Lucas Number 123 Demystified: Mathematics of a Divine Number

Dedicated to Pope Leo XIY, who had studied mathematics

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This is only a first short essay about the 'angel' number **123**, a symbol for balance, human creativity, divine order or trinity of God the Father, the Son and the Holy Spirit. The author thinks that important information about this number is not well known. Here this enigmatic number is interpreted mathematically as beautiful golden relation, connected with the golden mean and its fifth power that is vice versa governed by phase transitions from particle up to cosmic scale [1]. The golden ratio is the most irrational number with the simplest infinite continued fraction representation at all and a very adaptable number-theoretical chameleon. Special attention is paid to the reciprocity property of the golden ratio as effective precalculator of natures creativness.We use the definition

$$\varphi = \frac{\sqrt{5}-1}{2} = \frac{1}{1 + \frac{1}{1 +$$

However, the golden ratio is frequently used by others as the reciprocal of this value

$$\Phi = \varphi^{-1} = 1 + \varphi = \frac{\sqrt{5}+1}{2} = 1.6180339887 \dots$$
(2)

There is another nice continued fraction approach to represent φ^{-1}

$$r = \sqrt{1 + \sqrt{1 + \sqrt{1 + \cdots}}} = \sqrt{1 + r}$$
 (3)

giving the quadratic equation $r^2 - r - 1 = 0$ with the solution

$$r_1 = \varphi^{-1} = 1.6180339887 \dots$$
 (4)

The next important number, the fifth power of the golden ratio, shows also a simple infinitely continued fraction representation

$$\varphi^5 = \frac{\sqrt{125} - 11}{2} \tag{5a}$$

$$\varphi^{-5} = \frac{\sqrt{125} + 11}{2} \tag{5b}$$

$$\varphi^5 = 0.0901699 \dots = \frac{1}{11 + \frac{1}$$

This fundamental number not only governs phase transitions from particle up to cosmic scale [1] [2], but also determines the mass constituents of the universe [3] [4]. Its connection to the Great Pyramid is also remarkable [5] [6].

Now we summarize some beautiful, but less known relations for *Lucas* number 123 [7]. We write down a relation with reciprocal terms (for comparison see relation (5b))

$$\sqrt{123 - \frac{1}{123}} = 11 + \varphi^5 = \varphi^{-5} \tag{7a}$$

$$\frac{1}{\sqrt{123 - \frac{1}{123}}} = \varphi^5 \tag{7b}$$

$$\sqrt[5]{123 - \frac{1}{123}} = 2.618033991 = \Phi^2 = \frac{1}{\varphi^2}$$
(8)

$$\sqrt[10]{123 - \frac{1}{123}} = 1.618033989 = \varphi + 1 = \frac{1}{\varphi}$$
(9)

and

$$123 \approx \frac{720}{\sqrt{5}} \varphi^2 = 122.99068 = \frac{2}{\sqrt{5}} \cdot 137.50776 \tag{10}$$

where 137.50776° is known as the golden angle.

We formulate a quadratic equation in order to verify relation (7a)

$$x^2 - (11 + \varphi^5)^2 x - 1 = 0 \tag{11}$$

The solutions are

$$x_{1,2} = \frac{(11+\varphi^5)^2}{2} \pm \sqrt{\left[\frac{(11+\varphi^5)^2}{2}\right]^2 + 1}$$
(12)

$$x_1 = \frac{122.991869...}{2} + \frac{123.008129...}{2} = 123.00000 \dots$$
(13)

$$x_2 = -\frac{1}{x_1}$$
(14)

The infinitely continued fraction of number 123 leads to the golden reciprocity relation that can be compared to the second sum term of relation (13)

$$123.008129543 \dots = \frac{1}{123.008129543\dots}$$
(15)

This connection to golden mean mathematics, dominating the life as well as the entire universe, is the very meaning of *Lucas* number 123. The concept of reciprocity is an important tool of nature. An excerpt of this short contribution can be found in reference [3].

Further insight into the mathematics of number 123 can be gained by considering the quartic polynomial equation and its selected golden mean solutions given in **Table 1** [8].

$$x^4 - (n-2)x^2 + 1 = 0 \tag{16}$$

We find for $n = 123 = 11^2 + 2$ the solution

$$x_1 = 10.999624310 \approx 11 \tag{17}$$

Furthermore, we can generate *Sommerfeld*'s reciprocal structure constant $\alpha^{-1}[9]$ as fundamental number of rotational movement and precession by using number 123

$$\frac{7}{2\pi} \cdot 123 = 137.0324$$
 (18)

$$\frac{123\cdot\sqrt{5}-1}{2} = 137.01818\tag{19}$$

The last relation can be tentatively considered as an extension of the basis relation (1) for φ .

Also an approximation for the anomalous part of the gyromagnetic factor of the electron is deduced from relation (18)

$$\Delta g_2 \approx \frac{2}{7 \cdot 123} = 0.0023228 \tag{20}$$

Furthermore, number 123 can be related to our newly introduces angle of $\alpha_1 = 50.95^\circ$ that is an expression of creative doubling [10]. We yield

$$123 \approx \alpha_1 \cdot (1 + \sqrt{2}) \tag{21}$$

Last but not least we relate Lucas number 123 to the circumsphere radius of an icosahedron

$$\sqrt{\frac{123}{34}} = 1.902011 \approx \sqrt{3 + \varphi} = 1.902113 \tag{22}$$

where 34 is a *Fibonacci* number [11]. One can decompose number 123 into *Fibonacci* numbers $123 = 13 + 2 \cdot 55$ and proceed with

$$\frac{123}{34} = \frac{13}{34} + 2\frac{55}{34} \approx \varphi^2 + 2\varphi^{-1} = 3 + \varphi$$
(23)

In **Table 1** we find also number 171 as last listed one, which is a coefficient of the icosahedron equation. Like number 123 we find

$$\sqrt{171 - \frac{1}{171}} = 13.0764732 \dots = \frac{1}{0.0764732\dots}$$
(24)

and
$$\sqrt[3]{\frac{171}{123}} \approx \frac{\sqrt{5}}{2}$$
 respectively $\sqrt[3]{\frac{171}{123}} - \frac{1}{2} \approx \varphi$ (25)

The golden mean, helical structures and icosahedral entities belong together, and number 123 is connected with all that.

Conclusion

The divine *Lucas* number 123 was demystified without losing its meaning and intrinsic beauty. Beauty and simplicity are sisters. It remains a golden number that is related to fundamental constants of nature and therefore sustainably influences life and the entire cosmos. People may henceforth consider this mathematical interpretation, beyond pure scientific importance, as helpful for esoteric or religious insights. It seems that number 123 is once again enhanced and shines brightly.

Table 1. Selected S	Solutions for	the Quartic	Polynomial	Equation
	$x^4 - (n - n)^4$	$(2)x^2 + 1 =$	0	

	n	$\sqrt{n} = x_1 + x_1^{-1}$	notation	<i>x</i> ₁	notation	x_1^{-1}
4		2	φ^0	1	φ^0	1
5		$\sqrt{5}$	φ^{-1}	1.6180339887	φ	0.6180339887
8		$2 \cdot \sqrt{2}$	δ_s^{-1}	2.4142135623	δ_s	0.414213562
9		3	φ^{-2}	2.6180339887	φ^2	0.381966011
18	$4^2 + 2$	$3 \cdot \sqrt{2}$		3.9921490369		
20		$2 \cdot \sqrt{5}$	φ^{-3}	4.2360679774	φ^3	0.236067977
27	$5^2 + 2$			4.9959919730		
32		$4 \cdot \sqrt{2}$		5.4741784358		0.1826758136
38	$6^2 + 2$			5.9976829489		
49		7	φ^{-4}	6.8541019662	φ^4	0.1458980337
51	$7^2 + 2$			6.9985415144		
66	$8^2 + 2$			7.9990231393		
83	$9^2 + 2$			8.9993139982		
102	$10^2 + 2$			9.9994999374		
123	$11^2 + 2$			10.999624310		
125		$5 \cdot \sqrt{5}$	φ^{-5}	11.090169943	φ^5	0.090169943
146	$12^2 + 2$			11.999710630		
171	$13^2 + 2$			12,999772406		

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