Proof of Collatz Conjecture

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Abstract

Collatz Conjecture ($3 * x + 1$ problem) stats any natural number /x/ will return to number /1/ after $3 * x + 1$ computation (when /x/ is odd) and $\frac{x}{2}$ computation (when /x/ is even), then doing the same computation on the outputs. in this paper we proved that Collatz Conjecture is true.

In working on this problem, Richard Guy(1) said: “don’t try to solve these problems!”

Key words: Collatz Conjecture.

The problem of the research: Collatz Conjecture ($3* x$ +1 problem) is unsolved conjecture.

The important of the research: the important of this research lies in solving the Collatz Conjecture($3* x$ +1 problem) that remained for 85 years without proof or denial.

Introduction

1-Prove that if all odd integers verify the state of the conjecture satisfy the conjecture to be true

2-Prove that the sets B, C, D represent all the odd integers without duplicated numbers

3-prove that is no loop when we perform the function to the set /*A*/, except the loop which number /1/ makes it with itself

4- perform the function to the sets of numbers B, C, D

5-Prove that when we perform the function -again and again- to all integers, they will return to the integer /1/

1- Prove that if all odd integers verify the state of the conjecture satisfy the conjecture to be true:

Every even number /e/ can be written as the following form:

$e = 2^a * (2 * n + 1)$, Therefore _ according to the stats of the conjecture _ we will divide /e/ by /2/ for /a/ times, so... we will obtain an odd number /x/

$x = \frac{e}{2^a} = 2 * n + 1$

Conclusion1: it is sufficient to prove that all odd numbers satisfy the conjecture to be correct.

2-Prove that the sets $\boldsymbol{B}$, $\boldsymbol{C}$, $\boldsymbol{D}$ represent all the odd integers without duplicated numbers:

2-1-We have the following three sets of integers: $\boldsymbol{B}$, $\boldsymbol{C}$, $\boldsymbol{D}$

$B = 1 + 4 + 4^2 + 4^3 + 4^4 + \cdots + 4^{(a-1)} + 2 * 4^a * n$

$C = 3 + 4 * n$

$D = 3 + 10 * \left(1 + 4 + 4^2 + 4^3 + 4^4 + \cdots + 4^{(a-1)}\right) + 4^{(a+1)} * n$

$a = \{1, 2, 3, 4, 5, 6, \ldots.\}$

$n = \{0, 1, 2, 3, 4, 5, \ldots\ldots.\}$

2-2-Prove that the sets $\boldsymbol{B}$, $\boldsymbol{C}$, $\boldsymbol{D}$ present all the odd integers /$\boldsymbol{A}$/ without duplicated numbers:

2-2-1-proof that there are no duplicate integers within set $\boldsymbol{B}$:

we assume that there are two equal integers in the set B, they are:

$b_1 = 1 + 4 + 4^2 + 4^3 + 4^4 + \cdots + 4^{(a_1-1)} + 2 * 4^{a_1} * n_1 \qquad b_1 \in B$

$b_2 = 1 + 4 + 4^2 + 4^3 + 4^4 + \cdots + 4^{(a_2-1)} + 2 * 4^{a_2} * n_2 \qquad b_2 \in B$

$b_1 = b_2 \Rightarrow$

$1 + 4 + 4^2 + 4^3 + 4^4 + \cdots + 4^{(a_1-1)} + 2 * 4^{a_1} * n_1 = 1 + 4 + 4^2 + 4^3 + 4^4 + \cdots + 4^{(a_2-1)} + 2 * 4^{a_2} * n_2$

$a_3 = a_2 - a_1 \Rightarrow$

$1 + 4 + 4^2 + 4^3 + 4^4 + \cdots + 4^{(a_1-1)} + 2 * 4^{a_1} * n_1 = 1 + 4 + 4^2 + 4^3 + 4^4 + \cdots + 4^{(a_1-1)} + \cdots + 4^{(a_1+a_3-1)} + 2 * 4^{(a_1+a_3)} * n_2 \Rightarrow$

$2 * 4^{a_1} * n_1 = 4^{a_1} + 4^{(a_1+1)} + 4^{(a_1+2)} + 4^{(a_1+3)} + \cdots + 4^{(a_1+a_3-1)} + 2 * 4^{(a_1+a_3)} * n_2 \Rightarrow$

$2 * 4^{a_1} * n_1 = 4^{a_1} * (1 + 4 + 4^2 + 4^3 + 4^4 + \cdots + 4^{(a_3-1)}) + 2 * 4^{(a_1+a_3)} * n_2 \Rightarrow$

$2 * 4^{a_1} * n1 - 4^{a_1} * (1 + 4 + 4^2 + 4^3 + 4^4 + \cdots + 4^{(a_3-1)}) = 2 * 4^{(a_1+a_3)} * n_2 \Rightarrow$

$4^{a_1} [2 * n_1 - (1 + 4 + 4^2 + 4^3 + \cdots + 4^{(a_3-1)})] = 2 * 4^{(a_1+a_3)} * n_2 \Rightarrow$

$4^{a_1} * [2 * n_1 - (\frac{4^{a_3} - 1}{3})] = 2 * 4^{(a_1+a_3)} * n_2 \Rightarrow$

$\frac{6 * n_1 - 4^{a_3} - 1}{3} = 2 * 4^{a_3} * n_2 \Rightarrow$

$n_2 = \frac{6 * n_1 - 4^{a_3} - 1}{3 * 2 * 4^{a_3}} \Rightarrow$

$3 * 2 * 4^{a_3} * n_2 = 6 * n_1 - 4^{a_3} - 1$

$3 * 4^{a_3} * n_2 = 3 * n_1 - 2^{(2a_3 - 1)} - \frac{1}{2}$

The left side of the equation is an integer, but the right side is not; therefore, there is not two integer (n_1 , n_2) make equation to be correct $\Rightarrow b_1 \neq b_2$

Conclusion: there is no duplicate numbers in the set B

2-2-2- prove that there are no duplicate integers within set C:

$C = 3 + 4 * n$

It is clear that there is no duplicate integers within the set $/C/$

2-2-3- prove that there are no duplicate integers within set D:

we assume that there are two equal integers in the set D, they are:

$d_1 = 3 + 10 * (1 + 4 + 4^2 + 4^3 + 4^4 + \cdots + 4^{(a_1-1)}) + 4^{(a_1+1)} * n_1$

$d_2 = 3 + 10 * (1 + 4 + 4^2 + 4^3 + 4^4 + \cdots + 4^{(a_2-1)}) + 4^{(a_2+1)} * n_2$

$d_1 , d_2 \in D \quad , d_1 = d_2 \Rightarrow$

$3 + 10 * (1 + 4 + 4^2 + 4^3 + 4^4 + \cdots + 4^{(a_1-1)}) + 4^{(a_1+1)} * n_1 =$

$3 + 10 * (1 + 4 + 4^2 + 4^3 + 4^4 + \cdots + 4^{(a_2-1)}) + 4^{(a_2+1)} * n_2$

$a_3 = a_2 - a_1 \Rightarrow$

$3 + 10 * (1 + 4 + 4^2 + 4^3 + 4^4 + \cdots + 4^{(a_1-1)}) + 4^{(a_1+1)} * n_1 =$

$3 + 10 * (1 + 4 + 4^2 + 4^3 + 4^4 + \cdots + 4^{(a_1+a_3-1)}) + 4^{(a_1+a_3+1)} * n_2$
$\Rightarrow$
$4^{(a_1+1)} * n_1 + 10 * (1 + 4 + 4^2 + 4^3 + \cdots + 4^{(a_1-1)}) * [1 - 4^{a_1} + 4^{(a_1+1)} + 4^{(a_1+2)} + 4^{(a_1+3)} + \ldots + 4^{(a_1+a_3+1)})] = 4^{(a_1+a_3+1)} * n_2 \quad \Rightarrow$
$4^{(a_1+1)} * n_1 + 10 * (\frac{4^{a_1} - 1}{3}) * [1 - 4^{a_1} * (1 + 4 + 4^2 + 4^3 + 4^4 + 4^{(a_3+1)})] = 4^{(a_1+a_3+1)} * n_2 \quad \Rightarrow$
$4^{(a_1+1)} * n_1 + 10 * (\frac{4^{a_1} - 1}{3}) * [1 - 4^{a_1} * \frac{4^{(a_3+2)} - 1}{3}] = 4^{(a_1+a_3+1)} * n_2 \quad \Rightarrow$
$4^{(a_1+1)} * n_1 + 10 * (\frac{4^{a_1} - 1}{3}) * [\frac{3 - 4^{(a_1+a_3+2)} + 4^{a_1}}{3}] = 4^{(a_1+a_3+1)} * n_2$
$\Rightarrow$
$9 * 4^{(a_1+1)} * n_1 + 10 * (4^{a_1} - 1) * (3 - 4^{(a_1+a_3+2)} + 4^{a_1}) =$
$9 * 4^{(a_1+a_3+1)} * n_2 \quad \Rightarrow$
$9 * 2^{(2a_1+1)} * n_1 + 5 * (4^{a_1} - 1) * (3 - 4^{(a_1+a_3+2)} + 4^{a_1}) =$
$9 * 2^{(2a_1+2a_3+1)} * n_2$
$9 * 2^{(2a_1)} * n_1 + 5 * 2^{(2a_1-1)} * (3 - 4^{(a_1+a_3+2)} + 4^{a_1}) - \frac{15}{2} +$
$5 * (2^{(2a_1+2a_3+1)} + 2^{(2a_1-1)}) = 9 * 2^{(2a_1+2a_3)} * n_2$

The left side of the equation is not an integer, but the right side is an integer; therefore, there is not two integer (n_1, n_2) make equation to be correct $\Rightarrow$ $d_1 \neq d_2$

Conclusion: there is no duplicate integers in the set D

2-2-4-proof that there are no duplicate integers between the two sets B, C:

we assume that there are two equal integers, one of them $b \in B$ and the other $c \in C$

$b = 1 + 4 + 4^2 + 4^3 + 4^4 + \cdots + 4^{(a-1)} + 2 * 4^a * n_1 \qquad b \in B$
$c = 3 + 4 * n_2 \qquad c \in C \qquad ,c = b \quad \Rightarrow$
$3 + 4 * n_2 = 1 + 4 + 4^2 + 4^3 + 4^4 + \cdots + 4^{(a-1)} + 2 * 4^a * n_1$
$\Rightarrow$
$2 + 4 * n_2 = 4 + 4^2 + 4^3 + 4^4 + \cdots + 4^{(a-1)} + 2 * 4^a * n_1 \quad \Rightarrow$
$\frac{1}{2} + n_2 = 1 + 4 + 4^2 + 4^3 + 4^4 + \cdots + 4^{(a-2)} + 2 * 4^{(a-1)} * n_1$
$\Rightarrow$
$n_2 = -\frac{1}{2} + 4 + 4^2 + 4^3 + 4^4 + \cdots + 4^{(a-2)} + 2 * 4^{(a-1)} * n_1$

The left side of the equation is an integer, but the right side is not an integer; therefore, there is not two integer (n_1, n_2) make equation to be correct $\Rightarrow$ $c \neq b$

Conclusion: there is no duplicate numbers in the two sets C, B

2-2-5-proof that there are no duplicate integers between the two sets B, D:

we assume that there are two equal integers, one of them: $b \in B$, and the other: $d \in D$

$b = 1 + 4 + 4^2 + 4^3 + 4^4 + \cdots + 4^{(a_1-1)} + 2 * 4^{a_1} * n_1 \qquad b \in B$

$d = 3 + 10 * (1 + 4 + 4^2 + 4^3 + 4^4 + \cdots + 4^{(a_2-1)}) + 4^{(a_2+1)} * n_2$

$d \in D \quad ,b = d \quad \Rightarrow$

$1 + 4 + 4^2 + 4^3 + 4^4 + \cdots + 4^{(a_1-1)} + 2 * 4^{a_1} * n_1 =$

$3 + 10 * (1 + 4 + 4^2 + 4^3 + 4^4 + \cdots + 4^{(a_2-1)}) + 4^{(a_2+1)} * n_2$

$a_3 = a_2 - a_1 \qquad \Rightarrow$

$1 + 4 + 4^2 + 4^3 + 4^4 + \cdots + 4^{(a_1-1)} + 2 * 4^{a_1} * n_1 =$

$3 + 10 * (1 + 4 + 4^2 + 4^3 + 4^4 + \cdots + 4^{(a_1+a_3-1)}) + 4^{(a_1+a_3+1)} * n_2$

$\Rightarrow$

$2 * 4^{a_1} * n_1 = 3 + 10 * (1 + 4 + 4^2 + 4^3 + 4^4 + \cdots + 4^{(a_1-1)} + \cdots + 4^{(a_1+a_3-1)}) - (1 + 4 + 4^2 + 4^3 + \cdots + 4^{(a_1-1)}) + 4^{(a_1+a_3+1)} * n_2$

$\Rightarrow$

$2 * 4^{a_1} * n_1 =$

$3 + 10 * (1 + 4 + 4^2 + 4^3 + 4^4 + \cdots + 4^{(a_1-1)}) + 10 * (4^{a_1} + 4^{(a_1+1)} + 4^{(a_1+2)} + 4^{(a_1+3)} + \ldots + 4^{(a_1+a_3-1)}) - (1 + 4 + 4^2 + 4^3 + 4^4 + \cdots + 4^{(a_1-1)}) + 4^{(a_1+a_3+1)} * n_2 \qquad \Rightarrow$

$2 * 4^{a_1} * n_1 =$

$3 + (10 - 1) * (1 + 4 + 4^2 + 4^3 + 4^4 + \cdots + 4^{(a_1-1)}) + 10 * 4^{a_1} * (1 + 4 + 4^2 + 4^3 + 4^4 + \cdots + 4^{(a_3-1)}) + 4^{(a_1+a_3+1)} * n_2 \qquad \Rightarrow$

$2 * 4^{a_1} * n_1 =$

$3 + 9 * (\frac{4^{a_1} - 1}{3}) + 10 * 4^{a_1} * (\frac{4^{a_1} - 1}{3}) + 4^{(a_1+a_3+1)} * n_2 \qquad \Rightarrow$

$3 * 2 * 4^{a_1} * n_1 =$

$9 + 9 * (4^{a_1} - 1) + 10 * 4^{a_1}(4^{a_3} - 1) + 3 * 4^{(a_1+a_3+1)} * n_2 \qquad \Rightarrow$

$3 * 2 * 4^{a_1} * n_1 = 9 * 4^{a_1} + 10 * 4^{a_1}(4^{a_3} - 1) + 3 * 4^{(a_1+a_3+1)} * n_2$

$\Rightarrow$

$3 * 2 * n_1 = 9 + 10 * (4^{a_3} - 1) + 3 * 4^{(a_3+1)} * n_2 \Rightarrow$

$3 * 2 * n_1 = 10 * 4^{a_3} + 3 * 4^{(a_3+1)} * n_2 - 1 \quad \Rightarrow$

$n_1 = \frac{10}{6} * 4^{a_3} + 2^{(2a_3+1)} * n_2 - \frac{1}{6} \qquad \Rightarrow$

$n_1 = 2^{(2a_3+1)} * n_2 - \frac{10 * 4^{a_3} - 1}{6} \qquad \Rightarrow$

The left side of the equation is an integer, but the right side is not an integer; therefore, there is not two integer (n_1, n_2) make equation to be correct $\Rightarrow$ $b \neq d$

Conclusion: there is no duplicate integers in the two sets B, D

2-2-6-prove that there are no duplicate integers between the two sets $\boldsymbol{C, D}$:

$c = 3 + 4 * n_1 \qquad c \in C$

$d = 3 + 10 * \left(1 + 4 + 4^2 + 4^3 + 4^4 + \cdots + 4^{(a-1)}\right) + 4^{(a+1)} * n_2$

$d \in D \quad ,d = c \quad \Rightarrow$

$3 + 4 * n_1 = 3 + 10 * \left(1 + 4 + 4^2 + 4^3 + \cdots + 4^{(a-1)}\right) + 4^{(a+1)} * n_2$

$\Rightarrow$

$4 * n_1 = 10 * \left(\frac{4^a - 1}{3}\right) + 2 * 4^{(a+1)} * n_2 \quad \Rightarrow$

$3 * 4 * n_1 = 10 * (4^a - 1) + 3 * 2 * 4^{(a+1)} * n_2 \quad \Rightarrow$

$3 * 2 * n_1 = 5 * (4^a - 1) + 3 * 4^{(a+1)} * n_2$

$3 * n_1 = 5 * 2^{(2a-1)} - \frac{5}{2} + 3 * 4^{(a+1)} * n_2$

The left side of the equation is an integer, but the right side is not an integer; therefore, there is not two integer (n_1, n_2) make equation to be correct $\Rightarrow$ $c \neq d$

Conclusion: there is no duplicate integers in the two sets C, D

2-3-Proof : $\boldsymbol{B + C + D = A}$

/A/ is the set of odd integers: $A = 2 * x + 1$

2-3-1-prove that: $\boldsymbol{B}$ represent $\boldsymbol{\frac{1}{3}}$ of the numbers of the set $\boldsymbol{A}$

$B = 1 + 4 + 4^2 + 4^3 + 4^4 + \cdots + 4^{(a-1)} + 2 * 4^a * n$

$a = 1 \Rightarrow B_1 = 1 + 2 * 4 * n \Rightarrow$

B_1 represent $\frac{1}{4}$ of the numbers of the set A

$a = 2 \Rightarrow B_2 = 1 + 4 + 2 * 4^2 * n \Rightarrow$

B_2 represent $\frac{1}{4^2}$ of the numbers of the set A

$a = 3 \Rightarrow B_3 = 1 + 4 + 4^2 + 2 * 4^3 * n \Rightarrow$

B_3 represent $\frac{1}{4^3}$ of the numbers of the set A

$a = 4 \Rightarrow B_4 = 1 + 4 + 4^2 + 4^3 + 2 * 4^4 * n \Rightarrow$

B_4 represent $\frac{1}{4^4}$ of the numbers of the set A

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$B = B_1 + B_2 + B_3 + B_4 + B5 + \ldots\ldots \Rightarrow B$ represent $\frac{1}{4} * A + \frac{1}{4^2} * A + \frac{1}{4^3} * A + \frac{1}{4^4} * A + \ldots\ldots\ldots\ldots \Rightarrow$

B represent $A * (\frac{1}{4} + \frac{1}{4^2} + \frac{1}{4^3} + \frac{1}{4^4} + \frac{1}{4^5} + \ldots\ldots\ldots\ldots \Rightarrow$

B represent $\frac{1}{3}$ of the numbers of the set A

2-3-2-prove that: C represent $\frac{1}{2}$ of the numbers of the set A

$C = 3 + 4 * n \Rightarrow C$ represent $\frac{1}{2}$ of the numbers of the set A

2-3-3-prove that: D represent $\frac{1}{6}$ of the numbers of the set A

$D = 3 + 10 * (1 + 4 + 4^2 + 4^3 + 4^4 + \cdots + 4^{(a-1)}) + 4^{(a+1)} * n$

$a = 1 \Rightarrow D_1 = 3 + 10 * (1) + 4^2 * n \Rightarrow$

D_1 represent $\frac{1}{2*4}$ of the numbers of the set A

$a = 2 \Rightarrow D_2 = 3 + 10 * (1 + 4) + 4^3 * n \Rightarrow$

D_2 represent $\frac{1}{2*4^2}$ of the numbers of the set A

$a = 3 \Rightarrow D_2 = 3 + 10 * (1 + 4 + 4^2) + 4^4 * n \Rightarrow D_3$ represent $\frac{1}{2*4^3}$ of the numbers of the set A

$a = 4 \Rightarrow D_4 = 3 + 10 * (1 + 4 + 4^2 + 4^3) + 4^5 * n \Rightarrow$

D_4 represent $\frac{1}{2*4^4}$ of the numbers of the set A

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$D = D_1 + D_2 + D_3 + D_4 + D_5 + \ldots\ldots \Rightarrow$

D represent $\frac{1}{2*4} * A + \frac{1}{2*4^2} * A + \frac{1}{2*4^3} * A + \frac{1}{2*4^4} * A + \ldots\ldots \Rightarrow$

D represent $A * (\frac{1}{2*4} + \frac{1}{2*4^2} + \frac{1}{2*4^3} + \frac{1}{2*4^4} + \ldots\ldots \Rightarrow$

D represent $\frac{1}{6}$ of the numbers of the set A

$B + C + D$ represent $\frac{1}{3} * A + \frac{1}{2} * A + \frac{1}{6} * A \Rightarrow$

$B + C + D$ represent all the numbers of the set $A = 2 * n + 1$

Conclusion2: the sets (B, C, D) present all the odd integers /A/ without duplicated integers.

3-prove that is no loop (except the loop which number /1/ makes it with itself):

firstly: every integers $m \in M$ [M is the set of multiples of /3/, $M = (3 * x)$]. multiples of /3/ do not make a loop with other numbers, because these integers (M) cannot be produced when we perform the function to an another number $/x/$. the explanation is as follows:

$ƒ(x_1) = \frac{3x_1+1}{2^a} = 3 * x \Rightarrow 3 * x_1 = 2^a * 3 * x - 1 \Rightarrow x_1 = 2^a * x - \frac{1}{3}$

We note that $/x_1/$ is not an integer $\Rightarrow x_1 \notin A$ **,**Therefore, it is refused

<u>Secondly:</u> $/R/$ is the set of odd integers except the multiples of /3/ $\{M = (3 * x)\}$, $R = A \setminus M$

We divide $/R/$ into two sets of integers, they are: R_a, R_b

$R_a = 1 + 6 * n$ $\quad R_b = 5 + 6 * n$ $\quad R = R_a + R_b$

performing the function to the set $/H_1/$ will produce the set $/R_1/$

$H_1 = 21 + 6 * 2^6 * n = 3 * (7 + 2^6 * 2 * n)$

$R_1 = ƒ(H_1) = \frac{3 * H_1 + 1}{2^6} = \frac{3 * (21 + 6 * 2^6 * n) + 1}{2^6} = 1 + 3 * 6 * n \Rightarrow$

$R_1 = 1 + 3 * 6 * n$

performing the function to the set H_2 will produce the set $/R_2/$ $\Rightarrow$

$H_2 = 3 + 2 * 6 * n$

$R_2 = ƒ(H_2) = \frac{3 * H_2 + 1}{2^1} = \frac{3 * (3 + 6 * 2^1 * n) + 1}{2^1} = 5 + 3 * 6 * n \Rightarrow$

$R_2 = 5 + 3 * 6 * n$

performing the function to the set $/H_3/$ will produce the set R_3 $\Rightarrow$

$H_3 = 9 + 6 * 2^2 * n$

$R_3 = ƒ(H_3) = \frac{3 * H_3 + 1}{2^2} = \frac{3 * (9 + 6 * 2^2 * n) + 1}{2^2} = 7 + 3 * 6 * n \Rightarrow$

$R_3 = 7 + 3 * 6 * n$

performing the function to the set H_4 will produce the set R_4 $\Rightarrow$

$H_4 = 117 + 6 * 2^5 * n$

$R_4 = ƒ(H_4) = \frac{3 * H_4 + 1}{2^5} = \frac{3 * (117 + 6 * 2^5 * n) + 1}{2^5} = 11 + 3 * 6 * n \Rightarrow$

$R_4 = 11 + 3 * 6 * n$

performing the function to the set H_5 will produce the set R_5

$H_5 = 69 + 6 * 2^4 * n$

$R_5 = ƒ(H_5) = \frac{3 * H_5 + 1}{2^4} = \frac{3 * (69 + 6 * 2^4 * n) + 1}{2^4} = 13 + 3 * 6 * n \Rightarrow$

$R_5 = 13 + 3 * 6 * n$

performing the function to the set H_6 will produce the set R_6 $\Rightarrow$

$H_6 = 45 + 6 * 2^3 * n$

$R_6 = ƒ(H_6) = \frac{3 * H_6 + 1}{2^3} = \frac{3 * (45 + 6 * 2^3 * n) + 1}{2^3} = 17 + 3 * 6 * n \Rightarrow$

$R_6 = 17 + 3 * 6 * n$

$R_1 + R_3 + R_5 = 1 + 6 * n = R_a$

$R_2 + R_4 + R_6 = 5 + 6 * n = R_b$

$R_1 + R_2 + R_3 + R_4 + R_5 + R_6 = R_a + R_b = R$

Conclusion3: 1-the sets of numbers R_1, R_2, R_3, R_4, R_5, R_6 represent all odd integers except multiples of /3/ (that means they represent $f(A)$, as we will see later)

2-we can produce all the integers of the set $/R/$ by performing the function to the sets $\{H_1, H_2, H_3, H_4, H_5, H_6$ (which are a part from multiples of /3/)$\}$

we represent this with the following equation:

$$R = f(H) = \frac{3 * H + 1}{2^t} \qquad \left\{ \begin{array}{c} H = H_1 + H_2 + H_3 + H_4 + H_5 + H_6 \\ t \in T \quad ,T = \{1, 2, 3, 4, 5, 6\} \\ \{x = 2 * n + 1\} \end{array} \right\}$$

$$R = f(H) = \frac{3 * H + 1}{2^t} = \frac{3 * 3 * x + 1}{2^t}$$

$$R = \frac{3^2 * x + 1}{2^t} \qquad t \in T \ ,T = \{1, 2, 3, 4, 5, 6\}$$

We assume that the integers: $\{r_1, r_2, r_3, r_4, \ldots\ldots\ldots, r_s, r_{(s+1)}\}$ form a loop, this means that performing the function to $/r_1/$ produce: $/r_2/$, and applying conjecture to $/r_2/$ produce: $/r_3/$, and applying conjecture to $/r_3/$ produce: $/r_4/$, and so on …….. until $/r_s/$ produce: $/r_{(s+1)}/$

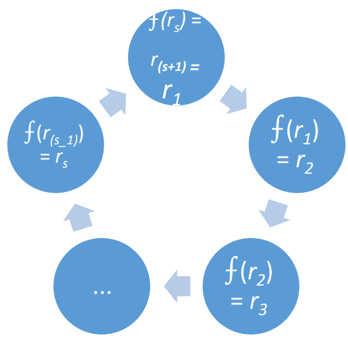
$$f(r_1) = r_2 \ , f(r_2) = r_3 \ , f(r_3) = r_4 \ , \ldots \ , f(r_s) = r_{(s+1)}$$

When have a loop, this means: $r_{(s+1)} = r_1$

$$r_1 = \frac{3^2 * x_1 + 1}{2^{t_1}} \qquad r_1 \in R$$

$$r_{(s+1)} = \frac{3^2 * x_2 + 1}{2^{t_2}} \qquad r_{(s+1)} \in R$$

$$r_1 = r_{(s+1)} \quad \Rightarrow \quad \frac{3^2 * x_1 + 1}{2^{t_1}} = \frac{3^2 * x_2 + 1}{2^{t_2}} \quad \Rightarrow$$



$$2^{t_2} * (3^2 * x_1 + 1) = 2^{t_1} * (3^2 * x_2 + 1) \quad \Rightarrow$$

$$3^2 * x_1 * 2^{t_2} = 3^2 * x_2 * 2^{t_1} + 2^{t_1} - 2^{t_2} \quad \Rightarrow$$

$$3^2 * x_1 = 3^2 * x_2 * 2^{(t_1 - t_2)} + 2^{(t_1 - t_2)} - 1 \quad \Rightarrow$$

$$x_1 = x_2 * 2^{(t_1 - t_2)} + \frac{2^{(t_1 - t_2)} - 1}{3^2}$$

$t_1 - t_2 = \{$-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5$\}$

when: $(t_1 - t_2) = \{$-5, -4, -3, -2, -1, 1, 2, 3, 4, 5$\} \Rightarrow x_1$ not an integer (so these values refused).

when: $(t_1 - t_2) = 0 \quad \Rightarrow \quad x_1 = x_2$ that means the integer $/r_1/$ make a loop with itself, as in the following equation:

$$f(r_1) = \frac{3 * r_1 + 1}{2^a} = r_1 \qquad , r_1 = \frac{3^2 * x_1 + 1}{2^{t_1}} \quad \Rightarrow$$

$$\frac{3 * r_1 + 1}{2^a} = \frac{3^2 * x_1 + 1}{2^{t_1}} \Rightarrow 2^{t_1} * 3 * r_1 + 2^{t_1} = 2^a * 3^2 * x_1 + 2^a \Rightarrow$$

$2^{t_1} * 3 * \frac{3^2 * x_1 + 1}{2^{t_1}} + 2^{t_1} = 2^a * 3^2 * x_1 + 2^a \Rightarrow$

$x_1 = \frac{1}{3^2} * (\frac{2^{t_1}}{2^{a-3}} - 1)$

When: $t_1 = \{1,2,3,4,5\} \Rightarrow x_1$ not natural number $\forall\, a \epsilon \{1,2,3,4,5, \dots.\}$ so: its refused.

When: $(t_1 = 6,\ a\, \epsilon\, \{1,3,4,5,6,7,8, \dots \dots\}) \Rightarrow x_1$ not natural number.

When: $(t_1 = 6,\ a = 2) \Rightarrow x_1 = 7 \Rightarrow r_1 = \frac{3^2 * 7 + 1}{2^6} \Rightarrow r_1 = 1$

Conclusion4: There is no any set of integers makes a loop, except the loop which integer /1/ makes with itself.

4- performing the function to sets B, C, D:

4-1- performing the function to set B:

$B = 1 + 4 + 4^2 + 4^3 + 4^4 + \cdots + 4^{(a-1)} + 2 * 4^a * n$

$$ƒ(B) = \frac{3 * B + 1}{4^a} = \frac{3 * (1 + 4 + 4^2 + 4^3 + \dots + 4^{(a-1)} + 2 * 4^a * n) + 1}{4^a} \Rightarrow$$

$$ƒ(B) = \frac{3 * (1 + 4 + 4^2 + 4^3 + \dots + 4^{(a-1)}) + 3 * 2 * 4^a * n + 1}{4^a}$$

$$1 + 4 + 4^2 + 4^3 + \cdots + 4^{(a-1)} = \frac{4^a - 1}{4 - 1} = \frac{4^a - 1}{3} \Rightarrow$$

$$ƒ(B) = \frac{3 * \left(\frac{4^a - 1}{3}\right) + 3 * 2 * 4^a * n + 1}{4^a} \Rightarrow ƒ(B) = 1 + 6 * n$$

Based on this equation we draw **Table 1**:

Table 1: $ƒ(B) = 1 + 6 * n$

		n	n=0	n=1	n=2	n=3	n=4	n=5	n=6	n=7	n=8
		$ƒ(x) = 1 + 6 * n$	1	7	13	19	25	31	37	43	…..
x	a=1	$1 + 2 * 4 * n$	1	9	17	25	33	41	49	57	…..
	a=2	$1 + 4 + 2 * 4^2 * n$	5	37	69	101	133	165	197	229	…..
	a=3	$1 + 4 + 4^2 + 2 * 4^3 * n$	21	149	277	405	533	661	789	917	…..
	a=4	$1 + 4 + 4^2 + 4^3 + 2 * 4^4 * n$	85	597	1109	1621	2133	2645	3157	3669	…..
	a=5	$1 + 4 + 4^2 + 4^3 + 4^4 + 2 * 4^5 * n$	341	2389	4437	6485	8533	10581	12629	14677	…..
	a=6	$1 + 4 + 4^2 + 4^3 + 4^4 + 4^5 + 2 * 4^6 * n$	1365	9557	17749	25941	34133	…..	…..	…..	……
	a=7	$1 + 4 + 4^2 + 4^3 + 4^4 + 4^5 + 4^6 + 2 * 4^7 * n$	5461	38229	70997	……	……	…….	……	…….	…….
	a=8	$1 + 4 + 4^2 + 4^3 + 4^4 + 4^5 + 4^6 + 4^7 + 2 * 4^8 * n$	21845	152917	…..	……	…..	…..	…..	……	…….
	a=9 …	……………………	….	…..	…..	……	…..	…..	…..	……	…….
		………………… …..	….	…..	…..	……	…..	…..	…..	……	…….

		$(1+4+4^2+4^3+4^4+\cdots+4^{(a-1)})+2*4^a*n$	$1+4+4^2+4^3+4^4+\cdots+4^{(a-}$	$(1+4+4^2+4^3+4^4+\cdots+4^{(a-}+2*4^a*1$	$(1+4+4^2+4^3+4^4+\cdots+4^{(a-}+2*4^a*2$	$(1+4+4^2+4^3+4^4+\cdots+4^{(a-}+2*4^a*3$	$(1+4+4^2+4^3+4^4+\cdots+4^{(a-}+2*4^a*4$	…….	…….	…..	…..
		…………………………..	….	…..	…..	……	…..	…..	…..	……	…….

Notice: we colored the inputs integers of the set V by red, and colored the outputs integer of the set C by yellow, and the inputs and outputs integers of the set $A\backslash(V+C)$ by white

4-2- performing the function to set C:

$C = 3 + 4 * n$

$$ƒ(C) = \frac{3*C+1}{2} = \frac{3*(3+4*n)+1}{2} \quad \Rightarrow$$

$ƒ(C) = 5 + 6 * n$

Table 2: $ƒ(C) = 5 + 6 * n$

n	$n=0$	$n=1$	$n=2$	$n=3$	$n=4$	$n=5$	$n=6$	$n=7$	$n=8$	$n=9$
$ƒ(x)=5+6*n$	5	11	17	23	29	35	41	47	53	….
$x = 3 + 4*n$	3	7	11	15	19	23	27	31	35	…..

We colored the inputs and outputs integers of the set C by yellow, and the outputs integers of the set V by green.

Notice1: the set $C = 3 + 4 * n$ is the only set which produce a bigger numbers when performing the function to its integers, and all the integers of the set $A\backslash C$ produce a numbers thinner integer when performing the function its integers.

4-3- performing the function to set D:

$$D = 3 + 10 * (1 + 4 + 4^2 + 4^3 + 4^4 + \cdots + 4^{(a-1)}) + 4^{(a+1)} * n$$

$$ƒ(D) = \frac{3*D+1}{2*4^a} = \frac{3*[3+10*(1+4+4^2+4^3+\cdots+4^{(a-1)})+4^{(a+1)}*n)]+1}{2*4^a}$$

$$ƒ(D) = \frac{3*D+1}{2*4^a} = \frac{10+10*[3*(1+4+4^2+4^3+\cdots+4^{(a-1)})]+3*4^{(a+1)}*n}{2*4^a}$$

$$1 + 4 + 4^2 + 4^3 + \cdots + 4^{(a-1)} = \frac{4^a-1}{4-1} = \frac{4^a-1}{3} \Rightarrow$$

$$ƒ(D) = \frac{10+10*3*(\frac{4^a-1}{3})+3*4^{(a+1)}*n}{2*4^a} \quad \Rightarrow$$

$$ƒ(D) = \frac{10*4^a+3*4^{(a+1)}*n}{2*4^a} \quad \Rightarrow$$

$ƒ(D) = 5 + 6 * n$

Based on these two equations ($ƒ(D) = 5 + 6 * n$, $ƒ(C) = 5 + 6 * n$), we draw Table 3:

Table 3: $ƒ(D) = 5 + 6 * n$ and $ƒ(C) = 5 + 6 * n$

		n	$n=0$	$n=1$	$n=2$	$n=3$	$n=4$	$n=5$	$n=6$	$n=7$	$n=8$
		$ƒ(x) = 5 + 6 * n$	5	11	17	23	29	35	41	47	....
		$3 + 4 * n$	3	7	11	15	19	23	27	31	.....
	a=1	$3 + 10 * (1) + 4^2 * n$	13	29	45	61	77	93	109	125	......
	a=2	$3 + 10 * (1 + 4) + 4^3 * n$	53	117	181	245	309	373	437	501	......
x	a=3	$3 + 10 * (1 + 4 + 4^2) + 4^4 * n$	213	469	725	981	1237	1439	1749	2005	.....
	a=4	$3 + 10 * (1 + 4 + 4^2 + 4^3) + 4^5 * n$	853	1877	2901	3925	4949	5973	6997	8021	......
	a=5	$3 + 10 * (1 + 4 + 4^2 + 4^3 + 4^4) + 4^6 * n$	3413	7509	11605	15701	19797	23893	.....	.....	......
	a=6	$3 + 10 * (1 + 4 + 4^2 + 4^3 + 4^4 + 4^5) + 4^7 * n$	13653	30037	46421	62805	......	....	......	....	....
	a=7	$3 + 10 * (1 + 4 + 4^2 + 4^3 + 4^4 + 4^5 + 4^6) + 4^8 * n$	54613	120149	.....	......	.....	.....	.....	......	....
	a=8	....	....	.....	.....	......	.....	.....	.....	......	....
	a=9	....	....	.....	.....	......	.....	.....	.....	......	....
	a=	$3 + 10 * \left(1 + 4 + 4^2 + 4^3 + 4^4 + \cdots + 4^{(a-1)}\right) + 4^{(a+1)} * n$	$3 + 10 * (1)$	$3 + 10 * (1 + 4 +) + 4^2 * 1$	$3 + 10 * (1 + 4 + 4^2 +) + 4^3 * 2$	$3 + 10 * (1 + 4 + 4^2 + 4^3 +) + 4^4 * 3$	$3 + 10 * (1 + 4 + 4^2 + 4^3 + 4^4) + 4^5 * 4$	....	....	.....	.....
		....	....	.....	.....	......	.....	.....	.....	......	....
		....	....	.....	.....	......	.....	.....	.....	......	....

Notice: we colored the inputs integers of the set V by red, and outputs integers of the set V by green and the inputs and outputs integers of the set C by yellow. And we colored the integers of the set V in the results by green, and the integers of the set $A \backslash (V+C)$ by white

5-Prove that the integers of set / $V = 5 + 2 * 6 * n$ / are present in all columns of the two tables (Table 1 and Table 3):

In Table1:

$B = 1 + 4 + 4^2 + 4^3 + 4^4 + \cdots + 4^{(a-1)} + 2 * 4^a * n$

$a = \{1, 2, 3, 4, 5, 6, \ldots\ldots\}$

$n = \{0, 1, 2, 3, 4, 5, \ldots\ldots..\}$
$(a = 1) \Rightarrow B_1 = 1 + 2 * 4 * n$
$V_1 \in B_1$
$V_1 = 17 + 3 * 2 * 4 * n \quad (a = 1,\ n = 2 + 3 * y) \Rightarrow$
$V_1 = 5 + 12 + 3 * 2 * 4 * n \Rightarrow V_1 = 5 + 12 * (1 + 2 * n) \Rightarrow$
$V_1 = 5 + 2 * 6 * n_1$
This means $V_1 \in V$
$(a = 2) \Rightarrow B_2 = 1 + 4 + 2 * 4^2 * n$
$V_2 \in B_2$
$V_2 = 5 + 3 * 2 * 4^2 * \mathrm{n} \quad (= 2\ ,\ n = 0 + 3 * y) \Rightarrow$
$V_2 = 5 + 12 * 2 * 4^2 * n \Rightarrow V_2 = 5 + 2 * 6 * n_2 \quad ,(n_2 = 2 * 4^2 * n)$
This means $V_2 \in V$
$(a = 3) \Rightarrow B_3 = 1 + 4 + 4^2 + 2 * 4^3 * n$
$V_3 \in B_3$
$V_3 = 149 + 3 * 2 * 4^3 * n \quad (a = 3, n = 1 + 3 * y) \Rightarrow$
$V_3 = 5 + 12^2 + 3 * 2 * 4^3 * n \Rightarrow V_3 = 5 + 12 * (12 + 3 * 2 * 4^3 * n)$
$V_3 = 5 + 2 * 6 * n_3 \quad (n_3 = 12 + 2 * 4^2 * n)$
This means $V_3 \in V$
The three previous steps means that: the integers of the set $/V/$ are present in the all columns of Table 1 (in the lines $\{a = 1, 2, 3\}$).
$V_s = 1 + 4 + 4^2 + 4^3 + \cdots + 4^{(a_s-1)} + 2 * 4^{a_s} * n_s = 5 + 2 * 6 * n_m$
$(V_s \in \mathrm{B}\ ,\ V_s \in V)$
$V_s + 4^{a_s} + 4^{(a_s+1)} + 4^{(a_s+2)} + 2 * 4^{a_s} * (4^3\_1) * n_s = 5 + 2 * 6 * n_m + 4^{a_s} + 4^{(a_s+1)} + 4^{(a_s+2)} + 2 * 4^{a_s} * (4^3\_1) * n_s$
$V_s + 4^{a_s} + 4^{(a_s+1)} + 4^{(a_s+2)} + 2 * 4^{a_s} * (4^3\_1) * n_s = 5 + 2 * 6 * n_m + 4^{a_s} * 21 + 2 * 4^{a_s} * 63 * n_s$
$V_s + 4^{a_s} + 4^{(a_s+1)} + 4^{(a_s+2)} + 2 * 4^{a_s} * (4^3\_1) * n_s = 5 + 2 * 6 *(n_m + 4^{(a_s-1)} * 7 + 2 * 4^{(a_s-1)} * 21 * n_s)$
$V_s + 4^{a_s} + 4^{(a_s+1)} + 4^{(a_s+2)} + 2 * 4^{a_s} * (4^3\_1) * n_s = 5 + 2 * 6 * n_r$
$n_r = n_m + 4^{(a_s-1)} * 7 + 2 * 4^{(a_s-1)} * 21 * n_s$
This means: $V_s + 4^{a_s} + 4^{(a_s+1)} + 4^{(a_s+2)} + 2 * 4^{a_s} * (4^3\_1) * ns \in V$

Conclusion5: the integers of the set $/V/$ are present in the all columns of Table 1.

each integer of the set $/V/$ -three lines below it- there is another integer of the set $/V/$

In Table 3:
$D = 3 + 10 * \left(1 + 4 + 4^2 + 4^3 + 4^4 + \cdots + 4^{(a-1)}\right) + 4^{(a+1)} * n$
$a = \{1, 2, 3, 4, 5, \ldots\}$

$n = \{0, 1, 2, 3, 4, 5, \ldots\}$
$(a = 1, n = 1 + 3 * y) \Rightarrow$
$V_4 = 29 + 3 * 4^2 * n \Rightarrow$
$V_4 = 5 + 24 + 3 * 4^2 * n \Rightarrow V_4 = 5 + 3 * 4 * (2 + 4 * n) \Rightarrow$
$V_4 = 5 + 2 * 6 * n_4 \qquad (n_4 = 2 + 4 * n)$
This means: $V_4 \in V$
$(a = 2, n = 0 + 3 * y) \Rightarrow$
$V_5 = 53 + 3 * 4^3 * n$
$V_5 = 5 + 48 + 3 * 4^3 * n \Rightarrow V_5 = 5 + 12 * (4 + 4^2 * n) \Rightarrow$
$V_5 = 5 + 2 * 6 * n_5 \qquad (n_5 = 4 + 4^2 * n)$
This means: $V_5 \in V$
$(a = 3, n = 2 + 3 * y) \Rightarrow$
$V_6 = 725 + 3 * 4^4 * n$
$V_6 = 5 + 720 + 3 * 4^4 * n \Rightarrow V_6 = 5 + 12 * (60 + 4^3 * n) \Rightarrow$
$V_6 = 5 + 2 * 6 * n_6 \qquad (n_6 = 60 + 4^3 * n)$
This means: $V_6 \in V$
The three previous steps means that: the integers of the set $/V/$ are present in the all columns of Table 3 (in the lines $\{ a = 1, 2, 3\}$).
$V_p = 3 + 10 * (1 + 4 + 4^2 + 4^3 + 4^4 + \cdots + 4^{(a_p-1)}) + 4^{(a_p+1)} * n_y =$
$5 + 2 * 6 * n_j \qquad (V_p \in D,\ V_p \in V)$
$V_P + 4^{a_p} + 4^{(a_p+1)} + 4^{(a_p+2)} + 4^{(a_p+1)} * (4^3 - 1) * n_y =$
$3 + 10 * (1 + 4 + 4^2 + 4^3 + 4^4 + \cdots + 4^{(a_p-1)}) + 4^{(a_p+1)} * n_y + 4^{a_p}$
$+ 4^{(a_p+1)} + 4^{(a_p+2)} + 4^{(a_p+1)} * (4^3 - 1) * n \Rightarrow$
$V_P + 4^{a_p} + 4^{(a_p+1)} + 4^{(a_p+2)} + 4^{(a_p+1)} * (4^3 - 1) * n_y = 5 + 2 * 6 * n_j$
$+4^{a_p} + 4^{(a_p+1)} + 4^{(a_p+2)} + 4^{(a_p+1)} * (4^3 - 1) * n_y \Rightarrow$
$V_P + 4^{a_p} + 4^{(a_p+1)} + 4^{(a_p+2)} + 4^{(a_p+1)} * (4^3 - 1) * n_y = 5 + 2 * 6 * n_j$
$+ 4^{a_p} * (1 + 4 + 4^2) + 4^{(a_p+1)} * (4^3 - 1) * n_y$
$V_P + 4^{a_p} + 4^{(a_p+1)} + 4^{(a_p+2)} + 4^{(a_p+1)} * (4^3 - 1) * n_y = 5 + 2 * 6 * n_j$
$+ 4^{a_p} * 21 + 4^{(a_p+1)} * 63 * n_y$
$V_P + 4^{a_p} + 4^{(a_p+1)} + 4^{(a_p+2)} + 4^{(a_p+1)} * (4^3 - 1) * n_y = 5 + 2 * 6 * n_j$
$+12 * 7 * 4^{(a_p-1)} + 12 * 21 * 4^{a_p} * n_y$
$V_P + 4^{a_p} + 4^{(a_p+1)} + 4^{(a_p+2)} + 4^{(a_p+1)} * (4^3 - 1) * n_y = 5 + 2 * 6$
$*(n_j + 7 * 4^{(a_p-1)} + 21 * 4^{a_p} * n_y)$
$V_P + 4^{a_p} + 4^{(a_p+1)} + 4^{(a_p+2)} + 4^{(a_p+1)} * (4^3 - 1) * n_y = 5 + 2 * 6 * n_c$
This means: $V_p + 4^{a_p} + 4^{(a_p+1)} + 4^{(a_p+2)} + 4^{(a_p+1)} * (4^3 - 1) * n_y \in V$

Conclusion6: -the integers of the set /V/ are present in the all columns of Table 3.

each integer of the set /V/ -three lines below it- there is another integer of the set /V/

6- proof that performing the function to set /$C = 3 + 4 * n$/ produce the set /$V = 5 + 2 * 6 * n$/:

$$ƒ(C) = \frac{3 * C + 1}{2} = \frac{3 * (3 + 4 * n) + 1}{2} \Rightarrow$$

$ƒ(C) = 5 + 6 * n$

We divide $ƒ(C)$ to two parts: $V_1 = 5 + 2 * 6 * n$ and $C_1 = 11 + 2 * 6 * n$

$\Rightarrow ƒ(C) = V_1 + C_1 = 5 + 6 * n$

$C_1 = 11 + 2 * 6 * n = 3 + 4 * (2 + 3 * n) \Rightarrow$

$C_1 = 3 + 4 * n_q$,($n_q = 2 + 3 * n$) This means: $C_1 \in C$

Since: $C_1 \in C$ this means that performing the function to /C_1/ also produce two sets of integers:

one of them: $C_2 \in C = 3 + 4 * n$, and the other: $V_2 \in V = 5 + 2 * 6 * n$

Conclusion7: performing the function to the set of numbers /C/ produce a series of integers (these series consist of one or several integers which belongs to the set ($C = 3 + 4 * n$) whereas the last integers in every single series produce- by performing the function to it- an integer $v \in V$: ($V = 5 + 2 * 6 * n$)

Conclusion8: performing the function to the set /C/ produce the set ($V = 5 + 2 * 6 * n$) which is present in all columns of Table1 and Table2 (Conclusion6)

Actually, when we perform the function to the set $C = 3 + 4 * n$ produce all the integers of the set $V = 5 + 2 * 6 * n$ because:

$ƒ(3 + 8 * n) = 5 + 2 * 6 * n$, $(3 + 8 * n) \in C$

v
…
….
…..
c_3
c_2
c_1

$\{c_1, c_2, c_3, c_4, …., …., \} \in C$, $v \in V$,example: number 31

161
107
71
47
31

7-Prove the all integers of the set /A/ return to integer /1/ when we perform the function to them:

In Conclusion8 we proved that performing the function to the set $/C = 3 + 4 * n/$ produce the set $/V = 5 + 2 * 6 * n/$

applying the function to the set / $V = 5 + 2 * 6 * n$ / produce the set /R/ infinitely many times for every single integer of the set /R / (because the integers of the set /V / appear in all the columns of both tables (Table 1, and Table 3), and in each column there are infinitely many integers of the set /V / as shown in. (Conclusion5 and Conclusion6))

clearing the previous idea in the Table 4:

Table 4: applying the function to the set $V = 5 + 2 * 6 * n$

$ƒ(V) = R$	1	13	11	31	5	49	29	67	19	85	47	103	7	121
V	5	17	29	41	53	65	77	89	101	113	125	137	149	161

$ƒ(V) = R$	65	139	37	157	83	175	23	193	101	211	55	229	119	247	1	..
V	173	185	197	209	221	233	245	257	269	281	293	305	317	329	341	..

in Table 4 we notice that every single integer of the set R appears infinity times in outputs.

since /V/ ϵ /R/ this means all integers of the set /V/ are present in the outputs we obtained, plus the along with all other integers of the set /R/

therefore, we will apply the function only to the integers of the set /R/ that do not belong to the set / $V = 5 + 2 * 6 * n$/

A subset of these integers is the set $C = 3 + 4 * n$ and we proved that applying the function to them produce the set $V = 5 + 2 * 6 * n$

the other integers in this set produce smaller numbers when we apply the function to them (Notice1), therefore: when we repeatedly apply the function to them, they eventually produce else number 1 (example: integer 113 ϵ V and the integer 51569 ϵ V)

or produce integers of the set $C = 3 + 4 * n$, and these integers (of the set C) produce -at the end- integers of the set $V = 5 + 2 * 6 * n$ (Conclusion8). (example: integer 65 ϵ V)

1
85
113

17
11
7
37
49
65

1
85
1813
38677
51569

at the end of the previous steps we obtain two kinds of outputs: the first is integer /1/, the second is: all the numbers of the set /V /, as in the following Table 5:

Table 5: applying the function to the set $V = 5 + 2 * 6 * n$ until obtain integer /1/ or integers of the set /V/

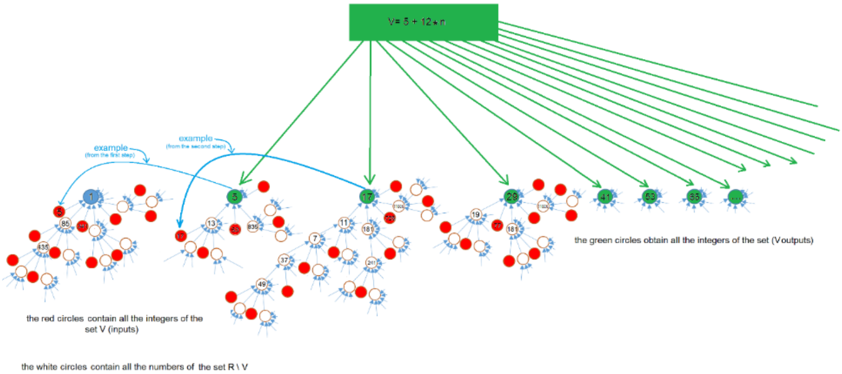
$f(R/V) =$													
$f(R/V) =$													
$f(R/V) =$													
$f(R/V) =$													
$f(R/V) =$				161		17							
$f(R/V) =$				107		11					161		
$f(R/V) =$				71		7					107	233	17
$f(R/V) =$		5	17	47		37		101	29	1	71	155	11
$f(V) = R$	1	13	11	31	5	49	29	67	19	85	47	103	7
V	5	17	29	41	53	65	77	89	101	113	125	137	149

						665							
			17			443		41					
137			11	89		395	53	109			125	65	….
91		209	7	59	125	263	35	145		317	83	43	….
121	65	139	37	157	83	175	23	193	101	211	55	229	….
161	173	185	197	209	221	233	245	257	269	281	293	305	….

Now: we have all the integers of the set / $V = 5 + 2 * 6 * n$ / in inputs. and in the outputs we have the integer 1 (infinitely many times) and also we have all the integers of the set / $V = 5 + 2 * 6 * n$ / in outputs. Between both (the set V (inputs) and the set V (outputs)) there is all the integers of the set R (as shown as in the Table5)

Now: under the integer 1 there is an infinity of integers of the set /V inputs/ (we colored them by red) (naturally these integers are present in the outputs set /V / we obtain (colored by green)), so, we will join these integers (which belong to the set /V outputs/) to their places at the series which's present under integer 1 as shown in the following scheme 1:

scheme 1: joining the integers (which belong to the set /*V* outputs/) to their places at the series which's present under integer /1/



Then we repeat this step on the new integers-which's belong to the set (V inputs (colored by red))- which became under number 1.

We will repeat this step- again and again- and all the integers of the set R will join to the integer /1/ one after the other in an ongoing process.

Performing the function to the set $M = 3 * x$ [M is the set of multiples of /3/, $(3 * x)$] produce integers belong to the set R (Conclusion3), so all the integers of the set $M = 3 * x$ return to number 1 when performing the function to them $A = R + M \quad \Rightarrow$

The integers of the set A (A present all the odd integers) return to number 1 when performing the function to them. and this means that all even integers return to number 1 when performing the function to them (Conclusion1)

Final conclusion: Through a step-by-step analysis, we demonstrated that applying the function to all integers in the set A eventually leads to the integer 1. By proving that specific subsets of A transition through intermediate sets before converging to 1, we established that the iterative process always terminates. This confirms that the Collatz Conjecture holds true for all integers.

Results: The Collatz Conjecture has been proven true.

References

(1) **GUY** R., 1983- **Don't try to solve these problems!**, *American Math*, Monthly 90, 35-41.