Order and Contingency

The Duhem Thesis on Origin of Physics in Christian Theology

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Abstract

The most important aspect of modern physics is the study of the unchanging order between measured quantities, such as that described by Newton's law of gravity or Ampère's law. This aspect makes physics useful, as the laws of physics are applied to develop a variety of technical devices and machines, being a crucial foundation of technical civilization. Additionally, this aspect reveals certain truths about the world, as accurate predictions calculated from these laws testify to their validity. Furthermore, when new experiments force a physical law to be modified, the description of relations between quantities is almost never refuted and becomes a special case of the new, more general theory, showing an incremental progress of knowledge.

How has this most critical part of physics originated? The source of it is theology, which produced core methodological principles that underpin this approach:

- The world is comprehensible and ordered according to universal, unchanging relations between measured quantities.
- The world is contingent, and any metaphysical possibility we conceive is possible, if only in agreement with experience.
- The highest goal of the rational mind is the pursuit and contemplation of truth.

These principles were essential to the development of modern physics. The success of this project provides evidence in favor of Christian Revelation, by means of heuristic power — prediction of essential knowledge that could not be obtained in any other way.

1 Extended abstract.

1.1 Order and Contingency.

The discovery of the above theses might be attributed to Pierre M. Duhem, Stanley L. Jaki, and in some specific cases to Augustin L. Cauchy, Leonhard Euler, and other scientists. By far the most important contribution belongs to Duhem, a physicist, methodologist, and historian of physics active at the turn of the 19th and 20th centuries, who wrote extensively on the method of physics as well as brought to light the legacy of medieval scholastic physics. Duhem's view on method of physics fits well with modern physics[1], according to the opinions of its creators[2],[3]. The method also finds support in the history of science. Duhem, as an outstanding historian of physics[4], is the main advocate of the historical continuity of physics. To see this continuity, one must cleanse physics of what cannot be established through experience. What must be left is what is necessary for the theory to predict new phenomena: an abstract, mathematical formulation of the theory and measurement procedures.

Thesis 1.1 *Physical theories describe an order among measured quantities that actually exists in nature*[5,6].

In this sense, scientific theories are true; according to Duhem, we can ascertain this through experience[7]. '"The supreme test to recognize a classification as true is to demand that it predict in advance things that will only become known in the future." When

[1] (Dugas, "La méthode physique au sens de Duhem devant la mécanique des quanta - translated by A. Aversa as Physical method according to Duhem in view of quantum mechanics")

[2] The eminent physicist Louis de Broglie (d. 1987), writing in 1952 in the foreword to (Pierre Duhem, *The aim and structure of physical theory*) p. ix, states that these views were adopted by many quantum physicists.

[3] On Duhem's influence on Einstein, see (Howard, "Einstein and Duhem")

[4] de Broglie in (Pierre Duhem, *The aim and structure of physical theory*), p. viii

[5] (ibid.), p. 28

[6] (López Ruiz and Woollard, "Pierre Duhem and scientific truth: contextual, partial and real"), p. 322, cited from "Filosofía de la Ciencia" by M. Artigas

[7] (Pierre Duhem, *The aim and structure of physical theory*), p. 28

predictions of the theory are confirmed by experiment, “we feel an increase in conviction that the relations established by our mind between abstract concepts do indeed correspond to relations between things”.

The descriptive, mathematical part of the theory is true— as far as it describes the true order of the world.

Therefore, Duhem is a realist, acknowledging that theories reveal the truth about nature. This view has been criticized: for example, Popper claimed that truth is beyond the reach of science; theories are conjectures that we put to test, attempting to refute them. This also denies us hope for the truth of theories, no matter how many times they have been proven correct so far. Duhem found a much more elegant answer to this problem[8]:

> When the progress of experimental physics goes counter to a theory and compels it to be modified or transformed, the purely representative part enters nearly whole in the new theory, bringing to it the inheritance of all the valuable possessions of the old theory, whereas the explanatory part falls out in order to give way to another explanation

The description of the order of the world is not subject to refutation, but we incrementally improve it. At the same time explanatory part is sometimes replaced. Here is the second thesis, according to which explanations of the laws of physics are contingent:

Thesis 1.2 *Explanations of causes and images are not fixed in the physical theory and are subject to revisions.*

Various explanations of physical phenomena are possible, as long as they can be reconciled with experience. Physical theories describe fixed relationships between measurable quantities, rather than explanations of the causes of these relationships. The latter Duhem describes as harmful, the former as only needed.

[8](Dugas, “La méthode physique au sens de Duhem devant la mécanique des quanta - translated by A. Aversa as Physical method according to Duhem in view of quantum mechanics”) p. 5, op. cit. (Pierre Duhem, *The aim and structure of physical theory*)

> Everything that is good in a theory, making the theory seemingly represent a law of nature and enables it to predict unknown facts, we can find in the descriptive part; all this was discovered by physicists when they did not remember to seek explanations. On the other hand, whatever is false in the theory and refuted by facts is found above all in the explanatory part.[9].

1.1.1 Some Details of Duhem's Theory.

We will outline some of the most important observations made by Duhem for the use of the reader who is more interested in physics or philosophy of physics and therefore would want to see connection of Theses 1.1 and 1.2 to modern physics (while others can skip it).

Duhem points that explanations dervied from theory are unavoidably doubtful and needlessly attach physics to metaphysics[10]. Images may be a valuable tool, but that does not make them true. Duhem, as a physicist, was a theoretical purist, interested in abstract systems of axioms. However, this was a personal choice, not a methodological principle[11]:

> The best means of promoting the development of science is to permit each form of intellect to develop itself by following its own laws and realizing fully its type; that is, to allow strong minds to feed on abstract notions and general principles,and ample minds to consume visible and tangible things.

We will now give few examples of application for Thesis 1.2.

Example 1.2.1 *The explanation of gravity as action at a distance has been replaced by the vision of curved spacetime, even though the successes of Newton's theory suggested validity of the old explanation. Previously, the curvilinear motions of the planets of the Solar System were explained by vortices of matter (proposed by Descartes) and crystalline spheres.*

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[9] (Pierre Duhem, *The aim and structure of physical theory*), p. 32

[10] (ibid.), pp. 9-15

[11] (Dugas, "La méthode physique au sens de Duhem devant la mécanique des quanta - translated by A. Aversa as Physical method according to Duhem in view of quantum mechanics"), p. 2

To consider explanations as true at all, one would have to reject the old, Newtonian explanations (as refuted) and adopt the new ones based on Einstein's theory. In practice, however, mechanical waves, planets of the Solar System[12], pendulums, cars, and many other systems are still described by Newton's equations. Moreover, engineers and physicists dealing with these systems still use classical physics intuitions, and the existence of the General Theory of Relativity is almost irrelevant to them[13]. Images created based on Newton's theory are still essential for solving problems, even though this theory is supposedly refuted. In conclusion, considering images as true or false would create contradictions..

Another example we will provide was important for the formulation of Duhem's philosophy:

Example 1.2.2 *Newtonian dynamics can be formulated on the basis of the principle of least action, in a way that is independent of the coordinate system, the concept of space, body, force, etc. The application of the variational principle (Lagrangian or Hamiltonian) allows the calculation of the equations of motion for any mechanical system.*

As a result, mechanics can be formulated in a purely abstract way, without reference to explanations, images, and causes. The variational principle was invented in the 18th century as a general principle from which the equations of many physical systems can be derived (without referring to existing theories). Prominent physicists of the 19th century (e.g., Maxwell, Poincare)[14] considered this as a confirmation of the old project of reducing physics to mechanics: if something can be described by a Lagrangian, it means that it is equivalent to a mechanical system.

Duhem opposed this point of view. In his opinion, it is the abstract formalism that creates a good theory, not mechanical ex-

[12]an exception is the apsidal precession of Mercury and a few similar phenomena used, for example, in satellites; but even these were calculated for a long time using Newtonian equations and only adjusting the discrepancy based on the General Theory of Relativity, see (Narlikar and Rana, "Newtonian N-body calculations of the advance of Mercury's perihelion")

[13]and if some effect of the General Theory of Relativity occurs, one simply consider a small correction based on Newton's theory

[14](José and Costa, "Duhem?s Critical Analysis of Mechanicism and His Defense of a Formal Conception of Theoretical Physics"), p. 42

planations. A "good theory" is one that can be tested and potentially falsified, whereas mechanical explanations are not falsifiable[15]. This very feature is crucial for progress, Duhem believes[16]:

> He [the physicist who is not content with knowing physics through the gossip of the moment] will see abstract theory, matured through patient labor, take possession of the new lands the experimenters have explored, organize these conquests, annex them to its old domains, and make a perfectly coordinated empire of their union. It will appear clearly to him that the physics of atomism, condemned to perpetual fresh starts, does not tend by continued progress to the ideal form of physical theory

The discoveries of the 20th century vindicated his method: the ether theory in its 19th-century form was refuted, particles of matter do not interact according to the principles of classical mechanics, and the variational principle can be used to formulate the General Theory of Relativity, quantum mechanics, and quantum field theory—without any connection to classical mechanics. Duhem was mistaken in being skeptical of the Special Theory of Relativity and statistical mechanics, but his principles triumphed in physics after a few decades.

As a physicist, he applied this principle to formulate thermodynamics based on variational formalism, thus involving himself in the prominent dispute in late 19th-century physics about whether atoms exist or not. He did not answer directly but pointed out that atoms are not necessary for thermodynamics.

An abstract, mathematical theory must be connected with reality: it must measure and interpret numerical values that are assigned to symbols. Duhem believes that:

Thesis 1.3 *A physical measurement must be not only a sensory experience but also a theoretical interpretation.*

Here is an example:

[15] (José and Costa, "Duhem?s Critical Analysis of Mechanicism and His Defense of a Formal Conception of Theoretical Physics"), p. 48

[16] (ibid.), p. 50

Example 1.3.1 *To measure temperature, sensory experience alone is not enough; first, through rational abstraction, we create the concept of hot or cold bodies: one body can be warmer than another, and a third body can be warmer than these two*[17]*. There exists an equivalence relation for bodies that are equally warm. Furthermore, the principle is introduced that bodies in thermodynamic equilibrium (e.g., in contact with each other) become equally warm, and the law of thermal expansion of liquids, which we will use for measurement. Then the temperature of a body can be measured using the height of a column of liquid in equilibrium with it (a thermometer).*

That's not all[18], because without a scale, the height of the liquid column informs us only about a relative relation: one body is warmer, another cooler. To measure temperature in an absolute sense, it is necessary to create a thermometer scale by referring to specific facts: for example, the freezing and boiling points of water, as is customary in the Celsius scale. This creates further theoretical connections (e.g., the phase transitions of water depend on pressure).

Another example Duhem borrows from Poincare[19]:

Example 1.3.2 *An experimenter built an electrical circuit with an acid battery, a light bulb, and a galvanometer connected in the circuit*[20]*. To the question "is current flowing," the assistant might answer, "no, because the galvanometer needle points to 0." He might also answer, "yes, because the light bulb is lit, and the battery emits gas," even though the galvanometer reads 0.*

Poincare notes that "the current is not flowing because the galvanometer reads 0" means something entirely different from its everyday French meaning; however, physicists talk this way among themselves for convenience. The assistant thinks about the entire

[17] (João, "Poincaré and Duhem: Resonances in Their First Epistemological Reflections"), p. 148

[18] (Pierre Duhem, *The aim and structure of physical theory*), p. 117

[19] (ibid.), p. 150

[20] A galvanometer measures the magnitude and direction of small electric currents; traditionally, the instrument contained a coil through which current flowed, placed between two magnets. The coil had a pointer or a mirror to measure deflection.

setup, not about the dependence of the current on the galvanometer readings. The galvanometer can provide a very precise value of the current, but it is a complex instrument that can fail (e.g. it can easily burn out if too great current flows through it). The light bulb and bubbles in the electrolyte are more reliable indicators of whether the current is flowing because they are simple (though they do not provide an exact value of the current).

Duhem summarizes[21] that the experimenter thinks about two different representations of his experiment; one is the specific setup he built and manipulates, the other is the theoretical interpretation: an abstract, idealized schematic of the setup to which he applies physical formulas. The progress of physics creates an increasingly perfect correspondence between real setups and abstract schematics.

This property seems peculiar to modern physics. We will see that ancient astronomy and geometry was successful when theoretical interpretation is simple: we only need to measure angles and distances. The above discussion on theoretical interpretation is closely related to another key observation by Duhem:

Thesis 1.4 *(Duhem's Thesis) If there is a conflict between theory and experiment in physics, it cannot be immediately clear what has been refuted.*

Poincare's assistant, seeing zero on the galvanometer and a lit bulb, cannot be immediately certain what has broken down. Is the circuit interrupted? Is the coil burned out? Is there another magnet nearby? Only when he tests the instrument in isolation or uses another meter can he discover the cause. We recently saw such a problem in the OPERA experiment in 2012, where initial reports of superluminal neutrino speeds were explained by issues in the clock synchronization system.

When an experimenter sees a clear anomaly that withstands all tests, the theoretical interpretation must be changed: data about the new phenomenon must be collected and an attempt is made to describe it. This is generally labor-intensive, because even then[22]:

[21] (Pierre Duhem, *The aim and structure of physical theory*), pp. 155-156

[22] (Ladyman, *Understanding Philosophy of Science*), p. 77, (Pierre Duhem, *The aim and structure of physical theory*), p. 183, p. 187

Thesis 1.5 *(Duhem's Thesis continued) A physical experiment does not determine the error of an isolated hypothesis, but of the entire theoretical framework.*

Relying on as simple tool as the galvanometer in the presented example requires assumptions: the validity of Ampere's law (for relation between magnetic force and current), Hooke's law of elasticity (the deflection of a coil is on a spring), the thesis that the battery produces constant voltage, and others. To describe the path of a comet using Newton's law[23], one must know its speed, the masses, and positions of the major bodies in the Solar System (and assume that there are no other significant masses). It is also necessary to assume the absence of drag and resistance, the absence of influence of the Sun's radiation (or its presence). When the result of an observation differs from expectations, any of these assumptions may be modified, which only further research can indicate. Thus physics is a complex system of interconnected and inseparable hypotheses, Duhem points out—and this distinguishes it from other fields of natural sciences.

The development of this system is possible at the cost of constant theoretical efforts, increasingly precise measurements, and ever more perfect interpretations of experience. All of this is possible only because the measurements and interpretations created long ago remain valid: there is an unchanging and universal order of the world, as indicated by Thesis 1.1, and physics would not exist if it were otherwise. Einstein expressed this truth as follows[24]:

> One could (yes one should) expect the world to be subjected to law only to the extent that we order it through our intelligence. Ordering of this kind would be like the alphabetical ordering of the words of a language. By contrast, the kind of order created by Newton's theory of gravitation, for instance, is wholly different. Even if the axioms of the theory are proposed by man, the success of such a project presupposes a

[23](Ladyman, *Understanding Philosophy of Science*), p. 77

[24](Albert Einstein, *A. Einstein, Letters to Solovine, translated by Wade Baskin, with an introduction by Maurice Solovine, p. 132-133.*)

high degree of ordering of the objective world, and this could not be expected a priori.

1.2 The Importance of Duhem's Method in the History of Physics

The above theses 1.1 and 1.2 — about the order of the world and the contingency of explanations — were not invented at the end of the 19th century. These are the foundations present in the work of various distinguished scientists such as Newton, Ampere, Euler, and later also Einstein and the discoverers of quantum mechanics. Duhem justified this in his historical works. Towards the end of his life, he also provided surprising answers as to how this method originated.

The Greeks, Babylonians, Egyptians, and other ancient nations knew basic arithmetic, geometry, and elements of astronomy. However, attempts to describe the rest of the world were not at all similar to astronomy and rarely differed from myths and fanciful speculations. The concept of an understandable organization of the world appeared occasionally, but it only became important with Plato, in his theory of forms and the theory of purposeful order. This change in orientation, however, decreased importance of experience; the visible world does not reveal anything certain and unchangeable about itself; Plato valued reason and intuition more than experience. Plato regarded the visible world as an imperfect reflection of the eternal and unchangeable world of forms.

Aristotle undertook the project of reconciling Plato's thought with reasoning based on experience and the reality of the visible world. Forms and purposeful order were the foundation of the new system. However, Aristotle's forms existed solely in things and could be discovered through the observation of things. For Aristotle, observations reveal the truth because the visible world is real. While Plato recognized only mathematics and the intuitive contemplation of forms, Aristotle proposed a new science based on experience and dealing with visible things. Things undergo changes, but due to purposeful order, there exist immutable, universal regularities. Discovering these regularities is the goal of Aristotle's physics.

This research program achieved successes, and some elements remain influential even today. It also had many flaws, and these flaws are related to Theses 1.1 and 1.2. Purposeful order as the striving of things and organisms towards what is best is an inaccurate explanation of the order in physics. For example, according to Aristotelians, planets should move in concentric circles because that is perfect, despite the difficulty of reconciling this thesis with observations[25]. Similarly, the moon should be an uniform body because celestial matter is perfect, despite observations showing that the moon has spots. Purposefulness restored faith in experience, but at the same time, it took priority over it when, for example, rare observations were dismissed as errors in nature.

Another problem is related to Thesis 1.2. A system of logical relationships between objects will never be a completely accurate physical theory. Neither in Aristotle's physics, nor in classical mechanics, nor in any other. This was misleading (Aristotle openly criticizes contingency as absurd[26]) and caused long stagnation of science, as wrong theory was deemed necessarily true. We will see that Aristotle's theory of natural motion is not only mostly consistent with experience but also coherently and precisely formulated; however, this is the cause of both its successes and failures.

The situation only changed when a thesis very similar to Thesis 1.2 was discovered. Here it is:

Thesis 1.6 *If I can think of X, then X is possible, as long as it can be reconciled with experience.*

For example: I could remove the assumption of curved spacetime from the General Theory of Relativity, as long as the predictions of the new theory were consistent with already known experiments[27]. I can even imagine that gravity creates a repulsive effect, not an attractive one, or that a spacetime has 4 spatial dimensions instead of 3. This is contrary to experience, but I can imagine another universe where this happens and create a theory of such a universe,

[25]Contrary to the Ptolemaic and Copernican systems, which are very accurate, but the planets do not move in concentric circles.

[26](W. D. R. Aristotle, *Metaphysics, translated by W. D. Ross*) 1047b

[27]For example, one can formulate a theory of gravity based on a variable speed of light, and such a theory would be equivalent to the first-order expansion of GR (Broekaert, "A Spatially-VSL Gravity Model with 1-PN Limit of GRT"), thus consistent with almost all observations. The difference would only be apparent for very strong gravitational fields, such as some pulsars.

as physicists do these days[28]. In such way Thesis 1.6 remains a tool of physics to this day. Here is the reasoning from which Thesis 1.6 was derived:

Subthesis 1.6.1 *If I can think of X, then God could create X (because He is omnipotent), so it is an error in faith to assume that X is impossible.*

Scholastic theologians developed the above thesis until the second half of the 13th century and applied it to all statements about logical impossibility in the visible world. As a result, some of Aristotle's theses were rejected, and others became just one among many, as physics transformed into a system of conventions and hypotheses. At the same time, the organic-purposeful view of the world, outlined by Plato and Aristotle, lost its relevance, giving way to a new image of the world operating according to immutable and comprehensible laws imposed by the Creator. This is also a conclusion of theology.

One of the first conclusions was the definition of a physical quantity. Through his theory of forms, Aristotle allowed recognition of properties and relations of objects as something real, paving the way for theoretical interpretations of Thesis 1.3. There were however key limitations. Theologians assigned numerical values to the intensity of any form and used this doctrine to formulate second-order forms (e.g., velocity) and the laws of proportionality between forms. In this way, they discovered, among other things, the law of distance in accelerated motion and the law of free fall.

These new principles became the foundation for the development of new science – dynamics, kinematics, and the cosmology of scholasticism. Buridan, Oresme, de Soto, da Vinci, and others made a series of key discoveries, paving the way for Newtonian dynamics. The influence of these discoveries on Galileo's school is also easy to demonstrate[29].

Thus, here is the conclusion, which we will call Duhem's historical thesis:

[28] This is nearly authentic example: today in physics, such unrealistic ideas are considered to help find solutions to theoretical problems. An example is the AdS/CFT correspondence from (Maldacena,). It is currently the most cited paper in physics.

[29] (Pierre Duhem and Aversa, *Galileo's Precursors: Translation of Studies on Leonardo da Vinci (vol. 3) by Pierre Duhem*), p. 252

Thesis 1.7 *Modern physics and the method of physics are products of scholasticism with prior foundation of Aristotle's physics. The existence of an understandable immutable order of the world (Thesis 1.1) and the contingency of natural explanations (Thesis 1.2) are conclusions of scholastic theology.*

2 Physics until the 13th Century

2.1 Ionian Philosophers

Here is the first question of the science of the ancient Greeks[30]: What is the world made of and how do separate things come from this? The Ionian philosophers of nature (physiologoi) were the first to attempt to answer what this is, the so-called "arche." Thales said "water," Anaximenes said "air," Anaximander said "the boundless".

Thales did not have in mind something like "pure H2O," a compound that we see in steam, ice, or liquid water. Rather, he referred to the liquid state and its role in transformations in nature as the element most susceptible to change[31]. This arises from a visual analogy – water poured into a pot always takes the shape of the container, disturbed by wind it waves, when spilled it spreads in all directions. For Thales, the fluid cosmic matter was in constant motion, says Windelbrand.

Vivid speculations dominate the science of the Ionian philosophers. They imagined a flat Earth floating on water, similar to a raft, or stars as holes in a great ring with fire beyond it– much like a fire seen through a wicker sieve. The "arche" matter was also the carrier of all changes and aspects of the animated world (Windelbrand says that Thales did not teach anything about the forces moving matter and different from it[32].). The Ionian philosophers were also influenced by Egyptian science[33]; the theorem about the proportions of triangles, now named after Thales, has likely Egyptian origin.

We know little about the science of the physiologoi. The writings of the authors have not survived; we know these doctrines from a few mentions. More is known about the famous philosophical dispute that arose shortly thereafter, based on the opposition between the traditional pantheon of anthropomorphic gods and the monism of the Ionian philosophers. The itinerant poet and philosopher Xenophanes proclaimed that human-like gods are a

[30] Written by (Windelbrand, *History of Ancient Philosophy*), p. 36
[31] (ibid.), p. 38
[32] (ibid.), p. 38
[33] (ibid.), p. 22

fabrication, made by people in their own likeness[34]. Xenophanes developed the doctrine of the physiologoi in his own way – he stated that the "arche," the beginning of all things, is a pantheistic Deity. Xenophanes' God is identical with the world, without beginning or end. All visible things are same universal essence that never changes. A certain problem arises: if the "essence of things" does not change, yet we see changes in nature, what are they then? Xenophanes did not provide an answer, but his thought was continued by Parmenides.

Parmenides pointed out that negative statements about existence are problematic. Thought relates to something existing. Non-existence cannot be conceived, so something that exists cannot not exist. As a result, it is impossible to think about transformation, as transformation is the transition from the non-existence of something to its existence (or vice versa). Thus, all the being is unchanging and eternal. Here are fragments from Parmenides' poem[35]:

> It needs must be that what can be thought and spoken of is; for it is possible for it to be, and it is not possible for, what is nothing to be. (...) In it are very many tokens that what is, is uncreated and indestructible, alone, complete, immovable and without end. Nor was it ever, nor will it be; for now it is, all at once, a continuous one. For what kind of origin for it. will you look for ? In what way and from what source could it have drawn its increase ? I shall not let thee say nor think that it came from what is not; for it can neither be thought nor uttered that what is not is.

The statement "an African elephant exists" is clear to us—we can point to an elephant in the zoo. It is harder to understand something like "Bigfoot does not exist." Someone might say: Bigfoot is some unknown species of ape, but that is still not precise. Another person might imagine Bigfoot as something else. Ambiguity arises when we want to talk about something we do not know, drawing conclusions from images in our thoughts. The Earth may drift in

[34] (ibid.), p. 47

[35] (Burnet, "Poem of Parmenides")

an infinite ocean according to Thales, or, as the Hindus preferred, rest on the back of four elephants, which in turn stand on a great turtle. Both are images painted from objects known from experience. However, we do not know the relationships between these objects: should a flat Earth float on water, or should it rather sink in it? The creation of objects from "arche" presents greater difficulties. Thinking about the non-existence of an elephant requires us to recall an existing elephant—that is precisely what Parmenides points out. It is impossible to think about transformation, even though transformations are all around us. As a result, experience testifies to something contrary to thought. Either thought or experience is mistaken. Parmenides believes that experience leads us astray.

Subsequent natural philosophers: Empedocles, Anaxagoras, and Democritus, tried to address the above problem[36]. They acknowledged that it is impossible to think that something which "is," i.e., essentially exists, did not exist before. But this is not relevant; when describing transformation, we can understand "is" differently. "John became a doctor" is a transformation. John existed before becoming a doctor and still he exists after it. Similarly, in any other transformation, we can assume that what we see as perishing and creation is actually a change in the properties of a substrate that exists eternally (such as matter, atoms, etc.).

2.2 From Heraclitus to Plato

The idea of an intelligible order of the world came not from natural philosophers but from the Pythagoreans, Heraclitus, Socrates, and Plato; philosophers mainly concerned with ethics and metaphysics. Heraclitus claimed that there exists a Logos, a rational reason governing the entire world[37]. According to Heraclitus, a person can understand the Logos by examining the depths of their mind and perceiving the hidden harmony in both moral and physical reality.

Socrates provided an original critique of the speculations of the natural philosophers. We noted that they offered explanations such as this: the Earth is stationary because it has a flat bottom, supported by the mass of air. Planets are small holes in the great ring

[36] (Meyer, *Ancient Philosophy: Plato and his Predecessors*)
[37] (Windelbrand, *History of Ancient Philosophy*), p. 56

of fire[38]. Socrates did not consider these to be true explanations; none of this answers the question of why it is *better* for the Earth to remain stationary, why it is *better* for the planets to move as they do. Socrates was not a proponent of studying nature either; he rather compared the physiologoi to people who have lost their sight by looking at a solar eclipse. This last comment is a jab at the existential meaninglessness of the materialistic world[39]. We know Socrates' opinion from Plato, who creatively developed it within his own philosophical system[40]. Plato argues that it is impossible to comprehend the world of biology based on chance and inanimate matter. A tree is more than just the matter from which it is made. If we chop a tree into sawdust, the pile of sawdust, composed of the same matter, will no longer be a tree. A tree grows, sprouts leaves, turns its leaves toward the sun, produces seeds and nuts from which more trees can grow. For Plato, a tree is primarily a form; the "design" of a tree that is realized again and again in matter. Thus, in the nature, there are teleological causes that "act intelligently to create what is good and desired."

Plato's contribution to philosophy includes especially the theory of forms; it is not limited to forms relating to the spatial or causal organization of things and living beings (the form of a cat, the form of a tree, the form of a house, etc.). Most important foundations is found in the Socratic dialogues[41] in questions about what justice, piety, courage, etc., are (these will be examples of forms). Plato then develops the doctrine that forms exist truly in a non-sensory world, immutable, eternal and beyond the space. Man, in turn, is an immortal spirit trapped in a body, and this spirit recalls the forms[42].

In "the Republic", Plato argues that justice or piety are examples of eternal forms in the world of ideas. He claims that sensory impressions are not knowledge; the visible world is like an imperfect reflection of the world of ideas—the world of ideas is more real, and the knowledge of forms is true knowledge. Famous is the

[38](Jaki, *The Relevance of Physics*), p. 10

[39](Jaki, *Science and Creation*), p. 105

[40](Jaki, *The Relevance of Physics*), p. 11

[41](Silverman, "Plato's Middle Period Metaphysics and Epistemology"), par. 1

[42](ibid.), par. 2

analogy of the cave[43]: prisoners chained in a cave, looking towards the wall, see only the shadows of people passing by the entrance. The shadows are the sensory world, merely a vague reflection of the true world of ideas.

This anti-empiricism does not help Plato to be interested in observations and experience. In “the Timaeus”, he provided a speculative, teleological cosmology[44], according to which the world is the product of a ”rational, purposeful, and beneficent causal force”. This cosmology is in large parts influenced by Pythagorean cosmology. Plato claims that the world has intelligence and a soul, the Earth is surrounded by 8 celestial spheres, and there are 4 elements: earth, fire, air, and water. He explains various aspects based on a teleological order: for example, each of the 4 elements consists of particles in the shape of regular polyhedra because that is most perfect[45].

Towards the end of his life, Plato also adopted a kind of heliocentrism, inspired by the doctrine of the Pythagoreans[46]. They believed that the center of the world contained fire, as it is more noble, and the Earth revolves around it. In this very center, Plato placed the World Soul, as in the most worthy place.

2.3 Aristotle’s Physics

So far, there is nothing on the philosophical horizon that resembles physics. The landscape is dominated by far-reaching speculations, doubts about the value of sensory knowledge, and lyrical literary forms. The closest to physics is Plato, who observes that the world is regular and organized. However, Plato is also little concerned with experience[47]. The essence of his argument is this: we want to acquire knowledge that is valuable, that is certain, precise, and immutable. Examples are the theorems of mathematics and geometry: they are necessarily true based on a proof and do not change, regardless of the facts in the world. However, the physical world

[43] (Silverman, “Plato’s Middle Period Metaphysics and Epistemology”), par. 13

[44] (Zeyl and Sattler, “Plato’s Timaeus”), par. 1

[45] (ibid.), par. 1

[46] (P. M. Duhem, *Le Systeme du Monde, t. 1*), p. 91

[47] (ibid.), p. 135

does not reveal such knowledge; things come into being and perish—they cannot be eternal, and sensory knowledge is inaccurate. Therefore, one should rather focus on the eternal and ideal world of forms. As a result, Platonists are only interested in mathematics and "intuitive contemplation of eternal forms"—theology.

Aristotle created a new system, recognizing the cognitive value of experience and the unambiguous realism of the visible world, while also adopting much from Plato (primarily the idea of teleological order and the theory of forms). Let us briefly summarize his main theses, which we will elaborate on in detail.

Claim 2.3.1 *The visible world exists and can be known through experience.*

Claim 2.3.2 *Forms exist solely in objects. We understand forms through our senses by observing objects. When we study many similar objects, the mind discovers the form common to them based on induction.*

Claim 2.3.3 *Nature is the internal principle of change; nature acts in a purposeful way; the goal of natural changes is the form.*

Extrapolating the above principles through logic and experience will be the method of Aristotle's physics. Among its fundamental conclusions, we will find theories of place, time, void, and many worlds—all of which will interest us later.

2.3.1 Forms and the Principle of Induction

At the foundation of the system lies Plato's theory of forms, but Aristotle's forms are different: they exist solely in visible objects. Aristotle rejected the idea of a non-sensory world of ideas[48]. Firstly, he argues, if forms truly exist in the world of ideas, they must exist separately from visible things. Therefore, what we see with our senses is not the thing itself; the thing is elsewhere. Thus, either the visible thing is a mirage, as Plato believes, or there are two things at once, not one, which is absurd.

[48](Aristotle, *Aristotle in 23 Volumes, translated by H. Tredennick*), 1077 b

> For the objects of astronomy will similarly be distinct from sensible things, and so will those of geometry; but how can a heaven and its parts (or anything else which has motion) exist apart from the sensible heaven? And similarly the objects of optics and of harmonics will be distinct, for there will be sound and sight apart from the sensible and particular objects.Hence clearly the other senses and objects of sense will exist separately; for why should one class of objects do so rather than another? And if this is so, animals too will exist separately, inasmuch as the senses will.

In the same passage, he points out that it is unclear how a mathematical object (which also exists in Plato's world of ideas) could exist other than in visible things:

> Further, body is a kind of substance, since it already in some sense possesses completeness; but in what sense are lines substances? Neither as being a kind of form or shape, as perhaps the soul is, nor as being matter, like the body; for it does not appear that anything can be composed either of lines or of planes or of points,whereas if they were a kind of material substance it would be apparent that things can be so composed.

Substance means something that actually exists, independently of other objects. He thus asks: in what sense are lines real—and it does not seem that there is an answer to this question.

The two difficulties mentioned above can be resolved by rejecting the world of ideas and recognizing the existence of visible things. If forms exist only in visible things, then sensory perception can be rehabilitated. We understand the general concept "tree" by observing many trees and discovering what is common among them. Aristotle proposes a kind of induction[49], indicating that it is indispensable for understanding generalities:

> Now demonstration proceeds from universals and induction from particulars ; but it is impossible to gain

[49](P. Duhem and Ariew, *Medieval Cosmology: Theories of Infinity, Place, Time, Void, and the Plurality of Worlds*), p. 133, (Aristotle, *Aristotle in 23 Volumes, translated by H. Tredennick*) 81 b

> a view of universals except through induction (since even what we call abstractions can only be grasped by induction, because, although they cannot exist in separation, some of them inhere in each class of objects (...))

How do we teach children arithmetic? Usually by counting sticks, fingers, or candies. Even mathematical abstractions are learned based on senses and induction, says Aristotle: eternal and unchanging mathematical knowledge in practice is not independent of the visible world. Aristotle then concludes that things that are not visible are beyond the reach of natural philosophy:

> ...and we cannot employ induction if we lack sense-perception, because it is sense-perception that apprehends particulars. It is impossible to gain scientific knowledge of them, since they can neither be apprehended from universals without induction, nor through induction apart from sense-perception.

We will see that Aristotle adopts various theses about things beyond the limits of perception, deriving generalities through induction and then deducing conclusions from these laws. However, these conclusions ignore the existence of things that elude experience.

In another passage cited by Duhem[50], Aristotle considers the example of a solar eclipse. We can see a solar eclipse standing on the Moon and seeing at a particular moment how the Earth obscures the Sun, but this does not mean we understand the causes. Only by observing the phenomenon many times we can comprehend it . This can be explained as follows: seeing how the Earth gradually obscures more of the solar disk, I can deduce the relationship of the eclipse to the relative positions of the Earth, the Sun, and the Moon.

2.3.2 The Problem of Arche

Book I of "Physics" begins with a somewhat mysterious passage about knowing from what is knowable to us to what is "knowable

[50](ibid.) 87 b

by nature"[51]:

> The natural way of doing this is to start from the things which are more knowable and obvious to us and proceed towards those which are clearer and more knowable by nature; for the same things are not 'knowable relatively to us' and 'knowable' without qualification. So in the present inquiry we must follow this method and advance from what is more obscure by nature, but clearer to us, towards what is more clear and more knowable by nature. Now what is to us plain and obvious at first is rather confused masses, the elements and principles of which become known to us later by analysis.

"Elements" refer to "principles," and "principles" to the aforementioned "arche." In the "Posterior Analytics"[52], Aristotle seems to say something opposite, but the context of the statement is different. There he speaks of knowing generalities from specifics, as we know the specifics through the senses, but we do not know the generalities. Here he talks about knowing things and their generalities first (because we know them better), and elements and arche later (because they are not visible). This is a key difference between physics (to this day) and speculative philosophy. The Ionian philosophers tried to guess the basic substrate of reality. Aristotle pointed that we should start with the study of visible things as wholes; only then can we uncover the secrets of how the world is organized. The above thesis appears to be based on further[53] criticism of philosophers dealing with the problem of arche. There is no agreement among them: some assume one arche, others two or three or more, others an unlimited number. Arche may undergo transformations or not. Aristotle points out that the latter opinion: the existence of one immutable being, is not the subject of natural philosophy, as it rejects its basic principles: Natural philosophers, he says, must take as a principle that at least some objects found

[51](Aristotle, *Aristotle's physics, translated by R. P. Hardie and R. K. Gaye*), 184a

[52]See cited paragraphs 81b and 87b.

[53](Aristotle, *Aristotle's physics, translated by R. P. Hardie and R. K. Gaye*),184a-186a

in nature undergo changes, as we know from experience. He further indicates that statements like "everything is unity" are imprecise because both "everything" and "unity" can be understood in various ways. Moreover, the statement itself is hardly comprehensible. For example, if we assume that everything contains things and qualities (horse, table, man, whiteness, heat), then everything is a multitude, not a unity. Or, if according to Heraclitus even opposing properties are the same, then everything might as well be nothing. Similarly, Aristotle refutes the arguments of the Eleatics[54] Melissus and Parmenides:

> Again, does it follow that Being, if one, is motionless? Why should it not move, the whole of it within itself, as parts of it do which are unities, e.g. this water? Again, why is qualitative change impossible? But, further, Being cannot be one in form, though it may be in what it is made of. (Even some of the physicists [i.e. Ionian philosophers] hold it to be one in the latter way, though not in the former.) Man obviously differs from horse in form, and contraries [differ] from each other.

> His [Parmenides'] assumption that one is used in a single sense only is false, because it is used in several. His conclusion does not follow, because if we take only white things, and if 'white' has a single meaning, none the less what is white will be many and not one. For what is white will not be one either in the sense that it is continuous or in the sense that it must be defined in only one way. 'Whiteness' will be different from 'what has whiteness'.

There are more critiques of various positions, as Aristotle's predecessors proclaimed things that contradicted common sense, and using logic allowed deriving various contradictions from their views. Toward the end of Book I, there appears an analysis of how something can arise from non-existence[55]. Eleatics argued that nothing could truly come into being or perish because everything

[54] (ibid.), 186a
[55] (ibid.), 191a

that comes into being could arise either from what does not exist or from what exists. Neither is possible, they claimed. Existing thing cannot come into being as they exist already, while no real thing could emerge from non-existent entities.

Aristotle responds that coming into being from non-existence is not "from non-existence" per se. A tree can arise "from non-existence" in the sense that there was no tree before, not in the sense that it comes into being without any components (i.e. seeds, water, soil, light, etc). Such coming into being is possible, despite the impossibility (according to Aristotle) of generation from absolute non-existence. This argument leads to a discussion of the distinction adopted from Plato between matter, form, and privation (or lack). Privation is absolute non-existence, while matter is the non-existence of form; matter has the potential to realize a certain form, as a tree, an animal, or a table might emerge from matter. In this way, Aristotle can remove the problematic assumption of Plato that something can arise from non-existence.

2.3.3 Four Causes

When discussing the cause of a lunar eclipse, we respond that the Earth obscures the Sun, which causes darkness on the Moon. This is the efficient cause, and addressing such causes was an important discovery by Aristotle. Through observations, individual phenomena can be linked in chains of cause and effect by referencing more general laws; this way, we can understand the world. Today, when explaining why a kicked ball flies, or why a radio works, or why a car drives burning gasoline, we mainly think of efficient causes—laws of mechanics, electromagnetism, thermodynamics, and the initial conditions from which the specific outcomes of these laws result (a foot striking a ball, a modulated signal flowing through a transmitter antenna, etc.). Besides efficient causes, Aristotle also lists three other types, and finding them is the subject of the natural philosopher. Aristotle believes that natural philosophy deals with answering questions "why"[56].

> Knowledge is the object of our inquiry, and men do

[56](Aristotle, *Aristotle's physics, translated by R. P. Hardie and R. K. Gaye*), bk. 2, par. 3

> not think they know a thing till they have grasped the 'why' of (which is to grasp its primary cause).

These questions can be answered in four ways[57]: by indicating the form, matter, purpose, and the aforementioned efficient cause:

> In one sense, then, (1) that out of which a thing comes to be and which persists, is called 'cause', e.g. the bronze of the statue, the silver of the bowl, and the genera of which the bronze and the silver are species.
>
> In another sense (2) the form or the archetype, i.e. the statement of the essence, and its genera, are called 'causes' (e.g. of the octave the relation of 2:1, and generally number), and the parts in the definition.
>
> Again (3) the primary source of the change or coming to rest; e.g. the man who gave advice is a cause, the father is cause of the child, and generally what makes of what is made and what causes change of what is changed.
>
> Again (4) in the sense of end or 'that for the sake of which' a thing is done, e.g. health is the cause of walking about. ('Why is he walking about?' we say. 'To be healthy', and, having said that, we think we have assigned the cause.)

Form and purpose serve a similar role as in Plato: they represent the structure of the object and the plan for which the object was created. Matter corresponds to what the thing is made of'. Consider the example: why does a saw cut wood?

- because it is made of iron[58] (matter);
- because it has teeth and a handle (form);
- because the teeth strike the wood repeatedly, chipping off particles (efficient cause);
- because a board is to be made (purpose).

[57] (ibid.), bk. 2, par. 3

[58] Aristotle uses the term matter in a relative sense, iron is the matter relative to the saw, but that does not mean that iron is matter without any form – see (Robinson, "Substance") 2.2.2

Summing up:

Claim 2.3.4 *(Aristotle) there are four causes of natural phenomena: formal, efficient, material, and final.*

Aristotle most important contribution is efficient cause and procedure to know causes from experience. Other causes appear in works of Plato (form and purpose) and the Ionian natural philosophers (matter). Aristotle criticizes the latter for focusing on material causes[59]: the physics of the Ionian philosophers is based on blind chance; it's all dead matter, interacting randomly. But chance does not create regularity, it does not repeat anything regularly.

2.3.4 Nature.

Aristotle begins Book II of "Physics" by pointing out the difference between human-made objects and natural objects[60]. Let's explain this difference with examples. Cats, dogs, horses, trees, and humans are endowed with the ability for spontaneous transformations: movement, growth, maturation, aging, changing shape, etc. Similarly, metals and minerals are endowed with tendencies for change. Copper tarnishes, iron rusts, a lump of salt dissolves in water, and rock undergoes erosion. Stones sink and fall, fire rises upward. Craftsmen are unable to bestow any such properties on their creations, nor to alter them. The blade of an axe falls downward just like any piece of metal. A marble statue of a boy will not grow to the size of an adult, although its shape is similar to that of a boy. If a wooden bed were to sprout shoots, a tree would grow from these shoots, not another bed. Aristotle states that objects created by nature have an internal tendency to change, which he calls the nature of the object.

Claim 2.3.5 *(Aristotle) The nature of an object is its internal principle of change or rest.*

This thesis allows for a good description of a very wide range of phenomena, and even today it is evidently wrong in only one part: it explains physics based on the same principle as the living world.

[59] (Jaki, *The Relevance of Physics*), p. 14

[60] (Aristotle, *Aristotle's physics, translated by R. P. Hardie and R. K. Gaye*), 2.1.

In Aristotle's time, it was rightly not considered a mistake; there were (almost) no experiments that would have refuted it. Moreover, such a conclusion fits well with commonly known concepts of rationality today, such as Occam's Razor or neopositivism (which maintains that differences between various scientific fields should be merely conventional). Why have separate principles of change for stones and for animals when it is better to have one?

Aristotle asks whether nature is form or matter, and answers that nature is primarily form: a thing is more accurately named by what it is when it is actualized. It is easy to see that for animals and plants: a horse exhibits its characteristic tendency to change when it most fully actualizes the form of a horse: a young and healthy horse eats grass and oats, gallops, neighs, etc. An old or sick horse loses its typical behaviors—impairment of form brings about impairment of the tendency of change.

Later[61] an important issue arises: whether nature operates with a purpose and thus belongs to the class of final causes, or not. Aristotle responds with certainty that nature must be a final cause, because many natural phenomena are regular, and there is no other explanation for this fact, other than a purposeful order. As a result, the world of nature is, in principle, regular and understandable—this observation allows Aristotle to study the laws of physics (unfortunately, organic and teleological explanations often will not be accurate). What the goal is that nature strives for? This goal is *form*. To summarize:

Claim 2.3.6 *(Aristotle) Nature is an internal principle of change, which purposefully strives to actualize a form.*

For instance, an oak grows from an acorn, first as a green stalk, then as an increasingly thick tree that produces leaves and branches. After about 20 years, the oak begins to produce acorns, from which new oaks can grow. In this way, the nature of the oak acts purposefully towards the actualization of the oak's form.

2.3.5 Natural Motion and Place.

Aristotle also applies the above theory to the motion. Solid bodies generally fall downward, volatile vapors rise upward. Metals sink,

[61] (ibid.), 2.8

air released under water escapes upward. These phenomena are very widespread and must be described within the framework of purposeful order and forms, consistently with other existing natures. A tree and a horse have their own natures: but at the same time, they are also endowed with a tendency to fall, just like stones. Therefore, there must exist a common component endowed with its own nature.

Aristotle[62] postulates prime matter, which has no form and is preserved through all transformations. This matter can adopt the property of being wet or dry and at the same time hot or cold. Thus, there exist 4 elements (or 4 principles) – fire (hot dry), air (hot wet), earth (cool dry), water (cool wet). These elements are mixed in material bodies.

The natures of these elements (principles of motion) presuppose a striving towards their natural place, and this place is their goal. A stone wants to fall to the center of the Earth, because that is its place. The fiery element rises up, towards the sphere of fire, which is located just below the celestial bodies.

This theory has a couple of problems. The periodic motion of celestial bodies does not strive towards any goal, because it always looks the same. Therefore, Aristotle must recognize an exception, relying somewhat on Plato and other philosophers: the heavens are made of a fifth element, an ideal heavenly matter, which is not subject to coming into being, perishing, and change, and its natural motion are celestial rotations[63].

Another key problem concerns dynamics on Earth. Aristotle defines progressive motion as "change of place“[64]. Moreover, the theory of natural motion relates to natural place as a goal (into which, for example, a stone is to fall). Place must therefore be something real, natural place even interacts with the body[65]:

> Further, the typical locomotions of the elementary natural bodies-namely, fire, earth, and the like-show not only that place is something, but also that it exerts

[62](Ainsworth, "Form vs. Matter"), 2

[63](Bodnar, "Aristotle's Natural Philosophy"), note. 34

[64](Aristotle, *Aristotle's physics, translated by R. P. Hardie and R. K. Gaye*), 208a

[65](ibid.), 208b

> a certain influence. Each is carried to its own place, if it is not hindered, the one up, the other down.

Place cannot be the form of a body, as it is not part of the body; it is external. Aristotle believes that being "in place is like being in a vessel, or a box: e.g., wine is in an amphora, a coin is in a box, just as a ball is "in place[66]. From this understanding, it follows that:

- Place is neither the form, nor the matter of the contents, nor a part of the contents. Place can be separated from the contents[67],
- Place directly surrounds its content[68],

Aristotle considers place to be the layer of matter surrounding a body. For example, the layer of water surrounding a boat is the place, the layer of air surrounding a person is the place. If a person raises their hand, without moving other parts of the body, their place changes. The above of course does not work for the celestial spheres: the last of them, the sphere of the fixed stars, has no place, because it is not surrounded by any matter (there is nothing outside). Progressive motion of the last celestial sphere as "change of place“ is a nonsensical statement, because there is no place (there is no matter outside the world). Aristotle postulates that rotational motion is possible, considering the Earth at its center as the place of the last celestial sphere.

2.3.6 Necessity, Errors of Nature, and Time.

Aristotle discusses the ideal heavens not only because he needs to fix an ad-hoc theory of natural motion for celestial bodies (where mentioned theory fails). The heavens play a fundamental role in his cosmological considerations, where he consistently tries to find the causes of all changes, applying the theories we outlined above. This leads him to conclude that changes in the world depend on the motion of the heavens. I. Bodnar writes about this as follows[69]:

[66] (ibid.), 210a
[67] (ibid.), 210b-211a
[68] (ibid.), 210b
[69] (Bodnar, "Aristotle's Natural Philosophy"), section. 4

> Aristotle argues at the opening of Physics bk. 8 that motion and change in the universe can have no beginning, because the occurrence of change presupposes a previous process of change. With this argument Aristotle can establish an eternal chain of motions and refute those who hold that there could have been a previous stationary state of the universe.

Aristotle's universe is eternal, which raises the following problem. If changes in the world had begun at some point in time X and caused a chain of changes lasting for a finite time Y, then after that time there would be no more changes. Change would then be accidental: changes might not occur indefinitely long, even though changes are possible. This (Bodnar points out) contradicts his thesis on necessity, from Book XI of "Metaphysics". It can be summarized as follows[70]:

Claim 2.3.7 *(Aristotle) If it were be true to say that a thing which was possible would not be, anything would be possible, and nothing impossible.*

This is not a simple contradiction (A) and (not A). Someone could devise a simple counterexample. A seed in the desert could sprout if watered, but most often it will not sprout because there is no water. In Aristotle's language, this would be possible per se (because the seed can sprout with the influence of water, so there is a possibility), but accidentally impossible (because just here, for this seed, there is no water). Aristotle accounts for such cases: in Book II of "Physics" he indicates[71], that events contrary to nature occur spontaneously.

Claim 2.3.8 *(Aristotle) Spontaneous events occur contrary to nature.*

In Claim 2.3.7, the issue is different: whether change can be accidental, that is, whether a possibility can be almost never realizable. For instance: changes were possible for a finite time in an

[70] (Aristotle, *Aristotle in 23 Volumes, translated by H. Tredennick*) 1047b – translator's note

[71] (Aristotle, *Aristotle's physics, translated by R. P. Hardie and R. K. Gaye*), 197b

eternal universe, meaning they were almost never possible. This reasoning seems to be as follows: nature acts with a purpose; at the same time, it must act through efficient causes. We know regularity from experience and it results from purposefulness; at the same time, only when there is regularity can we recognize efficient causes. The chain of efficient causes must be subordinated to the final cause, otherwise, there would be no regularity, and yet regularity exists. Aristotle will therefore reject the existence of potentiality that cannot be realized[72] - as in that case efficient causes are not truly subordinate to final cause. Bodnar phrases it as follows:

> Hence Aristotle postulates that the processes of the universe depend on an eternal motion (or on several eternal motions), the eternal revolution of the heavenly spheres, which in turn is dependent on one or several unmoved movers

This means, the eternal universe must have continually active causes of change, which depend on animate beings moving the spheres of celestial bodies[73]. This doctrine fits well with the organic vision of the world of the pagan Greeks. It is also a logical conclusion of Aristotle's most important theses and a striving for coherence in the system. It also has some empirical support: The Sun and the Moon have a fundamental impact on phenomena on Earth.

From the above discussion, it follows a conclusion that will outrage some scholastics: if the heavens were to stop, then all changes on Earth would also stop. Another source of this is the Philosopher's[74] theory of time, naturally connected with the above discussion. Shields[75] indicates that Aristotle, wanting to treat time as something existing, places it in the category of quantity: just as length exists in a line, so time exists in change: hence the definition "time is the quantity of movement, with respect to before and

[72](Bodnar, "Aristotle's Natural Philosophy"), section. 4

[73](W. D. R. Aristotle, *Metaphysics, translated by W. D. Ross*), 1073b-1074a, Aristotle postulates about 50 such movers as causes of planetary motions.

[74]In medieval commentaries, Aristotle was generally called "the Philosopher," and Averroes "the Commentator."

[75](Shields, "Aristotle"), 6.

after"[76]. It immediately follows that if there is no change, there is also no time, just as there is no length when there is no line. Moreover[77]. time is the motion of the celestial spheres, inasmuch as there must be one time, and that this motion is best suited as a measure of time (the measure of the time of motion is the same as time, by analogy to the length of a line).

2.4 Aristotle Through the Eyes of a Physicist.

Carlo Rovelli, a renowned theoretical physicist, recently marveled that according to contemporary history experts, Aristotle's physics "is either not science at all, or, to the extent it is science, is a failure."[78]. One of these experts[79] believes that although "traditionally" scholars considered Aristotle's science empirical, the current generation has completely denied this thesis. Aristotle's physics is "standardly" regarded as a "paradigm" of the dialectical method, understood as an "a priori" technique, opposed to empirical science. This is the view of Kuhn's supporters, with which, however, Rovelli disagrees[80]:

> I show that Aristotelian physics is a correct and non-intuitive approximation of Newtonian physics in the suitable domain (motion in fluids), in the same technical sense in which Newton theory is an approximation of Einstein's theory. Aristotelian physics lasted long not because it became dogma, but because it is a very good empirically grounded theory

and elsewhere[81]

> It is valid in the same sense in which Newton's theory is still valid: it is correct in its domain of validity, profoundly innovative, immensely influential and has introduced structures of thinking on which we are still building.

[76](Aristotle, *Aristotle's physics, translated by R. P. Hardie and R. K. Gaye*), 220a, Book IV
[77](ibid.), 223b
[78](Rovelli, "Aristotle's Physics: A Physicist's Look"), p. 1
[79](ibid.), p. 1
[80](ibid.), abstract
[81](ibid.), p.10

And we can endorse this thesis. For instance, Aristotle's physics states that heavier bodies fall faster than lighter ones. We all think that's untrue, right? Not so[82]:

> Aristotle's physic does not enjoy good press. It is commonly called “intuitive”, and at the same time “blatantly wrong”. For instance, it is commonly said to state that heavier objects fall faster when every high-school kid should know they fall at the same speed. (Do they??) (...) Why don't you try: take a coin and a piece of paper and drop them. Do they fall at the same speed?

Paper falls slower than a coin, that's obvious. Newton's physics can say the same, considering the fall with air resistance. Of course, Aristotle didn't mean the fall of bodies if we remove the air (writes Rovelli) — he meant the motion of real bodies in the real world, where water and air exist. Indeed, nearly all motion phenomena that an ancient scientist could observe are subject to significant resistances: a hay cart moves when pulled by a horse, a ship sails when pushed by oars or the current of a river, and a marble block moves, when moved by workers. There are resistances to motion and an action must be performed to overcome these resistances — there must be some "external agent" who performs this action. This is what Aristotle calls violent motion. Besides violent motion, there is natural motion, as we have already discussed (p. 31). A dropped stone falls, smoke rises up, water in a stream flows from higher to lower terrain, and rocks and mud sometimes fall from steep mountains. The four elements: fire, water, air, and earth correspond to four types of natural motion.

Rovelli notes[83], that such complexity is necessary to describe many complex phenomena. If all bodies fell, says Rovelli, one substance would suffice; yet fire rises upwards. This gives two different types of behavior. But that's not all: stones sink in water, but wood floats on it; yet wood falls through the air. This requires describing the relationships between different substances. Celestial bodies, which move in a way quite different from earthly matter, must also be described.

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[82] (ibid.), p.1, p.10
[83] (ibid.), p. 3

> If all things fell down, only one substance would be needed; but some things, like fire, move up. If there were only things moving upwards (like fire) or downward (like earth), two elementary substances would suffice: one with a natural tendency moving upward and one with a natural tendency moving downward. But observation teaches us that there are objects that move upwards in a medium but downward in another.

Aristotle thus divides motions as follows:

- Natural motion: in the absence of external causes, a body moves vertically towards its natural place[84].

- Violent motion: a body moves under the influence of an external causal cause.

Rovelli notes that a similar division is found in Newton's laws of dynamics[85]:

- If no net force acts on a body, it rests or moves with uniform motion

- If a net force acts on a body, it moves with accelerated motion.

The second law of dynamics describes forced motion, the first law describes free motion. Of course, Newton's forced motion is accelerated, and all gravitational and hydrostatic phenomena, which Aristotle considers as the types of natural motion, must be separately described. The distinction itself is very similar to Aristotle's.

Metaphysical reasonings, which we mentioned (p. 22), are not discussed here, but that does not matter. Rovelli states that the theory was empirically justified and based on experience, so it is unwarranted to question its scientific validity.

Moreover, while Aristotle was not interested in mathematical laws of nature, that does not mean he did not discover any. On

[84] I omitted the issue of celestial motion, to which we will return.

[85] (Rovelli, "Aristotle's Physics: A Physicist's Look"), p. 3

the contrary, he formulated the velocity of a falling body as proportional to the weight of the body and inversely proportional to the density, which Rovelli rewrites as follows:

$$|\vec{v}| = c\frac{W}{\rho} \tag{1}$$

where v is the velocity, W is weight (equivalent to the force of gravity), ρ is the medium's density and c is a constant.Today we also know that the velocity of a body moving with resistance depends on the properties of the medium (viscosity, density), the mass of the body, and other properties (e.g., shape). Generally, for motion in air or another not very viscous medium the value of the resistance force depends on the velocity squared and the density of the medium:

$$\vec{F}_o = c_1\rho|\vec{v}|^2,$$

where c_1 is a constant. A falling body will therefore accelerate until it reaches maximum velocity, at which point the resistance force balances the force of gravity.

$$c_1\rho v_{max}^2 = W,$$

as a result, we get an equation for the maximum velocity of a falling body.

$$v_{max} = \sqrt{\frac{W}{c_1\rho}}.$$

That would be the maximum velocity which is achieved after sufficient time for acceleration. It is not very useful for modeling falls from small heights in thin medium (such as air). In such case velocity will strongly depend on time[86], as the falling body accelerates with acceleration no greater than $g = 10$ m/s^2:

$$v(t) = \sqrt{\frac{W}{c_1\rho}}\tanh\left(\frac{1}{m}\sqrt{Wc_1\rho}t\right). \tag{2}$$

An iron object falling from a small height will never reach maximum velocity, but will move with nearly uniform acceleration. If

[86] (ibid.), equation 10

the term in the bracket in (2) is small (e.g., when the density ρ is high) we can approximate the hyperbolic tangent by the first order term:

$$v(t) \approx \sqrt{\frac{W}{c_1 \rho}} \frac{1}{m} \sqrt{W c_1 \rho} t = \frac{W}{m} t = gt$$

,

Aristotle didn't discover this, but his law (1) is very close to the actual formula for maximum velocity. This, and his other doctrines are a success, not a scientific failure. Rovelli concludes[87]:

> What Aristotle does not have is only the square root (...), which would have been hard for him to capture given the primitive mathematical tools he was using. His factual statements are all correct. Hard to claim this is not based on good observation. If the reader thinks all this is "intuitive" and "self-evident", he should ask himself if he would have been able today to come up with such an accurate and detailed account of the true motion of falling object.

The answer generally is "no." Few people even study complicated details of aerodynamics and hydrodynamics. The modern framework was only created in the 19th century (Reynolds numbers, Navier-Stokes equation).

2.5 Problems of Aristotle's Physics.

Let's add to the above discussion what evidently distinguishes the Aristotle of Stagire from a modern physicist. Law (1) makes him believe that a vacuum does not exist. Going to the limit $\rho = 0$ in the denominator gives v divergent to the infinity —this leads to the conclusion that bodies in a vacuum would fall with infinite velocity, thus, according to Aristotle, a vacuum is impossible. It is commendable that he analyzed his law in this respect[88], although the judgment is too far-reaching. Other arguments, which he presents in Chapter IV of "Physics", also probably played a

[87](Rovelli, "Aristotle's Physics: A Physicist's Look"), p. 5

[88]In this regard, he is more far-sighted than initially Galileo, who proposed a law in the form $v \approx cW - \rho$ (ibid.), p. 3

role. Interpreting it on the grounds of teleology, Aristotle states famously that the nature abhors a vacuum, which supports one of his hypotheses to explain the cause of motion: that, for example, an arrow released from a bow is pushed by the momentum of the air. Very important issue is the lack of a modern distinction between average and instantaneous velocity: this might be the reason Aristotle did not realize that heavy and dense bodies fall faster and faster. Aristotle did not recognize velocity as a form (Section 3.2) and did not consider motion at a point in time as a coherent concept. As a result, the evident effect of acceleration of a heavy iron ball would not have piqued his interest, as he would not have recognized acceleration as a coherent concept, even if he had thought of such a thing.

More shortcomings of Aristotelian physics can be pointed out, centered around an organic view of the world, treating physics as akin to other natural sciences, and the theory of purposeful order. Today, Aristotle's work on meteorology and geophysics is full of incorrect organic analogies. Although he had a correct vision[89] that the Sun induces air mass circulation and water evaporation, and that vapor at higher altitudes cools and condenses, creating rain, he did not develop this vision. His attention was absorbed by "natures", Stanley L. Jaki writes, dry and humid exhalations produced by the Earth, described as a great animal that grows, digests, and releases gas", similar to how horses and cows do. Falling stars are an example result of such dry exhalations, same for lightnings and comets. Heat and cold are also counted among natures[90]. As "opposites" and according to the general law on opposites they accelerate mutual reactions when they are placed together. Aristotle thus thought, quite absurdly, that water cools faster if it is previously heated[91].

Aristotle did not know what was discovered only after him, just like all innovators in world history. However, all this does not diminish Aristotle's contributions to physics and other fields of science. These merits will become more clear when we show how Aristotle's system was transformed nearly 1500 years later. For

[89](Jaki, *The Relevance of Physics*), p. 23

[90]Correct, quantitative form of heat was understood in the Middle Ages, p. 57

[91](Jaki, *The Relevance of Physics*), p. 25

that purpose, however, the method of purposeful order and organic vision of the world must be changed. We will not find such change among the rest of the ancient Greeks and Romans, for they followed similar paths as Plato and Aristotle, viewing the world as an organism. Even Ptolemy, co-author of accurate mathematical equations for planetary motions emphasizes that mechanical analogies provide only a superficial insight into the movement of celestial bodies and in reality, they move using vital principles. Stoic philosophers adopted organic physics: its main thesis is well illustrated by this quote provided by Cicero, attributed to Zeno of Elea[92].

> Nothing that is without a soul and reason can generate of itself anything endowed with life and reason; the world however creates beings with soul and reason; therefore, the world is living and possessed of mind.

After the 1st century, ancient philosophy turns towards the irrational and mystical Neoplatonism, rather barren in the study of nature. Prominent philosophers of nature of this period: Alexander, Themistius, Damascius, and Simplicius are mainly engaged in commenting and developing Aristotle's doctrine.

2.6 Everything According to Measure, Number, and Weight.

We have described the main assumptions of Aristotle's physics, pointing out, on one hand, its remarkable achievements, and on the other, its failures — both stemming from an imperfect conception of the world order and attempts to create a logical system of images.

About the science of other ancient civilizations, nothing better can be said. The Incas[93], Babylonians[94], Chinese[95], and Indians[96] adopt an organic or pantheistic view of the world and explain nature on this basis. Generally, their science is at a lower level than

[92](Jaki, *The Relevance of Physics*), p. 35
[93](Jaki, *Science and Creation*), p. 55
[94](Jaki, *Saviour of Science/Zbawca Nauki*), p. 37
[95](ibid.), p. 35
[96](Jaki, *Science and Creation*), p.16

that of the pre-Socratic Greeks; accurate observations mingle with myth. A comparative analysis can be found in Stanley Jaki's "Science and Creation"[97].

An exceptional case of somewhat different thinking is the Bible. In the books of the Old Testament, the following texts appeared[98]:

> When the Lord created his works from the beginning, and, in making them, determined their boundaries, The arranged his works in an eternal order, and their dominion for all generations. They neither hunger nor grow weary, and they do not abandon their tasks. They do not crowd one another, and they never disobey his word. Then the Lord looked upon the earth, and filled it with his good things. With all kinds of living beings he covered its surface, and into it they must return. (Sir 16, 26-30)[99]
>
> Even apart from these, people could fall at a single breath when pursued by justice and scattered by the breath of your power. But you have arranged all things by measure and number and weight. (Wis 11, 20)
>
> When he established the heavens, I was there, when he drew a circle on the face of the deep, when he made firm the skies above, when he established the fountains of the deep, when he assigned to the sea its limit, so that the waters might not transgress his command, when he marked out the foundations of the earth, then I was beside him, like a master worker;(Pro 8, 27-30)

Similar mentions can be found in the Books of Psalms, Job, Ecclesiastes, and others. The above verses suggest a world with (Sir 16) a fixed, (Wis 11) mathematical, and (Pro 8) rational order. The quote from (Pro 8) is spoken by Divine Wisdom. The world, as the masterpiece of Divine Wisdom, is orderly and understandable. God "draws" vaults and foundations like an architect.

[97](ibid.)
[98]Selection after (Jaki, *Saviour of Science/Zbawca Nauki*), p. 60-62
[99]Bible quotes from New Revised Standard Version Catholic Edition

These passages from the Holy Scriptures do not explicitly say "there are such and such laws of nature." but portray rational Creator that orders the world according to plan, giving it unchanging structure. In addition, priority of quantities is emphasized. The application of philosophy and related logical tendencies allowed many such interconnected passages to be interpreted and used to develop a worldview.

2.7 The Dynamics of John Philoponus.

Before the 10th century Arab renaissance, the development of Aristotle's natural philosophy was undertaken by a small number of pagan philosophers. Some supported Aristotle's theories against other currents of pagan philosophy, while others combined them with the prevailing Neoplatonism.

Among these early commentators on Aristotle was the Nestorian Christian John Philoponus (6th century). S. Jaki lists the following theses of Philoponus[100]:

> ...all bodies would move in a vacuum with the same speed, regardless of weight (mass); that bodies with greatly different weight falling from the same height hit the ground practically at the same time; that projectiles move across the air not because the air keeps closing behind them, but because they were imparted a certain "quantity of motion"

All this stands in contradiction to Aristotle's physics and is a correct qualitative description. Initially, Philoponus' groundbreaking theories did not gain followers, but over time his observations proved to be correct. Here's another revolutionary argument from him:

> Could the sun, moon and the starts be not given by God, their Creator, a certain kinetic force in the same way as heavy and light things were given their trend to move...?

[100](Jaki, *Science and Creation*), p. 186

In the above example, Philoponus tries to unify mechanics on Earth and in the heavens into one, to which he invents something akin to kinetic energy. This thought is rooted in theology. First, if there is a transcendental Creator, purposeful order can be explained without an organic world. The world could have been set in motion, like a mechanism created by man. Secondly, Philoponus is a step away from making the conclusion: I can think of X, therefore God could create X, therefore X is possible. The reference to God is not just a rhetorical figure.

2.8 Scholasticism

Under the influence of Aristotle, St. Boethius (d. 524-526), Eriugena (d. 877), St. Anselm (d. 1109), and Abelard in the theology and philosophy of the Latin thinkers, the scholastic method emerged. Logical and semantic analyses were used to examine the texts of Revelation. Text fragments were organized thematically and a common key to their meaning was sought. Care was taken to organize knowledge and to formulate thoughts precisely. Among the works that emerged were extensive lexicons and syntheses (Summae), discussions (Quaestiones), and elaborate commentaries.

In this way, reflection on the nature of the world was also revisited. Where did the idea to talk about "laws" of nature by analogy to a legal code come from? According to ancient theologians, both natural and moral laws have the same source. St. Anselm[101] compares the righteousness of God's moral order, to the proper arrangement of the created world (expanding on St. Augustine's thought). In Anselm's view, the Creator is the lawgiver of both the physical and the moral world. Thus, both justice and truth are subcategories of *rightness*. One relates to will, the other to perception. This idea has important predecessors. We have pointed out that Aristotle's physics began with philosophers who wondered why it was "better" for planets to move as they do.

It is also worth mentioning Robert Grosseteste (d. 1253), the Bishop of Lincoln and Chancellor of the University of Oxford[102]. According to Grosseteste, light is the primary substrate of the

[101] (Gwozdz, "St Anselm's Theory of Freedom.")
[102] (Jaki, *Science and Creation*), p. 222

world, and light obviously shows connections with geometry: it propagates in straight lines, undergoes reflection and refraction, and has something to do with heat, etc. In "De lineis," Grosseteste writes:

> The usefulness of considering lines, angles, and figures is the greatest, because it is impossible to understand natural philosophy without these. They are efficacious throughout the universe as a whole and its parts and in related properties, as in rectilinear and circular motion.

For discovering laws of nature, Grosseteste proposed testing hypotheses through experiments. He himself made progress in understanding the phenomenon of the rainbow (previously misunderstood as a result of reflection). Grosseteste also developed a philosophy of measurement[103]. He indicates that human measurements can never perfectly capture the quantity being measured; it is known to God, who sets everything "according to measure, number, and weight."

Another English pioneer of science is Roger Bacon (d. 1292). He was extensively involved in optics—optical experiments and the application of geometry in optics, continuing the legacy of Ptolemy, Alhazen, and Grosseteste[104]. He managed, among other things, to calculate the maximum elevation of the rainbow[105], invent gunpowder, and build a few simple optical devices. Bacon also wrote about "universal laws of nature"[106] (which included the laws of reflection and refraction of light); he claimed that experiments should confirm or refute theoretical theses and speculated about various technical devices that could be built in the future (e.g., telescopes, automobiles).

2.9 Philosophy of St. Thomas Aquinas

One of the most important figures of the scholastic rediscovery of Aristotle and arguably the most influential Catholic philosopher

[103] (Jaki, *Science and Creation*), p. 223, a summary by William of Alnwick cited

[104] (Hackett, "Roger Bacon"), 5.5

[105] (ibid.), 5.4.3

[106] (ibid.)

overall is St. Thomas Aquinas. In the field of physics, he appears to be mostly a typical supporter of Aristotle and Averroes, especially compared to works published after his death by Scotists and terminists (these will be our most important focus here). In general philosophy, however, a transformation that foreruns the revolution in physics is already in the making in his works.

Several doctrines of Aristotle and Averroes were contradictory to the Christian creed and needed modification. Furthermore, a new direction for development opened with Aristotelian methods and concepts applied to Christian philosophy and theology.

Christian view of the world and Creator. In "Summa Contra Gentiles" St. Thomas Aquinas described the differences between the world-picture of Catholicism and that of pagans[107]. According to St. Thomas, the world is a reflection of Divine Wisdom, so it must be understandable and intricately organized.

> First, because meditation on His works enables us in game measure to admire and reflect upon His wisdom. For things made by art are representative of the art itself, being made in likeness to the art. Now, God brought things into being by His wisdom; wherefore the Psalm (103:24) declares: "You made all things in wisdom." Hence, from reflection upon God's works we are able to infer His wisdom, since, by a certain communication of His likeness, it is spread abroad in the things He has made. For it is written: "He poured her out," namely, wisdom, "upon all His works"

Two things are important here: first of all, with Christianity there is no need for an animate, organic world, as conceived by the Stoics or the followers of Plato. We saw the former group (p. 42) assume that if the world generates highly complex, animate organisms, then the world itself must be animate and complex (and even more so than animals and people). Christians, believing in a wise Creator beyond the world, might easily conceive the world as simple and ordered. Secondly, the Christian vision of God (contrary to,

[107](Aquinas, *Summa Contra Gentiles Book II translated by J. F. Anderson*), book 2, chapter 2.

for example, the Muslim vision) clearly endorses rationality and wisdom seen in the order of all created things, suggesting that the created world is comprehensible to human reason.

Contingency of Creation Another important distinction between Aquinas and Aristotle is related to Thesis 1.2, concerning the logical necessity or, respectively, the contingency of creation. Christians, asserting the omnipotence of God, cannot assert that creation is subject to any kind of necessity. God could make it in any way He wanted (or even not make it at all). It also follows that God is not bound by any necessity and can freely overrule the typical order of things[108].

> ...through ignorance of the creature's nature something is subtracted from God's power in its working upon creatures. This is evidenced in the case of those who set up two principles of reality; in those who assert that things proceed from God, not by the divine will, but by natural necessity; and again, in those who withdraw either all or some things from the divine providence, or who deny that it can work outside the ordinary course of things. For all these notions are derogatory to God's power. Against such persons it is said: "Who looked upon the Almighty as if He could do nothing" (Job 22:17), and: "You show Your power, when men will not believe You to be absolute in power" (Wis. .12: 17).

Creation of the world. One of the most obvious problems with Aristotelian philosophy, as far as the Christian creed was concerned, was the eternity of the world. Aristotle and his followers believed that the eternity of the world could be demonstrated by philosophical argument; Christian faith stated that the world had a beginning in time.

This is connected to another difference regarding what God is supposed to be with respect to creation. For theistic Aristotelians[109],

[108](Aquinas, *Summa Contra Gentiles Book II translated by J. F. Anderson*), book 2, chapter 2.
[109](Pasnau, "Thomas Aquinas"), sec. 3

God as the first mover is merely "the initial, remote source of motions that has always existed," rather than the eternal being who created the universe anew. Aquinas believes Christianity holds the contrary opinion: all that exists was made by God from nothingness, and only God is capable of creation in the proper sense. The events of creation and perishing we see in the world are merely transformations of visible things.

Furthermore[110], God created the world freely, choosing to create it without any necessity. He could create a better universe, or He could create nothing at all. For this reason, arguments for the eternity of the world must fail. For Aquinas, there is no necessity in creation except for the fact that God, having created all things, by His goodness puts them in the most beautiful order (ST Ia, 25, 6, 3) (but God could always make it better by adding something).

> The universe, the present creation being supposed, cannot be better, on account of the most beautiful order given to things by God; in which the good of the universe consists. For if any one thing were bettered, the proportion of order would be destroyed; as if one string were stretched more than it ought to be, the melody of the harp would be destroyed. Yet God could make other things, or add something to the present creation; and then there would be another and a better universe.

The aim of the universe is truth In the beginning paragraphs of Summa Contra Gentiles, Book 1[111], Aquinas declares that truth is the ultimate purpose of the whole universe, being "the good of intelligence." This remark is supported by both the testimony of the Gospel and that of Aristotle, but Aquinas seems to elevate this principle much more than Aristotle and any other pagan: truth is the ultimate end of everything.

> The prime author and mover of the universe is intelligence, as will be shown later (B. II, Chap. XXIII,

[110](ibid.), sec. 3

[111](Aquinas, *Summa Contra Gentiles Book II translated by A. C. Pegis*), book 1, chapter 1.

> XXIV). Therefore the last end of the universe must be the good of the intelligence, and that is truth. Truth then must be the final end of the whole universe; and about the consideration of that end wisdom must primarily be concerned. And therefore the Divine Wisdom, clothed in flesh, testifies that He came into the world for the manifestation of truth: For this was I born, and unto this I came into the World, to give testimony to the truth (John xvii, 37). The Philosopher also rules that the first philosophy is the science of truth, not of any and every truth, but of that truth which is the origin of all truth, and appertains to the first principle of the being of all things; hence its truth is the principle of all truth, for things are in truth as they are in being.

For the above reason, there seems to be a difference between scholasticism and pagan and Muslim Aristotelian philosophy, as the number of relevant Latin scholastic authors is much greater in a much shorter period of time. Furthermore, similar sentiments are found among modern scientists, showing the importance of the pursuit of truth as the motivation for scientific discovery, together with the Christian origin of this idea. Here is a very analogous quote from the 19th-century mathematician Augustin L. Cauchy[112]:

> Yes, undoubtedly, gentlemen, the search for truth must be the sole goal of all science. It is towards this end that the efforts of true scholars are directed; it is to this alone that they devote their vigils. Should we be surprised? The human spirit, made to possess the truth, cannot find rest outside its domain. Man cannot live without the truth. It is one of the conditions of his existence, like the air he breathes and the bread that nourishes him."

Dynamics of Aquinas: Weight and Mass While departures from Aristotle's physics aren't frequent in Aquinas's works, he may

[112](A. Cauchy, *Sept lecons de physique generale par Augustin Cauchy*), p. 2

be counted among the very few early forerunners of Buridan's impetus theory[113], for he was first to conceive the modern weight vs. mass distinction. Aquinas exposes the opinion of Averroes on the fundamental principle of Aristotelian dynamics: that when we accelerate a massive body, the only resistance we feel is from the medium (such as air or water).

> But in regard to heavy and light bodies, when we subtract that which the mobile body has from the mover (meaning the form, which is a principle of movement and which the generator or mover gives), then nothing remains except matter, in regard to which no resistance to the mover can be considered. Hence it follows that in such things the only resistance is from the medium.

The body, according to Aristotle, is form and matter or form applied to matter. The form defines the principle of movement and change for this body. A stone falls, iron sinks in water, fire goes up. If we remove the form from the body, we get shapeless matter: the principle of motion is removed when the form is removed. In such a case, the matter should have no resistance to the mover, except for the resistance of the medium. Aquinas rejects this strongly:

> When the form, which the generator imparts, is removed from heavy and light things, a body with magnitude remains only in understanding. But a body has resistance to a mover because it has magnitude and exists in an opposite site [opposite to where the movement should lead] No other resistance of celestial bodies to their movers can be understood.

This is an "extremely brief" remark "but let us not allow its brevity to make us misunderstand its importance," says Duhem. The argument seems to be as follows: celestial bodies move endlessly in circular orbits and without medium resistance. Yet, they show some resistance to their movers (as they aren't moved immediately with great speed). But where does this resistance come from? It is not

[113](P. Duhem and Ariew, *Medieval Cosmology: Theories of Infinity, Place, Time, Void, and the Plurality of Worlds*), s. 378

from moving against nature, like we would do with a stone when we move it up. It is not from medium resistance (for celestial bodies there's no medium). Thus, the mass itself must show certain resistance to a mover. In this way, Duhem says, the human mind for the first time saw in a massive body the distinction between weight and mass (in modern terms), between motive force and moved thing. Furthermore, Aquinas was the first to think of mass as the quantity of matter that remains when we suppress all forms. This doctrine is invoked by Duhem in his chapter on theories of the void, as it is very helpful to overthrow one of Aristotle's arguments against the possibility of the void: that the lack of medium resistance would produce infinite acceleration.

Aquinas didn't carry this reasoning much further though, largely following Aristotelian dynamics elsewhere. In his commentary on Aristotle's "On the Heavens," which is one of his last writings, he subscribed to Averroes' theory that shaken air is the only cause that allows a projectile to continue its movement[114].

[114](Pierre Duhem and Aversa, *Galileo's Precursors: Translation of Studies on Leonardo da Vinci (vol. 3) by Pierre Duhem*), p. 199

3 Physics after the 13th Century

3.1 The Scientific Revolution of 1277

From the 12th century onwards, previously unknown writings of Aristotle, particularly on physics, metaphysics, and cosmology, as well as works of Muslim science, began to flow into Western Europe. Aristotle's philosophy contained several theses that were contradictory to Christian dogma: for example, the universe is eternal, human souls are not immortal, and divine providence does not exist[115]. Some Western philosophers adopted the views of Aristotle and Averroes, while others attempted to reconcile philosophy with faith.

As mentioned, the most important member of the conciliatory party was St. Thomas Aquinas[116]. He rejected few of theses of Aristotle that were explicitly contrary to the faith and adapted the rest of his (and Averroes's) philosophy. This did not yet breach Aristotle's and Averroes's physics, nor did it harm its increasingly broad claims to reality. We know that Averroes even rejected the most important theory that could be considered close to mathematical physics: Ptolemy's astronomical system[117]. In his view, the epicycles used by Ptolemy were "absolutely impossible" because circular motion could only occur around a material center. Alhazen and Bernard of Verdun attempted to remove the features of Ptolemy's system that were contradictory to Aristotelian philosophy[118]; however, they were unable to resolve this one issue.

This dispute bolstered the confidence of Franciscan theologians in what we will describe next; Ptolemy's system reigned[119] "supremely" among the scholars of the University of Paris. In the "seed" of accurate predictions of planetary movements, set against the "chaff" of Averroists' explanations, they saw much more than Ptolemy himself.

Let us recall the cited deduction:

[115] (McInerny, *A First Glance at St. Thomas Aquinas: A Handbook for Peeping Thomists*), 5
[116] (ibid.), 5
[117] (P. Duhem and Ariew, *Medieval Cosmology: Theories of Infinity, Place, Time, Void, and the Plurality of Worlds*), s. 142
[118] (ibid.), s. 180
[119] (ibid.), s. 180

- (Premise) I can conceive X without contradiction.
- (Premise) God is omnipotent (based on the dogma of Divine Omnipotence) and can create X.
- (Conclusion) Therefore, X is not impossible.

As a result: I can conceive of X, so X is possible, and anyone who wants to claim otherwise contradicts the dogma of faith. This reasoning was the main cause of the condemnations issued at the University of Paris in 1277 by Bishop Tempier. The Franciscan theologians not only rejected theses that were openly contrary to faith but demanded that the logical conclusions of Revelation be recognized over the conclusions of Aristotle. Consequently, a number of the Philosopher's views were condemned and banned from being taught under threat of ecclesiastical penalties. Here are some of the condemned theses related to the above deduction[120]:

1. The First Cause cannot create many worlds.
2. If the heavens stopped, fire would not burn flax because God would cease to exist.
3. God cannot move the universe in a straight line because it would leave a vacuum.

In (1) the Aristotelian view that the existence of many worlds is a contradiction is condemned. In (2) it refers to the belief that the passage of time would stop if the movements of celestial bodies stopped (see Section 3.3). (3) occurs according to the Aristotelians because they believe a vacuum is impossible. As a result, the key theses of Aristotle's physics were rejected in one fell swoop, and a methodological tool was created that made physics a field of continuous search for new images.

A few other rejected views[121] concern the organic view of the world, astrology, and Eternal Returns[122]:

[120](P. Duhem and Ariew, *Medieval Cosmology: Theories of Infinity, Place, Time, Void, and the Plurality of Worlds*), s. 450, s. 392, s.181

[121](Jaki, *Science and Creation*), s. 229 – paraphrases of Tempier's decree.

[122]The pagan belief that all events repeat themselves when the stars return to the same configuration every few tens of thousands of years.

- (73, 31, 32) Celestial bodies are living beings, and celestial matter is eternal and animated.
- (92) All the same events repeat every 36,000 years.
- (75) The celestial spheres are organs similar to eyes and ears.
- (105) Stars have a deterministic influence on humans from birth.

Thus, the vision of an organic universe loses significance, making way for the Christian vision of the ordered world. Duhem called these condemnations of 1277 the birth certificate of science. It is indeed the birth certificate of *physics according to Duhem's philosophy*.

3.2 Scholastic Theory of Physical Quantity

A long time ago, the balance scale was invented. The Egyptian Book of the Dead states that in the afterlife, a person's heart is weighed on a scale to check if it is lighter than a feather. Typically, polished stones of various masses served as weights, allowing the determination of the masses of small objects. A few people even knew the law of proportionality between the position of the balance point and the sizes of the suspended masses (Archimedes' lever law). However, this was not connected with the theory of motion, inertia, or gravity. In most cultures, such tendencies did not exist.

Aristotle considered quantity and quality as two separate categories[123]. By quality, he meant attributes such as heat, whiteness, and weight, and by magnitude, he meant length, surface area, or volume (as well as time, as a quantity of motion). The philosopher pointed out a significant difference. Quantity, or "magnitude," implies a relationship of composition or inclusion. A long rope is made up of smaller pieces of rope, a bag of sand contains many grains of sand, and the natural number 5 can be expressed as 1+1+1+1+1. Some magnitudes are discrete (the term "number" is used in such a situation): a handful of coins contains 30 coins. Other magnitudes are continuous: 1 gallon of water is 4 quarts.

[123](Pierre Duhem and Aversa, *Galileo's Precursors: Translation of Studies on Leonardo da Vinci (vol. 3) by Pierre Duhem*), p. 227

Aristotle's quantities take different values, but keep "the same nature.", corresponding to specific amounts of substance. A gallon of water is created by adding quarts of water together. A larger magnitude, such as 10 gallons, is created by adding more water. A 10-foot rope is made by cutting a 20-foot rope in half. The short rope exists physically within the long rope. Similarly, a quart of water exists within a gallon. It works the same for bushels of grain, yards of cloth, and dozens of eggs. Similarly, one can measure surface areas and distances. This idea of quantity corresponds well to the practical applications, such as trade, cartography, or inventory management.

But what about qualities like heat? Aristotle says that "a body becomes whiter or hotter without adding any whiteness or heat; the existing quality becomes more intense as it gets closer to its goal"[124]. This means that the quality or intensity of a form (e.g., heat) becomes more intense (the body becomes hotter) as it approaches its ideal form (extreme heat). Scholastics of the 13th century were generally supporters of this or a similar doctrine: for example, St. Thomas Aquinas or Henry of Ghent:

> (Aquinas) a body becomes whiter or warmer without any addition of whiteness or heat; but the preexisting quality becomes more intense because it is closer to its end.
>
> (Henry of Ghent) The augmentation of forms, is not done by an apposition of parts in their substance or essence; this is an increase in force (in virtute), through which the increased form becomes more efficacious in its own operation, which cannot produce the addition of a similar to similar; a warmth added to an equal warmth is no more heat

Durandus of Saint-Pourçain[125] similarly noted that even degrees of quality are not similar to the divisibility of a quantity into parts, but rather degrees of form denote approach greater and greater perfection of form." Thus, one can speak of a distance to the ideal,

[124](Pierre Duhem and Aversa, *Galileo's Precursors: Translation of Studies on Leonardo da Vinci (vol. 3) by Pierre Duhem*), p. 228
[125](ibid.), p. 231

but this *distance* does not obey the laws of addition. Imagine following: in the factory of porcelain, one can select the most precisely made items as category A+ (which can be sold at a higher price). Others that meet the basic quality standard are classified as category A. Those with certain minor defects are categorized as B. For such defined categories, the difference between B and A is different from the difference between A and A+, and it cannot be expressed in terms of adding some quality.

The above doctrine was dominant in the 13th century, although there were also opposing opinions. St. Thomas cites Aristotle's opinion against "some philosophers." The successors of these philosophers became active after the anti-Peripatetic condemnations of 1277. Franciscans Richard of Middleton and Blessed John Duns Scotus were the most important representatives of this trend. This thought was further developed by Scotus's "favorite pupil," John Bassols.

> By degree of charity or of any form, I understand a certain individual of this form; (...) I thus give the same sense, in the proposition before me, to the words: degree of form, and to the words: limited individual of that form; it is the same to compare a subject that has a greater degree of that form to another subject that has a lesser degree of it, or to say that we are dealing with a more perfect individual of this form and with less perfect individual.

The reader may recall the example given at the beginning of how to understand the measurement of temperature with a thermometer (p. 10). The first step is that we compare bodies to each other in terms of the degree of heat, which creates the concept of more or less heat; this is precisely Bassols's construction. He goes much further, though. A good illustration is the example given by Bassols[126].

> The two warm bodies here are something more than each of them; it is clear from the effect they produce, because, together, they generate in a third body a heat more intense than what each of them will generate in

[126](ibid.), p. 240

> isolation; so if we add the heat of one to the heat of the other, we produce something of greater intensity, as the effect of these two heats is more intense than the effect of each in isolation. This can be seen clearly by taking the example of weight; two stones or two weights taken together weigh more than one of them, extensively; but if you added the weight or gravity of one of these bodies to the weight or gravity or of the other, so as to make a single weight or gravity by the union of the two weights or gravities, the result would be heavier in intensity than each of the two weights in isolation; and this is natural, although neither of these weights, considered separately, is more perfect than the other.

These speculations are surprisingly modern. To better translate this to modern intuition, imagine two blocks of steel at a temperature of 50 degrees Celsius and a third block at a temperature of 0 degrees. If the first and third blocks are joined together and brought to thermal equilibrium, their temperature will settle at 25 degrees. If the first, second, and third blocks are joined together, the temperature will settle at 33.3 degrees (assuming the system is isolated). This is precisely Bassols's thought experiment. Although he did not know about temperature and specific heat, he wrote enough to infer that there is an *amount of heat*, not the *closeness to the perfection of the form of heat*.

The second example is equally intriguing. Aristotle's physics explains the effect of gravity by the natural tendency of bodies to seek their natural place. Solid bodies strive to be as close as possible to the center of the Universe, resulting in a stationary and roughly spherical Earth. In modern physics, one "tendency" is rather for a body to remain in motion or at rest, and another tendency is that masses attract each other. Inertial mass and gravitational mass determine the magnitude of both tendencies, and the equality of one and the other is an experimentally determined law. This tendency cannot be nature, because nature is one. Aristotelians could not conceive of this thinking about "natures," resulting in the erroneous conclusion that a vacuum cannot exist because bodies would fall infinitely fast. The new philosophy after 1277 was highly corrosive to such reasoning and filled with original ideas. If we can imagine a vacuum, then apparently it can exist. If we can imagine levitating stones or that the same stone can become three times heavier, then this state is possible. Gravity can thus be a property that we can take away, add, increase, or decrease.

These ideas spread widely and were refined in the first half of the 14th century. Duhem summarizes[127]:

> In the first half of 14th century, therefore, the most famous of Scotists and Nominalists conspired to completing the work that Richard of Middleton and John Duns Scotus inaugurated; abandoning the peripatetic doctrine, erasing the so entrenched distinction that it demarcated between the category of quantity and the category of quality, they established a close analogy between the increase of a quantity and the tension of a qualitative form; the increase of intensity, like the increase of a quantity, results from the addition of parts to other parts of the same species.

and further:

> This theory leads at once to a corollary of extreme importance: The intensity of a quality is henceforth susceptible to measure, as is the magnitude of a quantity; just as they apply to such magnitudes, the reasonings and operations of Arithmetic can combine the various intensities of forms of the same species

Scholastics then, "without defining this doctrine explicitly," quickly began to apply it.

In 1344, Gregory of Rimini used "dual forms," speaking of the *speed* "with which the intensity of the form is created" and distinguishing cases of uniform and non-uniform transformation. Similarly, Albert of Saxony wrote about local motion, expansion, and transformation[128]:

[127](Pierre Duhem and Aversa, *Galileo's Precursors: Translation of Studies on Leonardo da Vinci (vol. 3) by Pierre Duhem*), p. 244

[128](ibid.), p. 245

> If, for example, unequal subjects gain in an hour equal qualities, they are altered with an equal speed; if the acquired qualities are unequal, these subjects are not altered with an equal speed.

This development would be applied in the development of 14th-century mechanics.

3.3 Time and Abstractions

In dynamics, electrodynamics and many other theories, fundamental equations contain derivatives of quantities with respect to time. One such quantity is velocity, the derivative of the length of a path of motion with respect to time. In physics, time is considered an additional dimension, similar to the three spatial dimensions. By selecting a point in space and a point in time, we obtain an event. Connecting many such points, we get a sequence, which can be, for example, part of a motion path. For all of this, we need time as a parameter or dimension. However, in the past, the understanding of time was different, which created significant obstacles in the development of physics.

We mentioned that Aristotle attempted to find effective causes of phenomena, even though such causes often do not exist in the sense he assumed: this led him to the conclusion that all changes are caused by the motion of celestial bodies. A similar conclusion arises from his theory of time: time is the length of motion analogous to the length of a line. According to the Aristotelian theory, what we perceive as the passage of time is caused by the movement of celestial bodies, and if the heavens were to stop, all changes on Earth would also cease.

Today, it's easy to dismiss the above as nonsense. However, without reference to modern physics proving that time wouldn't stop when the celestial dynamics stop moving would be very difficult. The influence of moving masses on the spacetime actually had its role in modern science - we can point to the hypothetical phenomenon of electromagnetic ether drag or the microscopic effect of frame-dragging in General Relativity.

This theory of time is related to the problems of Aristotelian dynamics (or rather, the recognition that there are problems). In

Book VI of "Physics," Aristotle insists that at a given point in time, nothing moves nor rests[129]. From the definition of time as the length of motion, it is evident that the length of a point is a contradiction. This allows him to resolve Zeno's of Elea arrow paradox: Zeno claims that since a flying arrow is motionless at a given point in time, movement is contradictory. Aristotle proposes a solution: he rejects the premise that time consists of indivisible moments[130], but at the same time, he believes that there is no such thing as motion or rest "at a point in time." This is not a satisfactory solution because instantaneous velocities and other similar quantities are essential in physics.

Today, the solution is known because we understand small and infinitesimal (infinitely small) intervals of time. A "point in time" is a concept used, for example, when we say that a photo was taken at 10:00. In reality, it was taken over a short duration (the shutter speed), starting at a certain point in time. If we consider the instantaneous velocity of an arrow, it is clear that the smaller the time interval we consider, the shorter the distance it will cover – it will only be at rest if we reduce the interval to zero. This convergence to zero in time is crucial for differential calculus. If I consider increasingly smaller time intervals and increasingly smaller displacements of the arrow (in that interval), their ratio converges to a constant value. This ratio is the instantaneous velocity and also an example of the use of differential calculus.

In summary, to understand dynamics and second-order quantities (which we mentioned in Section 3.2), it was necessary to discard the Aristotelian theory of time – this was the main problem to overcome. Even up to the 13th century, followers of Aristotle – such as the Averroists or Robert Angelus[131] – maintained that without the daily movement of the Sun in the sky, there would be no time at all. The breakthrough came with the discovery and application of Thesis 1.6 in conjunction with several other fragments of

[129](Aristotle, *Aristotle's physics, translated by R. P. Hardie and R. K. Gaye*), 234a
[130](ibid.) 239b, (Cohen, *Lecture notes on History of Ancient Philosophy*), "Zeno Arrow Paradox"
[131](P. Duhem and Ariew, *Medieval Cosmology: Theories of Infinity, Place, Time, Void, and the Plurality of Worlds*) p. 297

Revelation[132]. Before 1277, attempts were made to reconcile Aristotelianism with theology through conservative adjustments, but after 1277, a radical revision occurred. Blessed John Duns Scotus was the first to propose the following thesis: even if the heavens were to stop, or even if they did not exist, time would still flow and measure motion. Moreover, even if all the motion ceased, time would still flow and measure the duration of rest.

Duns made a few distinctions. Firstly, according to the Aristotelians, the size of an object is the result of the measurement of the object ("essentially depends" on the measurement) in the sense that a stack of hay is made up of 1000 parts of such and such a size, and a rope 10 feet long is made up of 10 parts each 1 foot long. However, it is different when we use a rope marked with a scale to measure a person's height: in that case, the person's height (e.g., 6 feet) does not essentially depend on the measurement, i.e., a piece of rope 1 foot long. Therefore, Duns points out, the measurement of time using the Sun does not necessarily imply a real dependency[133]. We measure one motion based on another in an arbitrary way. Therefore, Duns points out, even if the Sun were to stop, time could still be measured based on the length of the day.

Secondly, he claims that the rest could exist even if motion (change) did not exist[134]. A given body can behave exactly as it does: it can stay motionless, do not change color, shape, temperature, and so on. Rest can exist in time that flows, even when no change exists. A rational being could perceive a piece of metal that neither moves nor changes in any way and mentally count successive intervals of this immobility. Thus, we obtain time independent of the motion of celestial bodies — potential and private. Potential, meaning it does not exist here and now but has the capacity of becoming. Private, meaning it is a product of human thought. However, Duns does not stop there — he asserts that time is not private. That is, a period of time can be known objectively as long as we establish a measure of that time, and the measure can be the movement of the Sun and the Moon, or something else (in our case, seconds, hours, etc.). Similarly, the length of cloth can be

[132](ibid.) p. 295

[133]Also related to the development of the theory of form – p. 55

[134](P. Duhem and Ariew, *Medieval Cosmology: Theories of Infinity, Place, Time, Void, and the Plurality of Worlds*), p. 296

measured by an arm's length or a fathom, and the measure may vary slightly if different people use their own arms — but this has no bearing on the existence of the cloth and the fact that its length is an objective quantity.

Replacing Aristotelian time with time existing in the mind would not seem like a step forward for physics; but Duns's time is universal, which also meets the conditions set in the 1277 decree[135]. The following views were condemned there:

- (79) If the heavens stood still, fire would not burn flax because time would not exist.
- (86) Time and eternity have no existence in reality, but only in the mind.

Duns was active slightly later (born in 1266) and probably his reasoning about compatibility with theology was not limited to Tempier's decree. Duns points out two other arguments:

- During Joshua's battle with the Philistines, the sun stood still, but this did not stop the passage of time.
- After the resurrection (at the end of the world), the heavens will stop, but change and motion will still be possible.

Peter Auriol (born in 1280) further developed the argument that time is not private. Auriol points out that time as a phenomenon and time as a measure of magnitude are different things. Time itself is, in his opinion, "nothing more than what was and what will be, to which we add continuity"[136]. That is, certain states of affairs follow one another in succession continuously, similar to what happens in motion or any other physical process dependent on time. Time thus defined, Duhem points out, has no parts (in the sense of the essence) because it is merely the succession of parts. However, it is different when we talk about the quantity of time — it has parts as the continuous quantity of time is connected with other quantities because we express it through other magnitudes. For example, "this period of time lasts two days," "this rope is

[135] (ibid.), p. 299
[136] (ibid.), p. 300

three meters long," and so on. This allows us to restate more clearly Duns's reasoning that time *is not* a creation of the mind (while the scale of time arises *in mind* by referring to existing magnitudes).

Another philosopher we will mention is Francis of Marchia, who died in 1344. In addressing the question of whether time is something else than motion, he made some intriguing observations[137], for example:

> How do place and the lodged body behave with respect to one another? (...) The term *place* expresses, not only the idea of volume, but also a relation of container to contained body; (...) I say as much about movement and time. (...) Similarly, when we call the movement of the first mobile "time" we do not consider it in relation to itself; we consider it in relation to other movements.

Compared to Aristotle, the term becomes more abstract: Aristotle thought of the magnitude of motion by analogy to the length of a line, as he felt strong urge to define all terms by the concrete objects. Scholastics do not care about such thing and prefer to make abstractions that describe the ordering of phenomena. De Marchia speaks of the relation of features to other features. Time is the relationship in which changes coexist: for example, the states of a clock on the wall remain in a relation of simultaneity to the states of a heating kettle or the states of a growing radish. We can assign a number to each class of simultaneous states so that all later states have a larger number and earlier states have a smaller number — and this is the concept of time.

Here is an example of the abstraction of concepts that is so typical of modern physics (compare with examples from p. 11). Aristotle defined everything he could through the concrete: late Scholastics go much further. This will be even more evident with the next author, the bishop of Malta, Nicholas Bonet (died in 1343). Bonet notes that it is necessary to consciously distinguish what kind of real objects we are talking about (or what kind of abstract objects). Does an absolutely immobile body exist, relative to which

[137] (P. Duhem and Ariew, *Medieval Cosmology: Theories of Infinity, Place, Time, Void, and the Plurality of Worlds*), p. 321

others move? Bonet[138] points out that it is pointless to look for such a real object because all bodies are capable of some motion. We can point to a certain volume of space and say — here, this is my reference point — but we are indicating not a real object, but a mathematical object that exists only in our minds. Such conceptual objects can have, he says, greater or lesser abstraction, a body can be thought of as having this or that substance or physical property.

Similarly, when we say "line," we mean something entirely different in the real sense and in the mathematical sense. The mathematician's line[139] is a certain segment of fixed length, unrelated to any matter or physical properties. A real line can be a stick, a rope, etc. — the length of such a line can change by stretching or cutting.

> The mathematician abstracts from the movement or change suffered by the subject; hence the line thus considered in no way changes by the effect of change in its subject. One therefore says rightly that mathematics deals with absolutely immobile things. One must say the same about the succesive line of time. The mathematician considers duration of a diurnal revolution and he separates by abstraction this successive line from all matter and all movement;

In this way, Bonet states that even if the Sun were to speed up or slow down one day, time would flow the same — the mathematical day would remain a period of 24 hours, while the physical day would change.

3.4 Theory of Impetus

We mentioned John Philoponus' hypothesis of "quantity of motion," which is akin to the modern concept of kinetic energy. Averroes mocked this hypothesis somewhat[140], but his writings likely helped Latin scholars hear about it. Initially, they were skeptical of

[138] (ibid.), p. 352
[139] (ibid.), p. 357
[140] (Pierre Duhem and Aversa, *Galileo's Precursors: Translation of Studies on Leonardo da Vinci (vol. 3) by Pierre Duhem*), p. xiv

the "quantity of motion." However, the situation changed in 1277 when the barriers of logical necessity crumbled once and for all, and some Scotists unearthed Philoponus' hypothesis. This hypothesis, combined with the theologians' thesis on contingency (Thesis 1.6) and subsequent adaptation of Aristotle's physics, gave birth to a new theory of dynamics. Jean Buridan of Bethune developed this theory in the 14th century: his work would become the foundation for the concepts of momentum and kinetic energy, and the discussion he presented would be repeated for the next several hundred years.

Below, we will present Buridan's theory of impetus based on his original writings cited by Duhem.

Antiperistasis In his commentary on Aristotle's "Physics," Buridan answers the following question: Is the projectile moved by air after it leaves the hand? If not, what moves it? Aristotle believed that the projectile is pushed by the air, which was set in motion by the hand. Buridan doesn't want to agree[141]:

> It seems,[says Buridan, that the projectile after leaving the hand that throws it] cannot be moved by the air; the air, indeed, that must be divided by this projectile, seems rather to resist its movement. In addition, you may say that the one who launches the projectile moves, at the beginning of the movement, not only this projectile, but also the nearby air, and this shaken air then moves the projectile up to a certain distance. But to that we will give this answer: What is it that moves this air once it is no longer driven by the one who launches the projectile? The difficulty is the same for this air as for the cast stone.

Aristotle has two attempts to answer this question, the first is called antipersistasis:

> The projectile quickly leaves the place where it was. Nature, which does not allow the existence of a vacuum,

[141](Pierre Duhem and Aversa, *Galileo's Precursors: Translation of Studies on Leonardo da Vinci (vol. 3) by Pierre Duhem*), p. 28

> sends with the same speed some air behind the projectile. This air, animated with a swift movement, meeting the projectile, pushes it forward; the same effect happens again until the body moves a certain distance.

While Aristotle didn't consider this theory sufficient, Buridan seeks to show that it is worthless and devises three thought experiments to refute it:

- A spinning top or a grindstone spins for a long time while staying in the same place, so it cannot be pushed by the air.
- In the case of a javelin with two sharp ends, the air cannot push the javelin because it is easily split by the rear end.
- A boat towed upstream cannot stop immediately; it continues to move for a long time. However, the helmsman feels the air drag from the front, not from the back. If the boat were loaded with hay or straw, we would see that the air is pushing it from behind.

Aristotle's Second Theory of Motion Aristotle prefers another explanation for the motion of a projectile. The air, when the projectile is thrown, is violently disturbed and can move the projectile and a new mass of air. The new mass of air moves the projectile again and another mass of air, the next mass of air moves the projectile, and so on.

However, Buridan points out that this still does not explain why blacksmith's grindstone spins when he sets it in motion and releases it[142]:

> ...if one were to cover the wheel completely with the help of a cloth that separates it from the ambient air, the wheel however would not cease to turn; it would continue for a long time to move; thus, it is not the air that moves it

A boat, which continues to move for some time after being pulled by a rope, is also not moved by the air. If the boat were

[142] (ibid.), p. 29

"covered with a tarp that one removes and, at the same time, removes the air that is contiguous to it, the boat would not stop for that".

Furthermore, air couldn't move heavy objects, if it makes so little resistance to them:

> ...however much the air moves, it remains easy to divide; it is therefore unclear how it could carry a stone of one thousand pounds launched by a slingshot or machine...

Moreover, Aristotle's theory would suggest that:

> ... you could throw a feather farther than a stone, and a body weighing less farther than a body of greater weight, their shapes and volumes being identical; however, we experience that this is false; and, the consequence following clearly from the principles, because the shaken air would support, carry, and move a feather more easily than a stone, a light body more easily than a heavy body[.]

With such examples, Buridan demonstrates that Aristotle's theory is insufficient for quite a lot of phenomena. Here is another interesting comment on mechanical waves, where Buridan refutes following doctrine of Averroes:

> By what air is it moved after what [the thing that] launched the projectile ceased to move it? To this question, the Commentator [Averroes] will answer that this air is driven by its levity, that it is in the nature of air to retain the motive force when it is shaken; thus, it is by this movement that sound, over time, propagates far away;

Buridan considers disturbance of air in analogy to waves on the water and concludes that such disturbance cannot move the projectile in any chosen direction. The levity is a property of upward motion, while a moving body can move in any direction. He further contemplates: was this levity already in the air, or did the person throwing the projectile impart this characteristic to the air? If it was already there, Buridan believes that the air should have the same driving force before and after the projectile is thrown (which is not consistent). But if not, then only option that remains is that the hand's movement throwing the projectile adds levity to the air. This would be rather strange, too.

Theory of Impetus From the above analysis it follows that air resists moving bodies and therefore cannot push them – in addition it's influence is too weak to lift large, heavy bodies. Buridan thus, having rejected opinion of Averroes concludes that the cause of motion is a characteristic of the stone itself. If, given Aristotle doctrine, we assume that the mover disturbs the air giving it the ability to carry the projectile, it is much better to assume that he gave it to the projectile, rather than air which seems to cause drag[143]:

> If, on the contrary, this lightness [levity] is (...) a new proper disposition for moving the air, which is impressed by the person who launches the projectile, we can and must also say that such a thing is imprinted on the stone or the thrown mobile, and that this thing is the virtue that moves this body; it is clear that it is better to make that assumption than to resort to air that would move the projectile; rather, indeed, the air seems to resist.

He follows up with his famous impetus theory:

> While the mover moves the mobile, it imprints on it a certain impetus, some power able to move this mobile in the same direction that the mover moves it, either upwards or downwards, or sideways, or circularly. The greater the speed that mover moves the mobile, the more powerful is the impetus that it imprints in it. It is this impetus which moves the stone after the one who throws it ceases to move it; but, by the air resistance, and also by gravity that inclines the stone to move in a

[143](Pierre Duhem and Aversa, *Galileo's Precursors: Translation of Studies on Leonardo da Vinci (vol. 3) by Pierre Duhem*), p. 29

> direction contrary to that which the impetus has power to move, this impetus weakens continuously;

It appears that impetus is combined like a vector. Buridan explains the effects of gravity through impetus as well[144]:

> This also seems to be the cause of why the natural fall of weights constantly accelerates. At the beginning of this fall, in fact, gravity moves only the body; it therefore falls more slowly; but, soon, this gravity impresses a certain impetus on the heavy body, an impetus which moves the body at the same time as gravity; the movement becomes faster; but the faster it becomes, the more the intense the impetus becomes; so, it can be seen that the movement will continually accelerate

Above is an attempt to include impetus in the description of the motion within the framework of Aristotelian physics. But what if I stop pushing, and the cart continues to move? Aristotelians say the cause is the motion of the air, while Buridan says it is the impetus. He correctly guesses the relationship between gravity and imparting impetus. Similarly, he also guesses that if a body contains more matter, the more impetus it can receive, and this is in *proportion* to the amount of matter. Therefore, a small piece of iron can be thrown farther than a piece of wood. A dense and heavy body receives "more intense impetus". In addition to that, Buridan notes that iron can receive more heat than the same amount of wood or water, seeing that all these changes are proportional to a single conserved quantity: an amount of raw matter in the body, which physicists now call "mass".

> We will say, for example: I can throw a stone farther than a feather and a piece of iron or lead that fits my hand farther than a piece of wood of the same size. I answer that the cause is the following: All forms and natural dispositions are received in the matter and in proportion to the [quantity of] matter; therefore, the

[144](Pierre Duhem and Aversa, *Galileo's Precursors: Translation of Studies on Leonardo da Vinci (vol. 3) by Pierre Duhem*), p. 31

more a body contains matter, the more it can receive this impetus, and the intensity is greater with which it can receive it; however, in a dense and heavy body, there is, all things being equal, in fact, more raw material than in a rarefied and light body; a dense and heavy body thus receives more of this impetus, and it receives it with more intensity [than a rarefied and light body]; similarly a certain amount of iron can receive more heat than an equal volume of water or wood. A feather receives so weak an impetus that it is destroyed immediately by air resistance. Similarly, if the one who launches projectiles moves with equal speed a light piece of wood and a heavy piece of iron, these two pieces also having the same volume and shape, the piece of iron will go farther because the impetus which is impressed in it is more intense.

Cosmology of Buridan Having developed his new theory, Buridan uses it to get rid of "divine intelligences", that Aristotelian philosophy conceived as the movers of the heavens, and that were prohibited by 1277 condemnation. He more or less correctly states that celestial bodies, moving without friction or resistance require no movers, but it is sufficient for God to impress a quantity of motion once in the past to keep them going for very long time.

It is not in the Bible that there are the intelligences responsible for communicating to the celestial orbs their own movement; it is therefore permissible to show that there is no need to assume the existence of such intelligences. You could say, in fact, that God, when he created the world, has moved as it pleased him each of the celestial orbs; he has impressed on each of them an impetus that has moved it since then; so that God no longer has to move these orbs, if not in exercizing a general influence, similar to that by which he gives support to all actions that occur; he could also rest on the seventh day from the work he had completed by entrusting mutual actions and passions to created things. These impetus, which God has impressed on celestial

> bodies, are not weakened or destroyed at a later time because there was, in these heavenly bodies, no inclination to other movements, and there was no longer any resistance that could corrupt and suppress these impetus

Similarly to the case of John Philoponus, the image of rational Creator and comprehensible creation, gets us picture of world that is very close to modern physics, getting rid of animate, organic nature of Aristotle and other pagans.

3.5 Place, the Plurality of Worlds, and Gravity

Place in Aristotle's Physics In Aristotle's physics, to conceive a motion, a stationary *place* is necessary[145], since rectilinear motion is a change in the location of a body. Place, as we have indicated, is the material surface surrounding the body (e.g., a layer of air, or water in the case of a floating body), according to Aristotle's idea to define all terms by existing objects. This definition allows for an elegant description of various movements and changes, demonstrating the generality of Aristotle's system. A person raising their hand, an inflating balloon, a tensing bow, a potter's wheel spinning, water leaking from a bucket—all are all examples of motion. The layer of air that surrounds a runner or a potter wheel or leaking water is precisely the place. When the motion occurs, the place is continuously replacemed by a new place.

On Earth, the definition works well; above Earth, a problem arises: the sphere of fixed stars, the outermost sphere of the universe (composed, according to medieval philosophers, of concentric celestial spheres), has no place because nothing surrounds it. Simultaneously, it moves, so it must be capable of motion. The motion is rotational, so it was initially assumed that the sphere is capable only of such motion. Aristotle therefore considered the material center of the spheres, that is, the Earth, to be the place of the celestial spheres, making an exception to his theory of place.

[145](P. Duhem and Ariew, *Medieval Cosmology: Theories of Infinity, Place, Time, Void, and the Plurality of Worlds*), p. 139

There were a few other proposals to improve this situation. Themistius and Al-Farabi or Ibn Bajja[146] maintained that the outer sphere of the universe has a place not from the outside but from the inside. The place is the layer of matter filling it, like the sleeve in a sliding bearing. Another approach assumes that beyond the sphere of fixed stars, there is another sphere that is immovable. Christian theologians (St. Bede, St. Anselm)[147] preferred this solution, as in their view, the heavens should not be suspended on Earth, as they are more noble and perfect than the Earth. In this case, the sphere of fixed stars has a place on the outside, which allows for rotational motion. None of the mentioned solutions permit the entire universe to move in a straight line, according to Aristotle's definition. For a long time, this was not a problem because there was no reason to suspect that the universe could move. This changed with the decree of 1277, which condemned the assertion that God could not move the universe in a straight line.

We have already seen the conclusions of Nicholas Bonet (p. 64): the Scholastics devised that there is no need for any material place to imagine motion; it is enough to introduce an imaginary point or surface relative to which motion or rest occurs. The first to suggest such a theory was John Bassols, continuing the earlier efforts of the Scotist school[148],

> In effect, the mathematician, with a view to the exposition of science and without pretending that it is so in reality imagines a line drawn from one part of heaven to another, passing through the center of the world which is in itself an imagined point. This line terminating in one part of heaven and the other, receives the name axis of the world; its extremities, or in other words, the points terminating it, are called the poles of the world. They are merely points that one imagines in heaven.It is with respect to such poles and such a center that place is said to be immobile.

Thus, the celestial sphere rotates around an imagined axis, and similarly, it can move in a straight line relative to an imagined

[146] (ibid.), p. 141
[147] (ibid.), p. 174
[148] (ibid.), p. 207

point. The reasoning leading to the above conclusion can be outlined with such an example: suppose a boat is anchored in the riverbed. The boat does not move, but it is continuously surrounded by masses of water, due to the current. The place is a certain layer of water and air surrounding the boat: but the place is not stationary; it is created by ever-changing masses of water, due to the current. So how can the boat have a stationary place to define its motion or rest? The place must not be a layer of water but rather a relation of equivalence to successive layers of water (or rather successive layers of anything). As a result, we get a place as an abstraction: a geometric surface surrounding the body. This doctrine was further developed by the Terminists. Abstract place that doesn't exist in reality is however another breach in Aristotle's theory of natural motion. In this theory, place acts on the body as a goal of the tendency of motion, but this is inconsistent, if the place is an abstraction, not a simple layer of matter.

Multiplicity of Worlds and Multiplicity of Planets. The effects of gravity were explained by Aristotelians through elements and the natures of these elements. The element of earth has its natural place at the center of the world, which is why a dropped stone falls and sinks. According to this hypothesis, the center of our planet is an absolutely distinguished point, around which all the stones and rocks of the universe gather.

This leads to several conclusions which today seem rather artificial. Firstly, many universes cannot exist. Earthly matter, having its natural place in one world, cannot have another natural place elsewhere, because then it would be no answer in which direction the matter should move. For a similar reason, there cannot be many planets similar to Earth; in such a case, there would be many natural places simultaneously[149]. Other planets of the Solar System were not recognized as planets by the ancients and there were no reasons for this. Observed with the naked eye, they appear as bright points similar to stars, hence they were called wandering stars. According to Aristotelians, they are made of a fifth kind of matter, called ether.

[149](P. Duhem and Ariew, *Medieval Cosmology: Theories of Infinity, Place, Time, Void, and the Plurality of Worlds*), p. 441

It is easy (for us) to come up with a simple objection: we can conceive many natural places, if a body would go to the one that is closest to it. Duhem notes that Aristotle considered similar a possibility. He asked: does the weight of a massive body change with distance from the center of the world? Averroes noticed that this question contains another: "Is weight the result of the attraction of similar elements (as believed by the Pythagoreans and Plato in "Timaeus")", or not?[150] The answer, according to Aristotle's system, is: no. If it were so, then greater distance from the natural place would change the nature of the body[151]. Saint Albert the Great wrote about this as follows:

> ...when an element is created, that which creates it gives it not only form, but everything that results from this form; it gives it, especially, natural motion and natural place, which is a consequence of the internal form. If proximity or distance from the natural place were to influence the substantial form of the element, the element would be composed of two forms having opposite properties; one of these forms would pull the body towards something nearest. This form would emanate from the attracting body similarly to the form that a magnet produces in iron.

In clearer terms, if something is the nature of an element, such as the nature of gold is to be dark yellow and shiny, we do not expect that something to change with position. Then it is not the nature of the element itself, but a property of a system of two or more bodies. Magnetism, as well as gravity in Newton's understanding, are indeed such mutual properties.

Attempts to reconcile the impossibility of multiple universes with the belief in the omnipotence of God date back to the early 13th century, when the works of Averroes were translated. Before 1277, among the doctors in Paris and Oxford, the prevailing opinion was that God could not create multiple universes[152]. After 1277, most scholars adopted the opposite view in accordance with the decree of Tempier, which required adjustments to the

[150] (ibid.), p. 446
[151] (ibid.), p. 447
[152] (ibid.), p. 455

theory of natural motion. The opinion of Godfrey of Fontaine can be summarized as follows: in every newly created universe, even one identical to ours, all elements would have their own natural places (e.g., their own center, towards which the earthly element would tend). Natural places in our world would have no influence on those worlds. This opinion is also echoed by William of Varon, John Bassols, and Thomas of Strasbourg. Averroists (e.g., John of Jandun) responded to this idea as follows: if the earth from one world has its natural place at the center of that world, and the earth from another world has its natural place in the center of another world, it means that the substantial forms are not identical[153]. Therefore, it is not possible to create another identical universe. This means, there can be another universe, which looks exactly identical in terms of the results of all experiments that can be conducted in it, but it will not be the same universe. Note that this in turn means that the problem exists only within the system of Aristotle, and not in visible reality.

This objection by the Averroists was undermined by William of Ockham[154], who at the same time created a very innovative development of the theory of natural motion. Fire and air in Aristotle's theory have natural places in the form of a sphere above the Earth – meaning, they have many natural places. However, they always tend towards one place, moving upwards. Therefore, an element can have many natural places and tend only towards one of them: as happens with fire and air, and so can also happen with earthly element. A body with a given substantial form can thus tend to different natural places depending on its position. Therefore, in a second universe, there can exist an Earth that is substantially identical, having at the same time a natural place at the center of the second universe.

Ockham also refutes the objection that a body cannot move away from one natural place, approaching another. This is exactly what fire does when rising upwards. It then approaches its natural place above the Earth, and moves away from another natural place on the opposite side of the Earth. The same should be generalized to the earthly element. This also means that Aristotle's theory of

[153](P. Duhem and Ariew, *Medieval Cosmology: Theories of Infinity, Place, Time, Void, and the Plurality of Worlds*), p. 460
[154](ibid.), p. 463

gravity is internally inconsistent, which is another nail in its coffin. The critique of Ockham is repeated by Robert Holkot[155]. Buridan, however, ignores it, and Albert of Saxony prefers to defend the Aristotelian position against Ockham: he claims that the sphere of fire, which fire tends towards, is still one place, because it is connected as a whole. Fiery matter thus tends to one place only by different paths. Interestingly, Albert, adopting the doctrine of Aristotle, cannot acknowledge the impossibility, due to the decree of 1277. This becomes the basis for noteworthy considerations[156]:

> Following Aristotle's doctrine, we conclude that the existence of several nonconcentric worlds is impossible naturally. It is no less true that God could create many worlds, since He is omnipotent. A last conclusion in accord with the preceding conclusions: By supernatural means, there can exist several worlds, simultaneous or successive, concentric or eccentric.

This doctrine was earlier stated by the zealous Averroist John of Jandun[157].

> All that does not say anything about divine power; one always safeguards its infinite freedom and infinite power to create several worlds, even though this reasoning cannot be derived from sensible things; and Aristotle derives his reasoning from sensible things.

Aristotle, therefore, reasons only "from sensible things" – he reasons postulating certain convention. Consequently, what Aristotle calls impossible, is not necessarily impossible. Thus, if his statement "there cannot be many worlds" is interpreted in light of the statement "God can create many worlds", and moreover "God created the world", it becomes meaningless, as clearly we can't know "other worlds" from experience, by the definition.

The effects of the decree of 1277 on the possibility of the existence of many worlds are as follows: critics discover serious problems in Aristotle's theory, and Aristotelians are forced to acknowledge their theory as limited to beliefs that can be deduced from

[155] (ibid.), p. 466
[156] (ibid.), p. 470
[157] (ibid.), p. 462

sensory things. The latter is more important than the former. For you do not know in advance what can be deduced from sensory things and what cannot: as a result, any conclusion of Aristotle becomes a hypothesis based on certain conventions. Aristotle's physics can no longer decree necessity and impossibility because nothing is necessary or impossible when the judgment itself is not certain. Thus, by Tempier's decree the old physics transitions into the new in its method, still retaining main theories of Aristotle's physics.

Nicole Oresme will take advantage of the new freedom to create hypotheses, from the critique of Ockham and older ideas, to develop a very innovative theory, upon which Nicolaus Copernicus and his successors will build. Here is what he writes[158]:

> Imagine a portion of the fiery element right at the center of our world, such that half of this portion lies on one side of the center, and the other half on the other side. Let a be the center, b one half, and c the other half. I assume that everything that could hinder the natural movement of fire is removed. Each part will tend towards opposite sides of the sphere surrounding Earth, separating from each other. But if these two parts of fire are joined in a sphere, such that they cannot be separated from each other and in the absence of other obstacles, this small sphere, or portion of fire will not move, because it will have no reason to move more in one direction than in another.

The law of natural motion must be preserved for both parts; neither can be distinguished, since Aristotle's physics does not speak of distinguished directions. This is an argument from the symmetry, similar can be found in physics to this day. Oresme notes: "This is fully consistent with the philosophy of Aristotle".

Let us recall the objection of Albert of Saxony against Ockham: that the entire sphere of fire is one natural place. The objection is not enough to protect against the problems indicated by Ockham. For the sphere of fire considered on its own we have the same

[158] (P. Duhem and Ariew, *Medieval Cosmology: Theories of Infinity, Place, Time, Void, and the Plurality of Worlds*), p. 473

problem as for two worlds and two natural places of the earthly element. Consequently, the example of two worlds is analogous to what is possible for the fiery element[159]:

> In like manner, one can say that, if a portion of earth were equidistant between two worlds and if it can be separated, one part would go to the center of one world and the other portion to the center of the other world. If the portion cannot be divided, it would not move at all because of the lack of inclination, being like a piece of iron halfway between two magnets of equal strength.

The magnet analogy refers to the earlier scholastic opinion[160], that weight results from the attraction of bodies by the center of the world, just as a magnet attracts pieces of iron. This allows conception of natural motion as an effect of an action at distance and the action at distance as a superposition of interactions of parts of the system. For example: the attraction of a piece of iron between two magnets cancels out to zero, whereas the attraction of a piece of iron by two magnets next to each other, with aligned poles, is stronger. Similarly, the sphere of fire, the natural place of the fiery element, can be divided into parts and considered separately: just as today we calculate gravitational interactions of masses. Aristotle and Averroes (and with them Albert) would not agree with this: for them, the movement of a stone results from the nature of the stone, just as a dog's search for food results from the dog's nature (that makes it feel hungry, smell food and strive to get it). Similarly the sphere of fire is considered by them as a whole, what matters is approaching the whole, not approaching or moving away from elements, but Oresme reasons quite differently. In addition, Oresme notes that the above-described state of equilibrium of the fiery element portion at the center of the world cannot last, since it is an unstable equilibrium; similarly, he says, a heavy sword cannot stand straight on the tip of its blade even for a moment.

Taking all this into account along with the earlier deconstruction of the concept of place, we can expect the elimination of the

[159] (ibid.), p. 474
[160] (ibid.), p. 471

concept of natural distinguished place, which in Aristotle's theory causes natural motion. This is exactly what Oresme did[161]. A natural, immovable place is, in his opinion, unnecessary: heavy bodies move downward, and light bodies upward, relatively to each other. Planet Earth is therefore a sphere of heavy matter surrounded by layers of lighter matter. The earthly element is in the center, water, air, and fire on the outside, in accordance with the general law that massive bodies move closer to each other.

> I say that a heavy body to which no light body is attached would not move of itself; for in such a place as that in which this heavy body is resting, there would be neither up nor down because, in this case, the natural law stated above would not operate and, consequently there would be not be any up or down in that place

A similar claim was proposed much earlier by Plato in "Timaeus" and taken up by Plutarch, but only Oresme had the opportunity to develop it, refuting the theses of Aristotle's system[162]:

> But I still doubt, and I imagine the case of a tile or copper pipe or other material so long that it reaches from the center of the earth to the upper limit of the region of the elements, that is, up to heaven itself. I say that, if this tile were filled with fire except for a small amount of air at the very top, the air would drop down to the center of the earth for the reason that the less light body descends beneath the lighter body. And if the tile were full of water save for a small quantity of air near the center of the earth, the air would move upward to heaven, because by nature air always moves upward in water. From these examples it appears that air can, by reason of its nature, descend and move upward to the distance of the semidiameter of the sphere of the elements.

This is a problem for Aristotle's physics because the air turns out to have two different natural movements, in opposite directions.

[161] (P. Duhem and Ariew, *Medieval Cosmology: Theories of Infinity, Place, Time, Void, and the Plurality of Worlds*), pp. 476-477
[162] (ibid.), p. 478

From this thought experiment, it follows that natural motion is not, in principle, in the direction of a fixed place, like the sphere of air or the center of the Earth, but its direction results from the distribution of matter. If, for example, there were a planet composed entirely of air (without the elemental water and earth) there would be no sphere of air, only a round cloud of air.

We mentioned that Aristotle's theory of natural motion dominated physics for 1500 years, providing an elegant description of the phenomena of gravity, hydrostatics, and aerostatics. Oresme was the first to provide an alternative theory that is just as good in every respect, while solving few relevant problems and bringing us closer to the understanding of gravity and the separation of material point and rigid body dynamics from fluid dynamics. Cosmological consequences are very spectacular, too. The Earth, detached from a distinguished point in space, can be mobile.

3.6 The Law of Free Fall

When neglecting air resistance, massive bodies fall with uniformly accelerated motion. Understanding this fact was a crucial step toward the discovery of Newtonian dynamics. First source of this law was work of a Dominican named Domingo de Soto in 1551[163]. He did not present it as his own discovery, but mentioned it as if it were widely known. Probably from de Soto, Galileo learned this law[164], says Wallace, as it explains other influences on Galileo.

The mathematical description of motion under acceleration was known almost 200 years earlier, hence it is as old as the mentioned quantitative forms discovered in the 14th century, likely during the time of Buridan[165]. It was presented in writing by Nicole Oresme (in the 14th century), along with a geometric proof of correctness, which later would be used by Galileo. The Canon of Rouen proves to be a great innovator in mathematics and physics. While previously philosopher considered simple proportions[166], Oresme

[163](Wallace, *Domingo de Soto and the Early Galileo: Essays on Intellectual History*), p. 119

[164](Wallace, "Duhem and Koyré on Domingo de Soto")

[165](Pierre Duhem and Aversa, *Galileo's Precursors: Translation of Studies on Leonardo da Vinci (vol. 3) by Pierre Duhem*), p. xvi

[166](Babb, "Mathematical Concepts and Proofs from Nicole Oresme: Using the History of Calculus to Teach Mathematics"), p. 4

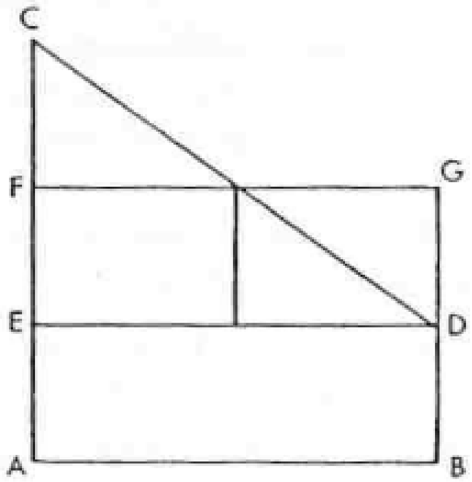

Figure 1: Geometric proof concerning distance travelled in accelerated motion.

examined (and applied) powers with rational exponents. He also calculated the sum of a geometric series and proved that the harmonic series is divergent. Oresme expressed quantitative intensities of forms in a geometrical way, on graphs, creating what we know as the Cartesian coordinate system. He also discovered the fact that the area under the speed curve represents the distance traveled[167], meaning, he discovered a primitive form of the integral and applied it to physics. In this way, Oresme proved the law that the distance in uniformly accelerated motion equals half the final velocity multiplied by the time of motion (or the average velocity times the time of motion). In Figure 3.6, we see his original drawing[168].

The area of the trapezoid ABDC is equal to the area of ABGF. This implies that the distance covered in accelerated motion is equal to the distance traveled at the average speed of that motion,

[167](Pierre Duhem and Aversa, *Galileo's Precursors: Translation of Studies on Leonardo da Vinci (vol. 3) by Pierre Duhem*), p. 284 – however, he did not provide a proof, probably considering it obvious.

[168](Mumford, *Course notes from "Math for non-math majors" on Brown University*), p. 7

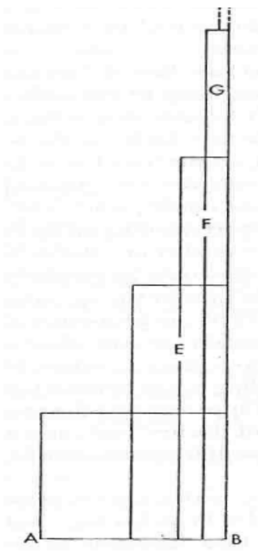

Figure 2: Oresme examines an improper integral.

over the same period of time. Oresme applies the same principle to any intensity of form (i.e. physical quantity)[169].

These types of analyses go beyond kinematics. Here is another interesting demonstration[170] in Figure 2: Each subsequent rectangle is one unit taller and half as wide, which can be expressed as a sum: $1 + \frac{1}{2} + \frac{1}{4} + ...$ This is the sum of a geometric series and it totals 2, which Oresme proves by folding the above rectangles into one. This sum is needed by Oresme to calculate an improper integral, which in modern notation would likely be $\int_{-1}^{0} \log_2 \frac{-1}{x} + \frac{1}{2} dx$. If we rotate this graph by 90 degrees, we get a bar chart of the function $x = 2^{-y+0.5}$. Expanding with respect to y, we get $y = \log_2 \frac{-1}{x} + \frac{1}{2}$. Computing the integral numerically yields 1.94, 0.06 less than Oresme's result.

A similar "improper integral" (or series) Oresme calculates

169(Pierre Duhem and Aversa, *Galileo's Precursors: Translation of Studies on Leonardo da Vinci (vol. 3) by Pierre Duhem*), p. 284

170(Mumford, *Course notes from "Math for non-math majors" on Brown University*), p. 7

in application to kinematics[171], demonstrating the possibility of dealing with some problems of non-uniformly accelerated motion. Imagine a motion over the course of 1 hour. For half the time, the body moved at velocity v, then for $\frac{1}{4}$ of the time at $2v$, then for $\frac{1}{8}$ of the time at $3v$, then for $\frac{1}{8}$ of the time at $4v$, and so on, continually increasing speed. Oresme shows that during the first half hour, the body covers a distance three times smaller than in the second half.

Let's refer here to Oresme's discussion about the construction of these graphs for any kind of quantity [172], which is an extension of the doctrine on the quantitative intensity of forms from Section 3.2:

> With the exception of numbers,any measurable thing has to be imagined in the manner of a continuous quantity. To measure it, one must imagine points, surfaces, lines; in the opinion of Aristotle, in fact, these objects are where the measure or proportion meet immediately; in other objects, the measure or proportion is known only by analogy, insofar as the reason compares these objects to those... So, any intensity that may be acquired in a successive manner should be imagined as a straight vertical line from each point of the space or subject that affects this intensity... Whatever the proportion between two intensities of the same species might be, a similar proportion must be found between the corresponding lines and vice versa.

This means that with a given variable (intensity, magnitude) we draw a bar chart, relative to another continuous value (e.g., time interval). In this way, on the vertical axis, we have the intensity of the form, and on the horizontal, the parameter. The proportions of the bar heights correspond to the proportions of the intensity of the same form.

> [The various intensities of a quality of a given species can therefore be imagined as straight lengths]; they can

[171] (Pierre Duhem and Aversa, *Galileo's Precursors: Translation of Studies on Leonardo da Vinci (vol. 3) by Pierre Duhem*), p. 284
[172] (ibid.), pp. 273-274

> especially, and in the most appropriate way, be represented by straight lines attached to the subject and vertically raised from its various points. The consideration of these lines helps and naturally leads to the knowledge of each intensity. ... Equal intensities are represented by equal lines, double intensities by lines where one is double the other, and so on, the intensities and lines always following the same ratio. And this representation extends, in a universal manner, to any imaginable intensity

Thus we saw the emergence of the geometric apparatus that would be crucial for physics until the end of the 17th century. The most famous work based on geometric proofs is Newton's renowned "Mathematical Principles of Natural Philosophy, book that introduced Newtonian theories of dynamics and gravity.

3.7 The Order of the World According to Newton

Until now, in support of the thesis about the existence of an understandable order in the world, I have quoted excerpts from Buridan, Oresme, and a few other scholastics, as well as passages from the Bible. Rarely it was it as emphasized as by our next author, Isaac Newton, who at the end of "The Mathematical Principles of Natural Philosophy" (Principia) included an essay "General Scholium". Newton recognizes the classification of observations along with the understanding of the final cause as the subject of his natural philosophy, while setting aside efficient causes. This gives a scientific method similar to the one we described in the beginning, which allows Newton to be counted among its precursors.

Here is what he writes[173]:

> the planets and comets will constantly pursue their revolutions in orbits given in kind and position, according to the laws above explained; but though these bodies may, indeed, persevere in their orbits by the

[173] (Newton, *Mathematical Principles of Natural Philosophy, translated by A. Motte*), pp. 960-961

> mere laws of gravity, yet they could by no means have at first derived the regular position of the orbits themselves from those laws. The six primary planets are revolved about the sun in circles concentric with the sun, and with motions directed towards the same parts, and almost in the same plane. Ten moons are revolved about the earth, Jupiter and Saturn, in circles concentric with them, with the same direction of motion, and nearly in the planes of the orbits of those planets; but it is not to be conceived that mere mechanical causes could give birth to so many regular motions, since the comets range over all parts of the heavens in very eccentric orbits; for by that kind of motion they pass easily through the orbs of the planets, and with great rapidity; and in their aphelions, where they move the slowest, and are detained the longest, they recede to the greatest distances from each other, and thence suffer the least disturbance from their mutual attractions.

Firstly, Newton seems to continue his critique of Descartes' theory, which he started earlier. Descartes' theory of vortices was intended to provide mechanical explanations for Copernicus' system, where planets move in circles with epicycles[174]. Thus, a large vortex around the Sun was meant to carry all the planets. In the orbits of these planets, there are smaller vortices, which correspond to the epicycles. Among other problems, Newton points out that it is impossible to explain how comets could pass through the system at high speed in such a vortex system (while his theory of uniform attraction can explain that by postulating interplanetary void).

Secondly, Newton discusses causes: mechanical (efficient) causes are not sufficient to explain the configuration of the Solar System (which also interested Descartes in his mechanical philosophy). Newton provided the laws of motion and gravity, but besides the laws, initial conditions (masses, positions, and velocities of planets, etc.) are necessary to describe the system. Newton's laws can thus describe many possible systems: some regular, others chaotic (e.g., the system of 3 equal bodies). For some initial conditions, planets

[174] small circles, used by Copernicus to approximate elliptical planet orbits by superposition of circles

will quickly crash into each other in a cosmic catastrophe, while for many others they simply drift away into interstellar void. He concludes as follows:

> This most beautiful System of the Sun, Planets, and Comets, could only proceed from the counsel and dominion of an intelligent and powerful being. And if the fixed Stars are the centers of other like systems, these, being form'd by the like wise counsel, must be all subject to the dominion of One; especially since the light of the fixed Stars is of the same nature with the light of the Sun, and from every system light passes into all the other systems. And lest the systems of the fixed Stars should, by their gravity, fall on each other mutually, he hath placed those Systems at immense distances from one another.
>
> This Being governs all things, not as the soul of the world, but as Lord over all: And on account of his dominion he is wont to be called Lord God Pantokrator, or Universal Ruler.

It appears to be a treatise on theology: but the title "General Scholium" and the rest of the essay's content point at the method of science, not at theology, and it is also not difficult for us to understand the argument, in light of earlier chapters. The world is orderly in an understandable and intricate way. The parameters seem intelligently fine-tuned: if stellar systems were close to each other, everything could quickly coalesce into one mass thanks to the influence of gravity. The fact that starlight from distant cosmos is the same suggests that the laws of physics are the same everywhere in the Universe. God governs everything "not as the soul of the world"—as Greek philosophies believed, but as ruling over the world from beyond the world. Only that could explain world that is ordered, yet not organic and animate (compare to the opinion of the pagans on p. 42). There must be other source for that order, as blind mechanical causes know no plans or goals. Newton writes further as follows[175]:

[175](Newton, *Mathematical Principles of Natural Philosophy, translated by A. Motte*), pp. 963-964

> We know him only by his most wise and excellent contrivances of things, and final causes: we admire him for his perfections; but we reverence and adore him on account of his dominion: for we adore him as his servants; and a god without dominion, providence, and final causes, is nothing else but Fate and Nature. Blind metaphysical necessity, which is certainly the same always and every where, could produce no variety of things. All that diversity of natural things which we find suited to different times and places could arise from nothing but the ideas and will of a Being necessarily existing. (...) And thus much concerning God; to discourse of whom from the appearances of things, does certainly belong to Natural Philosophy.

The universe has final causes, but differently than in Aristotle's view. On one hand, there is a great diversity of created things, which indicates the creativity and intelligence of the creative process. Such a plan realizes one of countless possible configurations: even a change in parameters such as the speed of light, or the distribution of stars and planets, would yield a different universe. On the other hand, there is also a kind of purposive order—the universe, for Newton, is beautifully intricate. This view of the world imposes a specific strategy for science: the scientist can expect a constant, universal order of the world. That "General Scholium" is a methodological essay is further demonstrated by the next paragraph. For, having laid out the elements of Thesis 1.1, Newton immediately moves to the thesis about the contingency of explanations and causes[176]:

> Hitherto we have explained the phenomena of the heavens and of our sea by the power of gravity, but have not yet assigned the cause of this power. This is certain, that it must proceed from a cause that penetrates to the very centres of the sun and planets, without suffering the least diminution of its force; that operates not according to the quantity of the surfaces of the particles upon which it acts (as mechanical causes use to do),

[176](Newton, *Mathematical Principles of Natural Philosophy, translated by A. Motte*), p. 964

> but according to the quantity of the solid matter which they contain, and propagates its virtue on all sides to immense distances, decreasing always in the duplicate proportion of the distances.(...) But hitherto I have not been able to discover the cause of those properties of gravity from phænomena, and I frame no hypotheses; for whatever is not deduced from the phænomena is to be called an hypothesis; and hypotheses, whether metaphysical or physical, whether of occult qualities or mechanical, have no place in experimental philosophy. In this philosophy particular propositions are inferred from the phenomena, and afterwards rendered general by induction. Thus it was that the impenetrability, the mobility, and the impulsive force of bodies, and the laws of motion and of gravitation, were discovered. And to us it is enough that gravity does really exist, and act according to the laws which we have explained, and abundantly serves to account for all the motions of the celestial bodies, and of our sea.

Newton separates what we know about gravity—the classificatory part of the theory—from what we don't know: the efficient causes of gravity. We know the *properties*: a force acting at a distance, proportional to the mass size and inversely proportional to the distance. What we cannot similarly demonstrate has no place in experimental philosophy.

Newton speaks of deducing from phenomena and "induction," which has been variously interpreted. One view is that Newton applies induction according to Francis Bacon's recipe, but there is no trace of this in the "Principia." Others, like Duhem, argued that Newton's method is hypothetico-deductive: he postulated a theory that reproduced known facts and tested it through precise predictions. This appears closer to the truth.

But why the word "induction"? In the context of the aforementioned essay, it is clear that reasoning about induction similar to Aristotle's (whom Newton undoubtedly studied) fits very well here; even the discussion of final and efficient causes. We've seen that Aristotle started from the problem of what kind of method can gain constant and certain knowledge about the world. The world changes, but at the same time, there's undeniably certain organization: the question is what it is, and how to understand the essence of that. Aristotle says it is a purposive order: humans, animals, stones, Earth, etc., have constant principles of transformation (natures), and we can understand how they work through observation: our senses indeed uncover the basic units of this organization (forms) and the relations between objects. We must first look broadly and outline the project, in order to start building it in detail. With Newton, it's similar, but with some changes: the final cause is no longer the "soul of the world." The world is ordered like a clock set in motion, according to once established principles. Efficient causes are not accessible, but we can know order among measured quantities that allows us to make precise predictions. We saw that Einstein aptly wrote that[177] the success of such endeavor as Newtonian physics presupposes very specific ordering of the world. And indeed, most probably Newton knew very well what he was looking for, having before his eyes God who made all things "according to measure, number and weight" and also the success of doctrines that started with 14th century scholasticism. That this is the only way to understand induction is even corroborated, by opinion of certain positivist philosophers like Reichenbach. Even this movement, hostile to metaphysics and religion, cannot find justification for induction without the reference to objective order of phenomena[178], induction can find the order of the world, provided that this order, in the given form, exists. The key issue, therefore, is whether it exists, and what kind it is. Thus to know that God ordered the universe by the constant relations among measured quantities was of prime importance.

3.8 Euler and Maupertuis' Variational Principle

In a similarly favorable tone, Euler and Maupertuis discuss the ordering of the world as a final cause on the occasion of one of the most astonishing discoveries in physics: the variational principle, a foundational construct of almost all modern theories of physics.

According to the basic classical dynamics to apply Newton's

[177] (Albert Einstein, *A. Einstein, Letters to Solovine, translated by Wade Baskin, with an introduction by Maurice Solovine, p. 132-133.*)
[178] (Henderson, "The Problem of Induction"), sec. 5.3

laws, we consider the system at a moment in time: to calculate the motion of an asteroid around the Sun, at each point of the path of this motion, we compute the acceleration vector a: $m\vec{a}(\vec{r}) = \vec{F}_S + \vec{F}_j + \ldots$ where m is mass of the asteroid, successive terms on the right are influences of the gravitational fields of the Sun $\vec{F}_S$, Jupiter $\vec{F}_j$, and other relevant bodies. The path of motion results from this very quantity calculated at each subsequent moment. It is not as if the body goes "somewhere", as in the case of Aristotle's natural place: the instantaneous state contains all relevant information.

It turns out that there is another way to approach that. Imagine that our asteroid follows a trajectory in space from point A to point B. What this trajectory should be, as a whole? We can imagine many random paths from A to B, some of them straight, some of them curved. Some of the imagined paths will be actual paths of physical motion, others would not. Is there something that distinguishes the actual path of motion from an imagined one—a special feature that the nature "prefers"? It turns out that there is such thing: we can integrate a certain scalar function (called the action) along this path of motion, and if the path is real, physical path then the result will be an extremal quantity. This is the principle of least action, or the variational principle. In different theories different extremal quantities are used. In classical mechanics, it is the difference between kinetic and potential energy—such a definition allows to derive Newton's dynamic. But the above principle also works for other theories: according to Fermat's principle a light ray moves along a path such that the travel time is shortest. A similar principle was discovered in General Relativity (the paths of freely moving bodies, temporal geodesics, are such that the passage of proper time is locally longest), and also in many other branches of modern physics; including but not limited to whole formulation of mentioned theories. The latter appears to be a great surprise and mystery. Interestingly, Euler nearly predicted such a turn of events in 1744[179].

> All the greatest mathematicians have long since recognized that the method presented in this book is not

[179](Oldfather, Ellis, and D. M. Brown, "Leonhard Euler's Elastic Curves"), p. 10

> only extremely useful in analysis, but that it also contributes greatly to the solution of physical problems. *For since the fabric of the universe is most perfect, and is the work of a most wise Creator, nothing whatsoever takes place in the universe in which some relation of maximum and minimum does not appear.* Wherefore there is absolutely no doubt that every effect in the universe can be explained as satisfactorily from final causes., by the aid of the method of maxima and minima, as it can from the effective causes themselves. *Now there exist on every hand such notable instances of this fact, that, in order to prove its truth, we have no need at all of a number of examples; nay rather one's task should be this, namely, in any field of Natural Science whatsoever to study that quantity which takes on a maximum or a minimum value, an occupation that seems to belong to philosophy rather than to mathematics.* (....) But one ought to make a special effort to see that both ways of approach to the solution of the problem be laid open; for thus not only is one solution greatly strengthened by the other, but, more than that, from the agreement between the two solutions we secure the very highest satisfaction. Thus the curvature of a rope or of a chain in suspension has been discovered by both methods; first, a priori, from the attractions of gravity; and second, by the method of maxima and minima, since it was recognized that a rope of that kind ought to assume a curvature whose center of gravity was at the lowest point. Similarly, the curvature of rays passing through a transparent medium of varying density has been determined both a priori, and also from the principle that they ought to arrive at a given point in the shortest time.

Similar opinion can be attributed to Maupertuis, who was first to formulate variational principle[180]:

> The laws of movement and of rest deduced from this

[180](David, *Idle Theory, cit. Maupertuis. Oeuvres.*)

> principle being precisely the same as those observed in nature, we can admire the application of it to all phenomena. The movement of animals, the vegetative growth of plants ... are only its consequences; and the spectacle of the universe becomes so much the grander, so much more beautiful, the worthier of its Author, when one knows that a small number of laws, most wisely established, suffice for all movements.

3.9 Science, Religion and the Enlightenment

3.9.1 Science Did Not Come from the Enlightenment

Few developments shaped the 19th century as strongly as the emergence of rigorous mathematical physics, electromagnetic theory, and modern chemistry. Despite the dominant atheistic and irreligious philosophy of the Enlightenment, the founders of these fields, such as Cauchy, Ampere, Faraday, Galvani, Volta, Fresnel, Maxwell, Lavoisier, Dalton, Riemann, and a few others, were overwhelmingly religious. These scientists were notably overrepresented among the pioneers of new, fruitful fields of research, while the established scientific orthodoxy remained preoccupied with declining research programs. Galvani and Volta pioneered the experimental study of electricity. Ampere gave a rigorous mathematical theory of electric current, while his friend Cauchy emerged as the founder of modern mathematical physics. Lavoisier established modern quantitative experimental chemistry, while Faraday discovered electromagnetic induction and distinguished himself as one of the most outstanding inventors and electrical engineers.

The scientific revolution of the turn of the 18th and 19th centuries was built by believers, not freethinking *philosophes*[181] of the Enlightenment. Moreover, the scientific productivity of the latter group is quite low and often oriented in the wrong direction. The chief reason for this appears to be that the *philosophes*, while always eager to appeal to "Reason" and "Progress" in the abstract, saw very little possibility for the development of physics beyond Newton's theory (which they deemed absolutely certain and valid)

[181] Anti-religious, left-wing intellectuals in 18th-century France, lit. fr. "philosophers"

and very little need for further experimentation in physics (as only physical theory they cared about was already absolutely certain). Furthermore, they saw the method of physics as applicable to all kinds of phenomena, including human affairs. Christian religion, in addition to the influences already discussed, gave scientists freedom from these mistakes.

Some more direct theological influences can be found, too. For example, the method of Andre Ampere in his discovery of Ampere's Law[182] accurately reproduces the method we presented in the beginning, as he seeks to classify measured quantities with the use of equations. Ampere had very strong personal devotion in his entire adult life[183]. Similar quantitative and experimental rigorization was introduced to chemistry by the efforts of Antoine Lavoisier[184]:

> He was the first to explain definitely, the formation of acids and salts, to enunciate the principle of conservation as set forth by chemical equations, to develop quantitative analysis, gas analysis, and calorimetry...

As a royal civil servant, Lavoisier ended his life having his head cut off by a revolutionary tribunal (the latter declaring that "The Republic needs no scientists," according to an anecdote, and Joseph Lagrange saying that there wouldn't be another such head in France for a century to come). His personal piety is briefly indicated in his private letters:

> To Edward King, an English author who had sent him a controversial work, he wrote, 'You have done a noble thing in upholding revelation and the authenticity of the Holy Scripture, and it is remarkable that you are using for the defence precisely the same weapons which were once used for the attack.'

In Cauchy's case, who was not only pious but also a vocal champion for the Catholic cause, similar influences are very explicit, and we will discuss these in the next section. Faraday, a devout member of

[182](Ampère, *Mathematical Theory of Electrodynamic Phenomena (English)*)
[183](Hofmann, *André-Marie Ampère: Enlightenment and Electrodynamics*)
[184](McKenna, "Antoine-Laurent Lavoisier")

one of the niche Protestant groups, was also a believer in the comprehensible harmony of the natural world, which underlined his approach to science, as C. A. Russell argued[185]. Luigi Galvani[186] allegedly never ended his lessons "without exhorting his hearers and leading them back to the idea of that eternal Providence, which develops, conserves, and circulates life among so many diverse beings" and in his final years was removed from the University for refusing the oath of allegiance to the new Napoleonic government.

James Clerk Maxwell (albeit criticized by Duhem for his moderate support for mechanistic models) in his 1873 paper outlines the following argument for the existence of a Creator[187], to refute arguments of the materialists and evolutionists. There is clear relation to Thesis 1.1, doctrine of creation of the world out of nothing, as well as similar arguments presented by Newton.

First he points to high degree of order in the matter and unchangeable properties of it, concluding that the basic building blocks of the world could not change and evolve. Indeed physics studied this unchanging character of the laws of universe in great accuracy.

> But in the heavens we discover by their light, and by their light alone, stars so distant from each other that no material thing can ever have passed from one to another, and yet this light, which is to us the sole evidence of the existence of these distant worlds, tells us also that each of them is built up of molecules of the same kinds as those which we find on earth. A molecule of hydrogen, for example, whether in Sirius or in Arcturus, executes its vibrations in precisely the same time. (...) No theory of evolution can be formed to account for the similarity of molecules, for evolution necessarily implies continuous change, and the molecule is incapable of growth or decay, of generation or destruction. None of the processes of Nature, since the time when Nature began, have produced the slightest difference in the properties of any molecule. We are therefore un-

[185](Russell, "Science and Faith In the Life of Michael Faraday")
[186](Fox, "Luigi Galvani")
[187](Maxwell, *Discourse on Molecules*)

> able to ascribe either the existence of the molecules or the identity of their properties to the operation of any of the causes which we call natural.
>
> On the other hand, the exact equality of each molecule to all others of the same kind gives it, as Sir John Herschel has well said, the essential character of a manufactured article, and precludes the idea of its being eternal and self existent.

Science of his days, he says, is necessarily limited, as it cannot "take molecule to pieces" to know anything about their origin and same holds about the organisms.

> Thus we have been led, along a strictly scientific path, very near to the point at which Science must stop. Not that Science is debarred from studying the internal mechanism of a molecule which she cannot take to pieces, any more than from investigating an organism which she cannot put together. But in tracing back the history of matter Science is arrested when she assures herself, on the one hand, that the molecule has been made, and on the other that it has not been made by any of the processes we call natural.
>
> Science is incompetent to reason upon the creation of matter itself out of nothing. We have reached the utmost limit of our thinking faculties when we have admitted that because matter cannot be eternal and self-existent it must have been created.

The contemplation of the order of the world, however, leads us to the Creator, who made all by the measure number and weight.

> It is only when we contemplate, not matter in itself, but the form in which it actually exists, that our mind finds something on which it can lay hold. They continue this day as they were created, perfect in number and measure and weight, and from the ineffaceable characters impressed on them we may learn that those aspirations after accuracy in measurement, truth in statement, and justice in action, which we reckon

among our noblest attributes as men, are ours because they are essential constituents of the image of Him Who in the beginning created, not only the heaven and the earth, but the materials of which heaven and earth consist.

3.9.2 Mathematics and Physics as Necessary Abstraction of World

Two greatest Enlightenment scientists: Jean R. D'Alembert and Pierre S. Laplace were firmly convinced for example, that Newtonian mechanics is only physical theory in the existence, to which all other hypotheses shall reduce. Variational principle, D'Alembert says, is merely a reformulation of Newton theory. As for Newton theory, they deemed it not based on any experiments whatsoever, but demonstrated by pure logic.

In this sense Euler and Maupertuis are not strictly "Enlightenment scientists" as they subscribed to none of this. Maupertuis and Alexis Clairaut were first to introduce Newton theories in France and initially they found allies in young accomplished mathematician D'Alembert and the leading *philosophe* Voltaire. That, however, turned out short-lived, as D'Alembert has set himself out to reform Newtonian physics. He believed[188] mechanics to be merely a subfield of mathematics, based on laws that were logical necessities. These laws should follow from reason alone and experimental evidence is not necessary, despite Newton's tedious efforts to put all the observations in agreement with his theory.

In such a way, the logical necessity of natural laws – an old mistake of Aristotelian philosophy – reemerged in the name of Science and Reason. We saw (on p. 77) that in the 14th century, Aristotelian physics was demoted from the level of logical truth to a set of conventions and hypotheses, even for die-hard supporters of Averroes. This, in turn, allowed physics to progress through the development of new, better hypotheses. The Enlightenment sought to reverse this by making a single theory into a certain logical necessity.

[188] (Bianchi, "Lecture notes on History of Computing and Information Technology")

No wonder that this effort put D'Alembert at odds with real Newtonians such as Clairaut (later famous for his accurate prediction of the return of Halley's comet), who criticized him for departing from experimental physics and dealing with fictions instead.[189]

> In order to avoid delicate experiments or long tedious calculations, in order to substitute analytical methods which cost them less trouble, they often make hypotheses which have no place in nature; they pursue theories that are foreign to their object, whereas a little constancy in the execution of a perfectly simple method would have surely brought them to their goal.

This program affected D'Alembert's mathematical career. He was very gifted and full of innovative ideas, but disregard for rigor and experiment produced more conflicts. Euler, initially on friendly terms with D'Alembert, was accused of stealing his ideas without credit, which was partly caused by[190] D'Alembert's work being too muddled to follow.

In general, mathematics of the Enlightenment was seen as merely an abstraction of physical world and this contribution, made by D'Alembert in "Preliminary Discourse" to Encyclopedia, secured his reputation as leading *philosophe*[191]. Here's how mathematics is constructed by abstraction from material things[192]:

> Hence we are led to ascertain the properties of extension simply as to shape. This is the object of Geometry, which facilitates its task by considering extension limited first by a single dimension, then by two, and finally by the three dimensions (...) Thus, by a few successive operations and abstractions of our minds we divest matter of almost all its sensible properties, in order to envisage in a sense only its phantom. (...) ... it is necessary to invent some means of achieving those combinations more easily; and since they consist chiefly

[189] (O'Connor and Robertson, "Jean le Rond D'Alembert - Biography")
[190] (ibid.)
[191] (Alexander, *Duel at dawn*), p. 49
[192] (d'Alembert, *Preliminary Discourse to the Encyclopedia of Diderot*)

> in calculating and relating the different parts of which we conceive the geometric bodies to be formed, this investigation soon brings us to Arithmetic or the science of numbers. This [science] is simply the art of finding a short way of expressing a unique relationship [a number] which results from the comparison of several others. The different ways of comparing these relationships [numbers] give the different rules of Arithmetic [addition, subtraction, etc.].

Quite naturally emerges the property of application of these formulae to generalized symbols, which will be strongly criticized by Cauchy as an invalid form of proof.

> Moreover, if we reflect upon these rules we almost inevitably perceive certain principles or general properties of the relationships, by means of which we can, expressing these relationships [numbers] in a universal way, discover the different combinations that can be made of them. The result of these combinations reduced to a general form will in fact simply be arithmetical calculations, indicated and represented by the simplest and shortest expression consistent with their generality.

Same construction, when applied to impenetrability of bodies, gives us mechanics:

> That is why, having so to speak exhausted the properties of shaped extension through geometric speculations, we begin by restoring to it impenetrability, which constitutes physical body and was the last sensible quality of which we had divested it. The restoration of impenetrability brings with it the consideration of the action of bodies on one another, for bodies act only insofar as they are impenetrable. It is thence that the laws of equilibrium and movement, which are the object of Mechanics, are deduced. We extend our investigations even to the movement of bodies animated by unknown driving forces or causes, provided the law

> whereby these causes act is known or supposed to be known.

It is easy to see this construction as plausible and rigorous from a common sense perspective. A geometer considers abstract geometric solids, while a physicist adds impenetrability, friction, and other similar properties to produce the laws of mechanics. Common sense, however, proves to be as poor an advisor as in the case of the Aristotelians, as this theory implies a denial of the Contingency Thesis (Thesis 1.2), suggesting that there could be no mathematical models different from those we infer from the physical world, contrary to the opinion of modern physics.

On the other hand, D'Alembert believed that the world is ordered by Supreme Intelligence, and we know that by perceiving this order. In one of his articles for the Encyclopedia[193], he writes as follows[194]:

> However the principal use that we should have obtained from Cosmology is to be able to raise ourselves from the authorship of the general laws of nature whose wisdom established these laws and who has allowed us to see what is necessary for our use or enjoyment and to have hidden the rest so as to teach us the use of doubt. Thus Cosmology as the science of the World or the Universe which is generally considered to be composed simply by the union and harmony of its parts; a complete whole which is governed by a supreme intelligence who winds the springs, puts the game in motion, all of which is handled by this intelligence.

3.9.3 All Fields of Science Allegedly Analogous to Physics

D'Alembert's peculiar research priorities were adopted by his close successor, Pierre Laplace, another outstanding mathematician and physicist of the Enlightenment tradition. As in his mentor's case,

[193] One of the chief literary works of Enlightenment philosophy, edited and in large part written by Diderot and D'Alembert

[194] (Denis Diderot, *Encyclopedia of Diderot and D'Alembert - Collaborative Translation Project*), "Cosmology"

neither the calculation of new experimental results nor rigorization of mathematics was a top priority. His chief interest and greatest achievement was demonstrating the stability of the solar system. He believed that mechanics is indeed foundational to all things, and he likely wanted to know whether mechanical systems are regular and predictable or chaotic. Having convinced himself that they are predictable, he concluded that all things and relations, including, but not limited to, human affairs, can be modeled and predicted. Here's what he writes in his "Philosophical Essay on Probabilities"[195]. Not so long ago, he says, superstitious crowds interpreted rare, singular events as signs of Divine wrath:

> Let us recall that formerly, and at no remote epoch, an unusual rain or an extreme drought, a comet having in train a very long tail, the eclipses, the aurora borealis, and in general all the unusual phenomena were regarded as so many signs of celestial wrath. Heaven was invoked in order to avert their baneful influence. No one prayed to have the planets and the sun arrested in their courses: observation had soon made apparent the futility of such prayers.

A long-tailed comet was counted among such signs, but learned men have expected that this phenomenon is regular and predictable, which is indeed the case as Clairaut has demonstrated.

> Thus the long tail of the comet of 1456 spread terror through Europe (...) Halley, having recognized the identity of this comet with those of the years 1531, 1607, and 1682, announced its next return for the end of the year 1758 or the beginning of the year 1759. (...) Clairaut then undertook to submit to analysis the perturbations which the comet had experienced by the action of the two great planets (...) he fixed its next passage at the perihelion toward the beginning of April, 1759...

That is now a known fact of celestial mechanics. Of course, similar predictability cannot be attributed to the rest of his examples:

[195](Laplace, *A Philosophical Essay on Probabilities*), p. 6

unusual rains[196], extreme droughts, or auroras. Laplace, however, thinks that all other things are analogous in this aspect to celestial mechanics:

> **The regularity which astronomy shows us in the movements of the comets doubtless exists also in all phenomena.** The curve described by a simple molecule of air or vapor is regulated in a manner just as certain as the planetary orbits; the only difference between them is that which comes from our ignorance.

Modern quantum mechanics typically suggests the falsehood of this observation, and modern physical research does not endorse speculation about single molecules from the behavior of planets, as much as it does not endorse such speculation about human affairs (contrary to ancient astrology). According to Laplace, however, this is supported by the principle of sufficient reason:

> Present events are connected with preceding ones by a tie based upon the evident principle that a thing cannot occur without a cause which produces it. This axiom, known by the name of the principle of sufficient reason, extends even to actions which are considered indifferent;

Leibniz, whom Laplace credits, would undoubtedly admit this principle, but he would mean something different by it: for instance, he would admit human free will. Laplace would not do that—on the contrary, everything in the universe is determined and can be calculated, given unlimited knowledge and computational capability.

> We ought then to regard the present state of the universe as the effect of its anterior state and as the cause of the one which is to follow. Given for one instant an intelligence which could comprehend all the forces by which nature is animated and the respective

[196] Weather is chaotic and only predictable on very short timescales, according to discoveries of E. Lorenz in the 1960s.

> situation of the beings who compose it—an intelligence sufficiently vast to submit these data to analysis—it would embrace in the same formula the movements of the greatest bodies of the universe and those of the lightest atom; for it, nothing would be uncertain and the future, as the past, would be present to its eyes.

The above passage is often interpreted as a thought experiment known as "Laplace's daemon", but the subsequent passage could also be read as referring to some form of deistic God that exists in reality:

> All these efforts in the search for truth tend to lead it back continually to the vast intelligence which we have just mentioned, but from which it will always remain infinitely removed. This tendency, peculiar to the human race, is that which renders it superior to animals; and their progress 5in this respect distinguishes nations and ages and constitutes their true glory.

Deist or not, it is certain that Laplace endorses hard determinism, with every future event determined by its past, coming quite close to ancient pagan cosmic fatalism. His strong conviction about the predictability of human affairs probably played a role in his career shift[197], when he obtained the post of Minister of the Interior. He proved himself a rather bad administrator, bringing to his new job "the spirit of infinitesimals" (as Napoleon wrote).

His views on the universal application of quantitative methods were endorsed before him by some of the *philosophes*, such as Voltaire, Holbach, or Condorcet[198]. Condorcet also shared a profound interest in Laplace's proof of the stability of the solar system, which warranted the success of science[199]:

> We learn that all bodies are subject to necessary laws, which tend of themselves to produce or maintain an equilibrium, which causes or preserves the regularity of their motions.

[197] (Jaki, *The Relevance of Physics*), p. 467
[198] (ibid.), p. 467
[199] (Condorcet, *Outlines of an Historical View of the Progress of the Human Mind*)

In this way, *philosophes*, having no answer as to why physics works (and disgusted with the only reasonable answer given by Euler and other faithful scientists, that God ordered the world according to measured quantities), come to the conclusion that there is indeed nothing special about it and that biology, sociology, or ethics might use similar methods.

3.9.4 Closer to the Atheism, Further From Science

D'Alembert (see p. 100) and Voltaire endorsed teleological argument for the existence of God[200]. Here are Voltaire's quotes on the matter:

> ...if a clock is not made to tell the hour, I will then admit that final causes are chimeras; and I shall consider it quite right for people to call me "cause-finalier," that is–an imbecile. ...It is, it seems to me, to stop one's eyes and understanding to maintain that there is no design in nature; and if there is design, there is an intelligent cause, there exists a God.

This opinion was not ubiquitously shared among the *philosophes*; some, such as the Encyclopedia's coeditor Denis Diderot, were atheists. Others, such as de Buffon, Kant, or Hume, would deny that the existence of God can be demonstrated from visible things. Interestingly, these very same people were openly dismissive of mathematics and physics. While for D'Alembert, a mathematician[201] saw the world more clearly and accurately from a vantage point, for de Buffon and Diderot he was a blind man, as mathematics was too detached from reality to contribute any real knowledge. Thus, more atheistic of the *philosophes* tend to openly deny that there is much use to mathematical methods and that world is ordered according to quantities.

The latter is evident for David Hume, the English *philosophe* famous to this day for allegedly showing that inductive reasoning cannot be rationally supported. By inductive reasoning, however,

[200](Arouet, *Voltaire's Philosophical Dictionary, selected and translated by H. I. Wolf*), "Final Causes"
[201](Alexander, *Duel at dawn*), p. 51-52

he means something different than Aristotle or Newton, who considered it with respect to the existing order of the world. Hume only conceives random relations of aspects that occur together and notes that there's no reason to expect these relations to be generally valid rules. For example, if I saw many swans and they were all white, this is not enough to claim that all swans are white, and we know that in Australia there is another black-feathered subspecies of a swan. But it does not show that we cannot find any generally valid rules in nature. Rather, some aspects of phenomena are ordered, and some are not. A surprising thing about physics is that a very high degree of order can be found when we consider very different phenomena in relation to each other. Earth magnetism, thunders, and electric engines, for instance, are subject to the same set of laws that are universally valid. This aspect is removed by Hume as irrelevant, and he approaches knowledge as if it had never existed[202]. He divides knowledge into "relations of ideas," such as mathematics or logic, and "real facts." Very clearly, physical or chemical theory is neither of these; furthermore, "relations of ideas" tell nothing about the physical world, while "real facts" do not tell us anything about the future. So, there's no point in expecting that[203] aspirin treats headaches based on the fact that it did treat them many times before.

Immanuel Kant shared opinion that Newton mechanics is the only theory of physics but saw it not as necessary decree of the Supreme Intelligence but rather set of categories of understanding that necessarily exist only in mind. In result, while D'Alembert, Laplace and Voltaire too deemed mechanics logical necessity, they at least admitted reality of objective order of world. Kant has no need for such things[204]:

> we ourselves bring into the appearances that order and regularity that we call nature, and moreover we would not be able to find it there if we, or the nature of our mind, had not originally put it there. [...] The understanding is thus not merely a faculty for making rules through the comparison of the appearances:

[202] (Morris and C. R. Brown, "David Hume"), 5.1
[203] example from (ibid.)
[204] (Rohlf, "Immanuel Kant"), s. 4.3

> it is itself the legislation for nature, i.e., without understanding there would not be any nature at all"

Indeed, laws of nature, he thinks, are merely made up by the observer. In a development that he called "Copernican Revolution in philosophy", he claimed that it is not our thought that conforms to experience, but the other way around[205]:

> Up to now it has been assumed that all our cognition must conform to the objects; but all attempts to find out something about them a priori through concepts that would extend our cognition have, on this presupposition, come to nothing. Hence let us once try whether we do not get farther with the problems of metaphysics by assuming that the objects must conform to our cognition, which would agree better with the requested possibility of an a priori cognition of them, which is to establish something about objects before they are given to us. This would be just like the first thoughts of Copernicus, who, when he did not make good progress in the explanation of the celestial motions if he assumed that the entire celestial host revolves around the observer, tried to see if he might not have greater success if he made the observer revolve and left the stars at rest. Now in metaphysics we can try in a similar way regarding the intuition of objects. If intuition has to conform to the constitution of the objects, then I do not see how we can know anything of them a priori; but if the object (as an object of the senses) conforms to the constitution of our faculty of intuition, then I can very well represent this possibility to myself.

Denial of the objective reality of the sensory world hardly endorses the systematic study of it[206], which is evident not only in the failures of ancient Greek pagan pantheism derived from Plato, but also in its partial resurrection by Kant's successors such as Hegel[207],

[205](Rohlf, "Immanuel Kant"), s. 2.2
[206](Jaki, *Angels, Apes and Men*), p. 32
[207](ibid.), p. 34

who, by the way, struggled to replace experimental physics with speculative a priori Naturphilosophie[208]. A similar program of this sort was more recently introduced by Thomas Kuhn, who in 1962 argued that physics is a social construct of mob psychology and gained much support (more on that in a while). Philosopher P. Lipton had an interesting observation on the foundation of the latter idea[209]:

> Kuhn, however, is Kant on wheels. Where Kant held that the human contribution to the phenomenal world is invariant, Kuhn's view is that it changes fundamentally across a scientific revolution. This is what he means by his notorious statement that, after a scientific revolution, 'the world changes'. This is neither the trivial claim that scientists' beliefs about the world change, nor the crazy claim that scientists can change the things in themselves simply by changing their beliefs. It is the claim that the phenomenal world changes because the human contribution to it changes.

The main problem for Kant's idea that Newtonian mechanics is a necessary structure of experience is that a major part of Newtonian mechanics was replaced by relativity and quantum mechanics. There is a solution, however: it is the world that changes with our theories, because "human contribution to it changes," meaning that the world as we see it is in part a changing social construct.

3.10 Cauchy's Revolution of Mathematical Physics.

The understanding of mathematics and physics by the great geometers of the 18th century was notably different from ours. Physics was not separated from mathematics, and mathematics wasn't seen as separate from the physical world. The most important tools of physics consisted of geometric demonstrations invented by 14th-century scholastics, who figured out how to apply Euclid's geometry to physical quantities. Simultaneously, algebraic formulae were often used to simplify calculations, but it was not clear what the

[208] (ibid.), p. 36
[209] (Lipton, "Kant on Wheels")

sense of the symbolic formulae was and what the criteria for their validity were.

From Theses 1.1 and 1.2 two things follow for mathematics and physics: first of all, calculations on real numbers are especially effective and important in the description of the real world. Second, mathematical models do not necessarily reflect anything that exists in the real world, as our world is merely one of many metaphysical possibilities. These observations are broadly accepted by modern scientists. One could prefer to see mathematics as completely "pure" and completely detached from the real world; but that, of course, would get us nothing in physics. We need both the freedom to formulate any systems of axioms and methods to connect formulae to real phenomena to make any use of them: that's why I am talking about mathematical physics.

A greatly successful doctrine of this sort can be attributed to Augustin Cauchy, recognized as the father of modern calculus and one of the most accomplished mathematicians who ever lived. Others who contributed to it were Cauchy's companion A. Ampère and Fr. B. Bolzano, who worked independently on related ideas.

Interestingly, Cauchy was a devout Catholic, an enemy of the Enlightenment, and a strong supporter of the Bourbon king, and these views affected his approach to mathematics. Let us have a look at the preface to the 1821 textbook "Cours d'Analyse." By 1821, the young professor of École Polytechnique had already been at war with the Enlightenment mathematicians[210] for several years, pushing his own version of mathematical analysis. The new textbook was supposed to "clarify any doubts" they had, and it indeed made clear that Cauchy sought to depose Enlightenment mathematics altogether, claiming it to be based on a seriously flawed foundation[211] (which we saw in D'Alembert's "Preliminary Discourse"):

> As for the methods, I have sought to give them all the rigor which one demands from geometry, so that one need never rely on arguments drawn from the generality of algebra. Arguments of this kind, al-

[210](Barany, "God, king, and geometry: Revisiting the introduction to Cauchy's Cours d'analyse")

[211](A. L. Cauchy, *Cauchy's Cours d'Analyse: an annotated translation*), p.2

> though they are commonly accepted especially in the passage from convergent to divergent series, and from real quantities to imaginary expressions, may be considered, it seems to me, only as examples serving to introduce the truth some of the time, but which are not in harmony with exactness so vaunted in the mathematical sciences. We must also observe that they tend to grant limitless scope to algebraic formulas, whereas, in reality, most of these formulas are valid only under certain conditions or for certain values of the quantities involved.

"Generality of the algebra" was a principle often employed in 18th-century mathematics. It stated that operations like addition or multiplication could be applied to mathematical symbols in the same way as they were applied to numbers, which aligned well with Enlightenment philosophy seeing mathematics as the abstraction of the world and algebra as the highest level of abstraction. Now it is broadly known that this is false; for example, the manipulation of terms of infinite divergent series doesn't produce consistent results. Cauchy, fully aware of this, proposes a different type of argument:

> In determining these conditions and these values and in establishing precisely the meaning of the notation that I will be using, I will make all uncertainty disappear, *so that different formulas present nothing but relations among real quantities*, relations which will always be easy to verify by substituting numbers for the quantities themselves.

For example, for Cauchy, there is no such thing as an imaginary number $i^2 = -1$. He only considers an "imaginary expression" that corresponds to two equations on real numbers. The meaning of the formulae of calculus can be found in corresponding operations on real numbers. All the proofs and theorems should connect symbols and formulae to operations on real quantities. As God made all things "according to measure, number, and weight," Cauchy makes real quantity a foundational entity of mathematics. That appears to be Cauchy's goal, judging by his own words[212]:

[212](Belhoste, "Practices and Principles in Cauchy's Work")

> We will similarly [to Bourbon king who regained throne] enthrone a rightful, lawful science, a science which will serve the Sovereign Lord and whose truth is confirmed by the entire Universe.

Cauchy's revolution quite easily made mathematics separate from other fields of knowledge. The only connection to reality that Cauchy's calculus had was the underlying calculation on real quantities; anything else was derived from freely posited axioms and definitions. In this way, the shackles of Enlightenment mathematics, which sought a foundation in the abstraction of the real world, are gone. This is related to Contingency Thesis 1.2 and allowed Cauchy to become an incredibly productive and innovative mathematician, as a new level of rigor went hand in hand with nearly limitless freedom of investigation. One crucial example of the effectiveness of his approach is Cauchy's theory of infinitesimals, which I elaborated on in more detail elsewhere[213]. Newton and Leibniz used the concept of a small quantity to calculate rates of change such as velocity:

$$v = \frac{ds}{dt},$$

which means the ratio of distance traveled ds in a very small interval of time dt to the duration of this interval of time. Leibniz conceived of infinitely small numbers, i.e., infinitesimals, which allowed him to simplify calculations involving derivatives as merely the division of infinitesimals and popularize his idea. Still, a paradox remained: there is no such thing as a real number that is less than all positive numbers and yet greater than zero, so the infinitesimal number was inconsistent.

Cauchy is recognized as the originator of modern approach[214], where continuity, derivative and integral were all defined by limits, while limits were rigorously defined through inequalities of real numbers. In Cauchy's terms, sequence x_i converges to 0, if subsequent terms come to 0 as close as we please. If we want sequence to come as close to 0 as c, there is N such that all elements $x_N, x_{N+1}, x_{N+2}...$ are found between $-c$ and c. Similarly, function

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[213] (Zawistowski, *Differential Calculus Made Clear by Its Original Inventor: Cauchy's Theory of Infinitesimals*)

[214] (Grabiner, *Origins of Cauchy's Rigorous Calculus*), p.78

$f(x)$ in modern terms is continuous if for decreasing increment of argument h, $f(x+h) - f(x)$ becomes as small as we please. In other words, for any positive δ, as small as we please, there can be found positive ϵ s.t. for all positive $\nu < \epsilon$ following holds

$$|f(x+\nu) - f(x)| < \delta. \tag{3}$$

Similar definition for derivative was first given by Ampere[215] in 1814, and Cauchy and Ampere were close collaborators, as fellow lecturers of calculus.

This standard reading of Cauchy neglects one important aspect of his work: Cauchy did not remove infinitesimals, nor he did consider them necessarily inconsistent. In fact he uses infinitesimals repeatedly and interchangeably with limit apparatus, either in "Cours d'Analyse", "Lessons on Differential Calculus"[216] or elsewhere. In the preface to the latter work he sets following goal[217] :

> My principal aim has been to reconcile the rigour, which I had set in my Course in Analysis, with the simplicity produced by the direct consideration of infinitely small quantities.

Infinitesimals are typically easier and more intuitive to use. The definition of continuity we gave in (3) could be stated as follows: $f(x)$ is continuous if for an infinitesimal increment of an argument, the change of value of the function is infinitesimal too. Moreover, they were widely employed by 18th-century physicists. It is doubtful that Cauchy could afford to ignore infinitesimals altogether without sharing the fate of Fr. Bolzano, whose work received little interest during his lifetime. Thus, he needed to reconcile infinitesimals with mathematical rigor. What he came up with was a variation of an idea that happened to be popular among 14th-century scholastics. Peter the Spaniard, author of a popular 13th-century treatise on logic (often identified with Pope John XXII), introduced the following doctrine[218]: infinity is understood in one sense as a

[215](ibid.), p. 129
[216](A. L. Cauchy, *Leçons sur le calcul différentiel*)
[217](ibid.), p. I
[218](P. Duhem and Ariew, *Medieval Cosmology: Theories of Infinity, Place, Time, Void, and the Plurality of Worlds*), p. 50

number bigger than all finite numbers, but we can also understand it differently: as a succession of numbers that increase indefinitely. The former we would call categorematic infinity, which describes some hypothetical "thing"; the latter, syncategorematic infinity, lacks denotation. For example, "infinite number of people running" could mean a crowd of people that grows and we see no end to it, and "infinite amount of funds" would be an amount that increases without limit. Indeed, we don't think of a number greater than all real numbers in these cases, but rather we consider a certain relation of increase without end. In a similar way, we could conceive syncategorematic infinitely small, which is a variable quantity that decreases indefinitely to zero. This is precisely the definition that Cauchy uses for his calculus[219]:

> When successive numerical values of the same variable, being assumed to be very small, decrease indefinitely so as to fall below any given number, this variable becomes what we call an infinitely small or an infinitesimal quantity. A variable of this type has zero as its limit. This is the variable a in the preceding calculations.
>
> When the successive numerical values of the same variable grow more and more, so as to rise above any given number, we say that this variable has as its limit the positive infinity indicated by the sign ∞, if it is a positive variable; and the negative infinity indicated by the notation $-\infty$, if it is a negative variable(...).

In this way, infinitesimal calculus becomes nearly identical logically to modern limit-based calculus, while allowing the explanation and use of basic concepts and theorems much more easily. Moreover, Cauchy can use some of the computational procedures of his predecessors to make rigorous proofs. One of such procedures[220] was conceived in 16th century by Stevin: given function $f(x)$ on an interval x_a, x_b, and $f(x_a) < 0, f(x_b) > 0$, we seek to find x s.t. $f(x) = 0$. We divide our interval in half $x' \frac{x_a + x_b}{2}$. If $f(x') < 0$ we substitute x' for x_a, if $f(x') > 0$ we substitute x' for x_b, which

[219] (A. L. Cauchy, *Leçons sur le calcul différentiel*), p. 4
[220] (Zawistowski, *Differential Calculus Made Clear by Its Original Inventor: Cauchy's Theory of Infinitesimals*), p. 7

gives us same problem with interval half as big. Cauchy says: if by this procedure we get variables x_a, x_b as close as we please to each other (infinitesimally close), and if then $f(x)$ is continuous, (that is infinitesimal change in x produces infinitesimal change of $f(x)$) then positive $f(x_b)$ is infinitesimally close to negative $f(x_a)$, so both are infinitesimally close to 0, so the solution exists.

Interestingly, this concept of infinity and infinitesimal fell out of fashion with some of Cauchy's successors: Cantor, Weierstrass or Dedekind, who used limit based formulation everywhere(see Borovik & Katz[221], while Abraham Robinson, who continued development of infinitesimal calculus in 20th century saw infinitesimals as extension of real numbers, and with him a hypothesis emerged, that Cauchy in fact is dealing with actual infinitesimals and infinities (see Bascelli et al.[222]), based on possibility to fix some rare errors in Cauchy's proofs. This however, seems not the case, Cauchy was strongly opposed to real infinities, which was another point of his religious and anti-Enlightenment worldview. In his "Seven Lessons on General Physics" he writes[223]:

> Moreover, this fundamental proposition that we cannot admit a sequence or series really composed of an infinite number of terms, can be demonstrated by mathematics in a thousand different ways; (...) I will add that it is precisely because of having admitted the existence of series composed of an infinite number of terms that very skilful geometers have several times been led to inaccurate results; and, if all the genius of Lagrange could not succeed in founding the theory of analytical functions on solid foundations, this is due to the fact that in the assumptions of this theory we find the sum of the terms considered as determined. if any series is extended to infinity.

This is closely related to religion as (below he quotes cardinal Gerdil) it proves that claims about the eternity of world are false:

[221] (Borovik and M. G. Katz, "Who Gave You the Cauchy–Weierstrass Tale? The Dual History of Rigorous Calculus")

[222] (Bascelli et al., "Cauchy's Infinitesimals, His Sum Theorem, and Foundational Paradigms")

[223] (A. Cauchy, *Sept lecons de physique generale par Augustin Cauchy*), p. 25

> geometry provides a rigorous, demonstrative proof of the falsity of the fundamental principle of atheism, I mean, of the necessary and therefore eternal existence of the universe

He is convinced that infinities are nonsense and a cause of failures in mathematics, and Lagrange's failure to establish a theory of calculus seems to be one of his favorite topics. He mentioned it in "Cours d'Analyse," and he also does in "Lessons on Differential Calculus"[224]:

> [To reconcile infinitesimals with rigor], I believed it necessary to reject the developments of functions in infinite series, whenever the series turns out convergent. It follows, for example, that Taylor's formula cannot be admitted as general, unless it is reduced to a finite number of terms, and completed by a remainder. I am not unaware that by first ignoring this remainder, the illustrious author of Analytical Mechanics [i.e. Lagrange] took the formula in question as the basis of his theory of analytic functions (...)[on which] most geometers now agree in recognizing the uncertainty of the results(...). There is more: Taylor's theorem seems, in certain cases, to provide the development of a function into a convergent series, although the sum of the series differs essentially from the proposed function...

As we mentioned, Cauchy's successors did not adopt his definition of infinitesimal, and subsequent generations of mathematicians did not consider infinitesimals a rigorous concept, even if they, like Toeplitz (see Katz & Polev[225]), saw "pedagogical" utility in infinitesimals. This reasoning seems to be based on a hidden premise that no other type of infinitesimal than an infinitesimal number should be considered, which seemingly follows from the use of axiomatic structures such as the field of real numbers or the field of complex numbers (originally introduced by Dedekind). A

[224](A. L. Cauchy, *Leçons sur le calcul différentiel*), p. 4

[225](M. Katz and Polev, "From Pythagoreans and Weierstrassians to True Infinitesimal Calculus")

field is a set of elements with operations such as addition or multiplication, which map one element of the set to other elements. A sum of complex numbers is another complex number, etc. To Cauchy, however, there is no such thing as complex numbers, but merely complex expressions reducible to operations on real numbers, and there is no infinitesimal number, but only an infinitesimal quantity—a variable that converges to the limit 0, taking successively smaller and smaller real values. So, for example, Dedekind would build an axiomatic definition of the real continuum and then axiomatize the field of complex and real numbers, and Robinson would do that with the field of "hyperreal numbers" (i.e., real numbers with added infinitesimals and infinities). To Cauchy, there is only one real continuum, and everything else is made of operations on real numbers: complex expressions, infinitesimal variables, infinities, and so on. In such a way, Cauchy's foundation is easier to work with and much closer to what a physicist or an engineer would, in fact, do. Clearly, there are no "complex numbers," but merely an abstract symbol to which we substitute a phase and amplitude to simplify some of the calculations.

His remark about the eternity of the world is also referenced in an argument that Christian religion, being necessarily the true one, possesses important heuristic power in science, making him very important forerunner of the ideas that this book seeks to outline[226].

> Secondly, [a scientist] must reject, without hesitation, any hypothesis that contradicts revealed truths. This point is crucial, not only in the interest of religion but also in the interest of science, since truth can never contradict itself. It is due to neglecting this rule that some scientists have had the misfortune of wasting precious time on futile efforts, time that could have been better spent making useful discoveries. Indeed, what remarkable works could have been added to the important memoirs included in our scientific collections if religion had always guided the pens of those authors who believed, at some point, that the zodiacs of Denderah

[226](A. Cauchy, *Sept lecons de physique generale par Augustin Cauchy*), p. 16

and Esneh were twelve thousand years old, that man descended from the polyp, that he had existed on Earth for all eternity, that the flood was a fable, that the creation of man and animals was an effect of chance, and that even today we see them emerging from the earth on the islands of the great Ocean, that the Americans formed a species of men distinct from ours, and so on.

3.11 Method according to Einstein and Weinberg

The presented method was adopted by many famous physicists, including modern physicists. We already quoted Einstein's opinion on the ordering of the world (p. 13), Dugas[227] on the application of Duhem's method to quantum physics, as well as de Broglie's testimony on the adoption of it (p. 6).

Indeed, much more evidence can be found. Einstein and Weinberg, despite being hostile to Christian religion, reproduce very similar methodological views. This is evident when they need to confront critics. In 1949, Einstein penned such a response to confront positivists[228] as well as followers of the Copenhagen Interpretation of Quantum Mechanics (CIQM). First, he confronts the question of whether a given atom has any definite time of decay. Response of a CIQM theorist is that it would make no sense to ask about time of decay if it cannot be empirically measured. To this, Einstein responds[229]:

> *The justification of such constructs does not lie in their derivation from what is given by the senses. Such a type of derivation (in the sense of logical deducibility) is nowhere to be had, not even in the domain of prescientific thinking. The justification of the constructs,*

[227](Dugas, "La méthode physique au sens de Duhem devant la mécanique des quanta - translated by A. Aversa as Physical method according to Duhem in view of quantum mechanics")

[228]Positivists were a group of atheist and left-wing intellectuals active until the 1950s, who shared views inspired by the Enlightenment *philosophes.* They believed in the empirical verification of all statements (which is what Einstein criticizes here) and universal application of single scientific method to all sciences.

[229](Einstein, "Reply to Criticisms in Albert Einstein: Philosopher-Scientist")

> *which represent "reality" for us, lies alone in their quality of making intelligible what is sensorily given* (the vague character of this expression is here forced upon me by my striving for brevity). Applied to the specifically chosen example this consideration tells us the following: One may not merely ask: "Does a definite time instant for the transformation of a single atom exist?" but rather: "Is it, within the framework of our theoretical total construction, reasonable to posit the existence of a definite point of time for the transformation of a single atom?" One may not even ask what this assertion means. One can only ask whether such a proposition, within the framework of the chosen conceptual system — with a view to its ability to grasp theoretically what is empirically given — is reasonable or not[230].

The mathematical order of the world is not only real but also gives meaning to the concepts of physics. If we say "time of decay," we actually think about something that can be calculated and predicted by our theoretical models, which in this way make the world comprehensible for us. In this way, we see that physics concerns itself mainly with universal relations between measured quantities, as Thesis 1.1 states.

Einstein proceeds to argue against the positivist (Reichenbach's) opinion that there is any empirical verifiability of concepts. He adopts Poincaré's opinion, which is very close to what Duhem held: physicists freely conceive concepts, assemble them into theory, and only theory as a whole can be corroborated or refuted, according to Thesis 1.2 and the Duhem thesis (Thesis 1.4). No other opinion is possible because even the most basic concepts of geometry cannot be verified independently of a number of other physical theories that *presuppose already* the concepts of geometry.

> Poincare: The empirically given bodies are not rigid, and consequently can not be used for the embodiment of geometric intervals. Therefore, the theorems of geometry are not verifiable. (...)

[230](ibid.) - this quote is a fictitious dialog conceived by Einstein for the exposition of argument.

Reichenbach: I admit that there are no bodies which can be immediately adduced for the “real definition” of the interval. Nevertheless, this real definition can be achieved by taking the thermal volume-dependence, elasticity, electro- and magnetostriction, etc., into consideration. That this is really [and] without contradiction possible, classical physics has surely demonstrated. (...)

Poincare: In gaining the real definition improved by yourself you have made use of physical laws, the formulation of which presupposes (in this case) Euclidean geometry. (...)

Reichenbach: (...) Should we not, on the basis of this astounding fact, be justified in operating further at least tentatively with the concept of the measurable length, as if there were such things as rigid measuring-rods. In any case it would have been impossible for Einstein de facto (even if not theoretically) to set up the theory of general relativity, if he had not adhered to the objective meaning of length. (...)

Non-Positivist: If, under the stated circumstances, you hold distance to be a legitimate concept, how then is it with your basic principle (meaning = verifiability) ? Do you not have to reach the point where you must deny the meaning of geometrical concepts and theorems and to acknowledge meaning only within the completely developed theory of relativity (which, however, does not yet exist at all as a finished product)? Do you not have to admit that, in your sense of the word, no “meaning” can be attributed to the individual concepts and assertions of a physical theory at all, and to the entire system only insofar as it makes what is given in experience “intelligible?” Why do the individual concepts which occur in a theory require any specific Justification anyway, if they are only indispensable within the framework of the logical structure of the theory, and the theory only in its entirety validates itself?

In a similar mood, Einstein responds to Bohr and Pauli, who accuse him of "rigid adherence to classical theory." It is not clear, however, he says, what they mean by classical theory. Original Newtonian theory was abandoned with the introduction of electromagnetic fields, and original Maxwellian theory ceased to be useful shortly after. "Consequently there is" no such thing as "classical field theory" in this sense, but there is something different:

> Nevertheless, field-theory does exist as a program: "Continuous functions in the four-dimensional [continuum] as basic concepts of the theory." Rigid adherence to this program can rightfully be asserted of me.

The rationale for such a theory is beyond our scope—the important thing is that Einstein can introduce whatever concepts he may deem necessary, as the concepts of the theory are "free conventions"; the adoption of quantum mechanics changes nothing. All these examples are applications of the Contingency Thesis (Thesis 1.2).

Here is a passage discussing that, showing, by the way, an omission that freethinking scientists often are forced to make:

> The theoretical attitude here advocated is distinct from that of Kant only by the fact that we do not conceive of the "categories" as unalterable (conditioned by the nature of the understanding) but as (in the logical sense) free conventions. They appear to be a priori only insofar as thinking without the positing of categories and of concepts in general would be as impossible as is breathing in a vacuum.

Einstein credits Kant, saying that his attitude is different only in conception of unalterable categories replaced by free convention, but there's much more to it: only some free conventions work, while other do not, and the reason is that there is an order of world "according to measure, number and weight", the truth which Kant plainly rejected.

A similar response to criticism, albeit a more general one, was once penned by Weinberg, Nobel laureate, particle theorist, and outspoken atheist, in response to Thomas Kuhn's doctrine of scientific revolutions. Kuhn saw the theory of physics as chiefly a social construct, with concepts such as the empirical evaluation of the theory, or the truth of the theory, as meaningful only within such a construct (thus denying them objective meaning).

> Even more radical than Kuhn's notion of the incommensurability of different paradigms is his conclusion that in the revolutionary shifts from one paradigm to another we do not move closer to the truth. To defend this conclusion, he argued that all past beliefs about nature have turned out to be false, and that there is no reason to suppose that we are doing better now.

That, however, ignores many facts about science and technology. "We certainly don't regard Newtonian and Maxwellian theories as simply false," says Weinberg. But what is true and established about physical theory? Weinberg's opinion is nearly identical to Duhem's. There is a hard part of the theory that describes the order of the world (Thesis 1.1) better and better, which is a cumulative approach to truth, and a soft part that consists of a vision of reality that we use to explain why equations work, which is subject to refutation (Thesis 1.2):

> It is important to keep straight what does and what does not change in scientific revolutions, a distinction that is not made in Structure.5 There is a "hard" part of modern physical theories ("hard" meaning not difficult, but durable, like bones in paleontology or potsherds in archeology) that usually consists of the equations themselves, together with some understandings about what the symbols mean operationally and about the sorts of phenomena to which they apply. Then there is a "soft" part; it is the vision of reality that we use to explain to ourselves why the equations work. The soft part does change; we no longer believe in Maxwell's ether, and we know that there is more to nature than Newton's particles and forces.
>
> The changes in the soft part of scientific theories also produce changes in our understanding of the conditions under which the hard part is a good approximation. But after our theories reach their mature forms,

> their hard parts represent permanent accomplishments. If you have bought one of those T-shirts with Maxwell's equations on the front, you may have to worry about its going out of style, but not about its becoming false. We will go on teaching Maxwellian electrodynamics as long as there are scientists. I can't see any sense in which the increase in scope and accuracy of the hard parts of our theories is not a cumulative approach to truth.

Thus, top atheist physicists do not want to believe in positivist or Kantian natural philosophy in their own field, but rather refute it by adopting doctrines of scholastic theologians and Christian scientists. And thus, the theses of theology allow for the dispersal of the mirages of false doctrines in the 14th century, the 19th century, as well as today

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