the gravitational attractive force is equal to the electric repulsive force [11].

$$F_e = F_g \quad (33)$$

$$1 = \frac{F_g}{F_e} = \frac{\frac{Gmm}{r^2}\cdot\frac{1}{\left(1-\frac{2G(m+m)}{rC^2}\right)}}{\frac{Kq^2}{r^2}} = \frac{Gm^2}{Kq^2}\cdot\frac{1}{1-\frac{4Gm}{rC^2}} \quad (34)$$

$$1 = \frac{6.67\cdot10^{-11}\frac{Nm^2}{kg^2}(9.11\cdot10^{-31}kg)^2}{8.99\cdot10^{9}\frac{Nm^2}{C^2}(1.6\cdot10^{-19}C)^2}\cdot\frac{1}{1-\frac{4\cdot6.67\cdot10^{-11}\frac{Nm^2}{kg^2}\cdot9.11\cdot10^{-31}kg}{r(3\cdot10^{8}\frac{m}{s})^2}} \quad (35)$$

$$1 = 2.405\cdot10^{-43}\cdot\frac{1}{1-\frac{2.7\cdot10^{-57}m}{r}}m \quad (36)$$

Using a series expansion [12]

$$r = (2.7\cdot10^{-57} + 6.49\cdot10^{-100})m \quad (37)$$

$$r \approx 2.7\cdot10^{-57}m \quad (38)$$

The density under these conditions is:

$$\rho = \frac{2m}{V} = \frac{2m}{\frac{4}{3}\cdot\left(\frac{r}{2}\right)^3\pi} = \frac{2\cdot9.11\cdot10^{-31}kg}{\frac{4}{3}\left(\frac{2.70\cdot10^{-57}}{2}m\right)^3\cdot3.14}$$

$$= 1.77\cdot10^{140}\,\frac{kg}{m^3} \quad (39)$$

The distance (37) where the gravitational attractive force equals the electric repulsive force, for two point-like electrons.

Note that there are real solutions to this equation where the gravitational force is, for example, $10^{50}$ times stronger than the electric force. That is, for $r = (2.7\cdot10^{-57} + 6.49\cdot10^{-150})m$ this is the distance where the gravitational attractive force is $10^{50}$ times stronger than the electric repulsive force, for two point-like electrons.

**Conclusions**

Note that this equation (29):

1. Provides accurate predictions for conditions similar to those on the surface of the Earth.
2. Provides accurate predictions for the photon sphere radius.
3. Suggests, at least in part, that gravity is the force that accelerates distant galaxies away from us.
4. Offers the possibility to think about the interior of quarks as objects packed by their own relativistic gravity, as it is capable of overcoming any other known force.
   More on this in the ***Supplementary Material on the Interior of the d (and anti-d) Quark.***

## References

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## Interior of the d Quark

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There are many papers on a similar topic [1], [2], [3], [4], etc.

My opinion is:

That deep inside the d quark, relativistic gravitational conditions prevail, which hold it together and make it what it is.

I believe that the d quark is a photon bound and deformed by its own internal gravitational field.

Only a small internal part of that photon (the d quark) is under conditions where $r \approx r_s$ and $r > r_s$, so that the gravitational attractive force is in equilibrium with the electric repulsive force and the magnetic repulsive force (37) in the “Falling Apples, Quarks, and Expanding Galaxies”.

Reasons and justifications for such reasoning:

1. The d quark could be a photon, because during the annihilation of a particle and its antiparticle, only photons can be released. "Positron Annihilation," "Electron-Positron Pair Production" https://shorturl.at/I2qpe

2. There is a sort of gravitational redshift during annihilation:
By calculation [5],[6],

$$mC^2 = hf = h\frac{C}{\lambda} \quad (1)$$

$$\lambda = \frac{h}{mC} = \frac{6.626 \cdot 10^{-34} Js}{8.9 \cdot 10^{-30} kg \cdot 3 \cdot 10^8 \frac{m}{s}} \approx 2.48 \cdot 10^{-13} m \quad (2)$$

Equation (2) gives an approximate size of the d quark, while it is measured to be around $10^{-18}m$ [7]. This is what I expect, since $2.48 \cdot 10^{-13}m$ is the length of the "released" d quark, while the length is $10^{-18}m$ when it is bound and compressed by its own internal, extremely strong gravitational field.

3. Deformed, because even if you are electromagnetically balanced (like a photon), existence and movement around your own relativistically dense interior most likely leads to an imbalance that manifests as a nonzero electric and magnetic field. In a way, this is similar to what causes a nonzero electromagnetic interaction between two conductors through which an electric current flows.

„How Einstein saved magnet theory“, Fermilab,

https://youtu.be/d29cETVUk-0?si=gcn7AbEjFdL3POu-&t=108

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"…the size of quarks is ~10^−18 m"

https://www.sciencedirect.com/topics/chemistry/up-quark#:~:text=While%20the%20size%20of%20protons,composed%20of%20smaller%20particles%20–%20preons