A Kantian System of Knowledge in SI Units

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In a Kantian approach to cognition, the magnitudes in SI units of physical constants and other significant physical quantities have been judged according to mathematical rules conceived by this cogniser. A system of knowledge has been constructed, in which the magnitudes of particularly significant masses, lengths and times, on the smallest to the largest scales, assume a consistent intelligible form in terms of the rules.

1. Introduction

In the *Critique of Pure Reason* [1], and specifically in the *Prolegomena* [2], Immanuel Kant addressed the question, "How is Metaphysics Possible as Science?" Kant argued that although knowledge begins with experience it is gained independently of any particular experience, in the synthesis of sensible (i.e., mediated through the senses) intuition and concept. An intuition is a singular representation that relates immediately to an object and a concept is a general representation originally generated in the understanding. Neither a concept without a corresponding intuition nor an intuition without a concept can yield cognition. "...we must not seek the universal laws of nature in nature by means of experience, but conversely must seek nature, as to its universal conformity to law, in the conditions of the possibility of experience, which lie in our sensibility and in our understanding." [2], 319. Scientific knowledge must constitute a single system of knowledge, under one idea, determined *a priori* (i.e., independent of experience) [1], A832.

The cognitive scheme used here requires the numerical value of the magnitude in SI units of a physical quantity to be a rational power of $\pi/2$. This requirement is met by converting the numerical value of the magnitude into a power of $\pi/2$ and then locating the exponent of the power in an infinitely-divisible grid of rational numbers [3]. If the physical quantity is judged significant, i.e., worthy of attention, the exponent will typically assume the value of an integer or a fraction of low denomination. The numerical value of the magnitude in SI units of a physical quantity deemed to be of particular significance is judged to be an integer power of $(\pi/2)^{25}$, if such a value is logically consistent with the values of all other magnitudes in the phenomenal world; otherwise, an intelligible multiplicative factor, typically α , the fine structure constant, is judged necessary to meet the requirement of logical consistency. Magnitudes thus cognised, i.e., on sure foundations, are fixed in value and are objectively valid for everyone. Kant's philosophy of transcendental idealism explains how the

magnitudes of physical constants cognised in recent times can have always had the same values: space and time are pure (i.e., not mediated through the senses) intuitions that structure our experiences.

The conceptual rules set out above have been applied to the cognition of the magnitudes of many physical quantities, on scales from the subatomic to the cosmological. It is the mean evaluation of magnitude that is judged, with no regard to uncertainty. The mean evaluations of some of the quantities have changed since first cognition, which, together with the necessary logical interconnectedness of all cognised magnitudes, has given rise to the small deviations from integer value of the exponents of $\pi/2$ in the formulae for some magnitudes.

The particle data used are the current (as of June 2025) evaluations of the Particle Data Group [4]. All cosmological data are from the review of Astrophysical Constants and Parameters by the Particle Data Group [5]. 2022 CODATA recommended values of the fundamental physical constants and the hertz-kilogram relationship have been used [6].

Mass magnitudes, length magnitudes and time magnitudes are covered below, in that order. A discussion and some conclusions follow.

2. Mass magnitudes

2.1 Particle masses and related mass scales

The mass of the up quark (2.16 MeV) has been judged in Planck units to be equal to $\alpha(\pi/2)^{-100} m_{\rm P}$ [7], and has subsequently been judged in SI units to be of value

$$m_{\rm u} = (\pi/2)^{-150} \,\mathrm{kg}$$
 (1)

To be precise, $m_{\rm u} = (\pi/2)^{-149.98}$ kg.

Consequent on the expressions for the up quark mass in Planck units and in kilograms, the Planck mass $(2.176434 \times 10^{-8} \text{ kg})$ is given by

$$m_{\rm P} \approx \alpha^{-1} (\pi/2)^{-50} \, \mathrm{kg}$$
 (2)

To be precise, $m_{\rm P} = \alpha^{-1} (\pi/2)^{-49.96}$ kg.

The mass of the charm quark (1.2730 GeV) has been judged in Planck units to be equal to $\alpha^2 (\pi/2)^{-75} m_P$ [8]. Consequent on equation (2),

$$m_{\rm c} \approx \alpha (\pi/2)^{-125} \, \rm kg \tag{3}$$

To be precise, $m_c = \alpha (\pi/2)^{-124.96}$ kg.

The mass of the top quark (172.56 GeV) has been judged in Planck units to be equal to $\alpha(\pi/2)^{-75} m_{\rm P}$ [7]. Consequent on equation (2),

$$m_{\rm t} \approx (\pi/2)^{-125} \, \rm kg$$
 (4)

To be precise, $m_{\rm t} = (\pi/2)^{-124.98}$ kg.

The electron mass in Planck units was derived from the Bohr radius, which had been cognised in Planck units [7]. The electron mass (0.51099895069 MeV) was then known to be equal to $\alpha^{-1}(\pi/2)^{-125} m_{\rm P}$. Consequent on equation (2),

$$m_{\rm e} \approx \alpha^{-2} (\pi/2)^{-175} \, {\rm kg}$$
 (5)

To be precise, $m_{\rm e} = \alpha^{-2} (\pi/2)^{-174.97}$ kg.

The GUT¹ scale $(2 \times 10^{16} \text{ GeV})$ has been judged in Planck units to be equal to $\alpha^{-1}(\pi/2)^{-25} m_{\rm P}$ [7]. Consequent on equation (2),

$$m_{\rm GUT} \approx \alpha^{-2} (\pi/2)^{-75} \, {\rm kg}$$
 (6)

To be precise, $m_{GUT} = \alpha^{-2} (\pi/2)^{-75.06}$ kg.

The masses of the W and Z bosons were found in a striking symmetrical arrangement when judged in Planck units [9]. However, individually, neither mass could be cognised satisfactorily when expressed in kilograms. Instead, the W and Z boson masses have been cognised in MeV [10]. The W boson mass (80.3692 GeV) has been judged to be of value

$$m_{\rm W} = (\pi/2)^{25} \,{\rm MeV}$$
 (7)

To be precise, $m_{\rm W} = (\pi/2)^{25.01}$ MeV.

¹ Grand Unified Theory

The Z boson mass (91.1880 GeV) has been judged to be of value

$$m_{\rm Z} = \pi^{10} \,\,{\rm MeV} \tag{8}$$

To be precise, $m_{\rm Z} = \pi^{9.98}$ MeV.

The variant cognitive scheme used here enables both the W and Z boson masses to assume notable values while retaining the notion of association. Powers of $\pi/2$ and π (and of e) are usually calculated together during the cognitive procedure; since $(\pi/2)^{25} \sim \pi^{10}$, the 'intersecting' powers $(\pi/2)^{25}$ and π^{10} , are judged special powers, as are integer powers of these two quantities, such as $(\pi/2)^{125}$ and π^{50} .

Adventitiously, while investigating the relationship between atomic mass and radius for period 4 transition metal nuclides in Planck units, the Higgs boson mass (125.20 GeV), $m_{\rm H}$, was cognised in relation to the Bohr radius, a_0 ; that is, $m_{\rm H} = 2^{25}/a_0$, where $\hbar = c = 1$ [11]. It follows that

$$m_{\rm H} = 2^{25} \alpha m_{\rm e} \tag{9}$$

To be precise, $m_{\rm H} = 2^{25.00} \alpha m_{\rm e}$.

From equations (5) and (9),

$$m_{\rm H} \approx 2^{25} \alpha^{-1} (\pi/2)^{-175} \,\rm kg$$
 (10)

To be precise, $m_{\rm H} = 2^{25} \alpha^{-1} (\pi/2)^{-174.96}$ kg. In Planck units, $m_{\rm H} = 2^{25} (\pi/2)^{-125.00} m_{\rm P}$. With a factor $2^{25} \alpha$ in mind from equation (9), the Higgs field vacuum expectation value (v = 246 GeV) was judged to be of value

$$v = 2^{25} \alpha \,\mathrm{MeV} \tag{11}$$

To be precise, $v = 2^{25.01} \alpha$ MeV. The following relationship results from equations (9) and (11).

$$m_{\rm H}/v \equiv m_{\rm e}/{\rm MeV}$$
 (12)

That is, the ratio of the Higgs boson rest mass energy to the Higgs field vacuum expectation value is approximately equal to the numerical value of the electron mass in units of MeV.

The relationship in (12) implies that $m_{\rm H}/v \sim 0.511$. With $m_{\rm H} = 125.20$ GeV and v = 246 GeV, $m_{\rm H}/v = 0.509$.

2.2 Cosmological mass values

The cosmological parameters used in this section have been taken from [5]; they were derived in a 6-parameter Λ CDM cosmology fit to *Planck* data.

The mass equivalent of the total mass/energy content of a critically dense observable universe of radius 46.5 Glyr and Hubble constant 67.4 km s⁻¹ Mpc⁻¹ (3.04×10^{54} kg) has been judged in Planck units to be equal to $\pi^{125} m_{\rm P}$. To be precise, the value is equal to $\pi^{125.00} m_{\rm P}$. With baryon (light matter) density 4.93% and cold dark matter density 26.5%, the total mass of light matter in the observable universe is 0.150×10^{54} kg and the total mass of dark matter is 0.806×10^{54} kg. The total mass of all matter in the observable universe (0.956×10^{54} kg) has been judged to be of value

$$M_{\rm OU,LM+DM} = (\pi/2)^{275} \, \rm kg \tag{13}$$

To be precise, $M_{OU,LM+DM} = (\pi/2)^{275.24}$ kg. Separately, the light matter content and dark matter content of the observable universe have been judged to be of value

$$M_{\rm OU,LM} = \alpha^{-2} (\pi/2)^{250} \, \rm kg \tag{14}$$

$$M_{\rm OU,DM} = \alpha^{-2} \pi^{100} \,\mathrm{kg} \tag{15}$$

To be precise, $M_{\rm OU,LM} = \alpha^{-2} (\pi/2)^{249.35}$ kg and $M_{\rm OU,DM} = \alpha^{-2} \pi^{99.83}$ kg. As we saw for the W and Z boson masses in equations (7) and (8), the cognitive scheme used here enables both quantities (light matter content and dark matter content) to assume notable values, while retaining the notion of association (i.e., $(\pi/2)^{250} \sim \pi^{100}$).

The dark energy content, mass equivalent, of the observable universe $(2.08 \times 10^{54} \text{ kg})$ has been judged to be of value

$$M_{\rm OU,DE} = e^{125} \,\mathrm{kg} \tag{16}$$

To be precise, $M_{OU,DE} = e^{125.07}$ kg.

3. Length magnitudes

3.1 Atomic and subatomic lengths

The inverse Rydberg constant (the Rydberg wavelength), $1/R_{\infty}$ (1/10973731.568157 m) has been judged in SI units to be of value

$$\frac{1}{R_{\infty}} = \alpha (\pi/2)^{-25} \,\mathrm{m} \tag{17}$$

To be precise, $1/R_{\infty} = \alpha (\pi/2)^{-25.00}$ m.

Since $1/R_{\infty} = 4\pi a_0/\alpha$, the Bohr radius (5.29177210544 × 10⁻¹¹ m), which had previously been judged in Planck units to be equal to $(\pi/2)^{125} l_p$ [7], is given in SI units by

$$a_0 \approx \frac{\alpha^2}{4\pi} (\pi/2)^{-25} \,\mathrm{m}$$
 (18)

To be precise, $a_0 = (\alpha^2/4\pi) \cdot (\pi/2)^{-25.00}$ m. The Bohr radius was the first length scale to be cognised (in Planck units); its magnitude is equal to $(\pi/2)^{125.0006} l_p$.

The pion charge radius (0.659 × 10^{-15} m), which had previously been judged to be equal to $(\pi/2)^{100} l_p$ [7], is given in SI units by

$$r_{\pi} \approx \frac{\alpha^2}{4\pi} (\pi/2)^{-50} \,\mathrm{m}$$
 (19)

To be precise, $r_{\pi} = (\alpha^2/4\pi) \cdot (\pi/2)^{-50.01}$ m.

Consequent on the expressions for the Bohr radius in Planck units and in metres, the Planck length $(1.616255 \times 10^{-35} \text{ m})$ is given in SI units by

$$l_{\rm P} \approx \frac{\alpha^2}{4\pi} (\pi/2)^{-150} \,\mathrm{m}$$
 (20)

To be precise, $l_{\rm P} = (\alpha^2/4\pi) \cdot (\pi/2)^{-150.00}$ m.

3.2 Astronomical and cosmological distances

The distance from earth of the stunning bubble-shaped emission nebula RCW 120 (1.34 kpc [12]) has been judged in SI units to be of value

$$d_{\rm RCW\,120} = (\pi/2)^{100} \,\rm{m} \tag{21}$$

To be precise, $d_{\text{RCW } 120} = (\pi/2)^{100.02}$ m.

The distance from earth of the Andromeda galaxy (765 kpc [13]) has been judged in SI units to be of value

$$d_{\text{Andromeda}} = \alpha (\pi/2)^{125} \,\mathrm{m} \tag{22}$$

To be precise, $d_{\text{Andromeda}} = \alpha (\pi/2)^{124.97}$ m, and also $\pi^{45.00}$ m.

The radius of the observable universe (46.5 Glyr) has been judged in SI units to be of value

$$R_{\rm OU} = \alpha^{-1} (\pi/2)^{125} \,\mathrm{m} \tag{23}$$

To be precise, $R_{\rm OU} = \alpha^{-1} (\pi/2)^{124.96}$ m.

From equations (14) and (23), although it is a judgement in itself,

$$M_{\rm LM,OU}/\rm kg = (R_{\rm OU}/\rm metre)^2$$
(24)

That is, the numerical value of the mass, in kilograms, of light matter in the observable universe has been judged to be equal to the numerical value of the square of the radius, in metres squared, of the observable universe. The concept behind equation (24) has drawn from the holographic concept used to relate the dark energy density in Planck units, ρ_{Λ} , to the theoretical value, $\rho_{\rm P}$, as in the equation,

$$\rho_{\Lambda}/\rho_{\rm P} = (R_{\rm OU}/l_{\rm P})^{-2} \tag{25}$$

where the quantity $(R_{\rm OU}/l_{\rm P})^{-2}$ is of value 1.3×10^{-123} [14], which is consistent with measurements of the dark energy density.

4. Time magnitudes

The hertz-kilogram relationship² (1 Hz \equiv 7.372497323 \times 10⁻⁵¹ kg [6]) has been judged to be

$$1 \text{ Hz} \equiv \frac{1}{4\pi} . (\pi/2)^{-250} \text{ kg}$$
 (26)

To be precise, $1 \text{ Hz} \equiv (1/4\pi) \cdot (\pi/2)^{-250.02}$ kg. The quantity 2π was in mind during cognition since the energy of a photon of frequency f equals $2\pi\hbar f$; \hbar , rather than h, is

 $^{^2}$ A photon of frequency 1 Hz has an energy equal to the rest mass energy of an object of mass 7.372497323 \times 10^{-51} kg.

normally used in this study. However, it was the quantity 4π that had previously been found in this study: in equations (18) – (20) above.

Equations (2) and (26) imply that the Planck time, $t_{\rm P}$, is given by

$$t_{\rm P} \approx \frac{\alpha}{8\pi^2} (\pi/2)^{-200} \,\mathrm{s}$$
 (27)

To be precise, the Planck time is equal to $(\alpha/8\pi^2)$. $(\pi/2)^{-200.05}$ s.

To digress, the speed of light ($c = l_P/t_P$) can be written in terms of this cogniser's rules using equations (20) and (27); its magnitude is then approximately equal to $2\pi\alpha(\pi/2)^{50}$ m s⁻¹. To be precise, the speed of light is equal to $2\pi\alpha(\pi/2)^{50.05}$ m s⁻¹. The approximate magnitudes of many other physical constants can also be written intelligibly in terms of this cogniser's rules.

The mean life of the π^0 meson (the lightest hadron) from direct measurement (8.97 × 10^{-17} s) was judged to be

$$\tau_{\pi^0} = 2\pi\alpha(\pi/2)^{-75} \,\mathrm{s} \tag{28}$$

To be precise, $\tau_{\pi^0} = 2\pi\alpha(\pi/2)^{-75.00}$ s. The factor $2\pi\alpha$ in (28) also appears in the above expression for the speed of light $(2\pi\alpha(\pi/2)^{50} \text{ m s}^{-1})$, which had been produced immediately beforehand; that is, a factor $2\pi\alpha$ was in mind when it was judged to be in the expression for the mean life of the π^0 meson. Subsequently, the mean life of the π^{\pm} meson (2.6033 × 10^{-8} s has been judged to be

$$\tau_{\pi^{\pm}} = (2\pi\alpha)^2 . \, (\pi/2)^{-25} \, \mathrm{s} \tag{29}$$

To be precise, $\tau_{\pi^{\pm}} = (2\pi\alpha)^2 . (\pi/2)^{-25.02}$ s.

The mean lives of the W, Z and H bosons have been calculated from the decay widths evaluated by the Particle Data Group [4]. The mean life, τ , is calculated using the equation $\tau = \hbar/\Gamma$, where Γ is the decay width.

The mean life of the W boson $(3.076 \times 10^{-25} \text{ s})$, which was calculated from a decay width of 2.14 GeV, has been judged to be of value

$$\tau_{\rm W} = (\pi/2)^{-125} \, {\rm s} \tag{30}$$

To be precise, $\tau_W = (\pi/2)^{-124.99}$ s. Subsequently, the mean life of the Z boson (2.638 × 10^{-25} s), which was calculated from a decay width of 2.4955 GeV, was found to be equal to $(\pi/2)^{-125.33}$ s. That the mean lives of the two bosons, when written in terms of the cogniser's rules, are not symmetrically arranged about a notable value, such has been found many times for closely related values (e.g. see [8]), reflects the absence of any expectation that they are closely related.

The mean life of the Higgs boson $(1.78 \times 10^{-22} \text{ s})$, which was calculated from a decay width of 3.7 MeV, has been judged to be of value

$$\tau_{\rm H} = \alpha (\pi/2)^{-100} \, {\rm s} \tag{31}$$

To be precise, $\tau_{\rm H} = \alpha (\pi/2)^{-100.00}$ s. The uncertainty in the Higgs boson decay width evaluation is considerable (+1.9, -1.4 MeV), which means that in future the precision of equation (31) will probably change.

Three more time values have been cognised, each of which lies at the highly notable 'intersection' of two particularly significant lengths of time that are close in value.

The day (86,400 s) has been judged to be of a value that lies at the intersection of π^{10} s and $(\pi/2)^{25}$ s. That is,

$$t_{\rm day} \approx \pi^{10} \, {\rm s} \tag{32}$$

and

$$t_{\rm day} \approx (\pi/2)^{25} \, \mathrm{s} \tag{33}$$

The value of the intersection is defined as the mean value of π^{10} s and $(\pi/2)^{25}$ s, which is of value $\pi^{9.93}$ s and $(\pi/2)^{25.18}$ s. Cognition has given rise to the values $t_{day} = \pi^{9.93}$ s and $(\pi/2)^{25.17}$ s.

The time elapsed $(7.704 \times 10^9 \text{ s})$ between May 1781, when the Critique of Pure Reason, 1st edition, was published, and 5 June 2025 (the present day), has been judged to be of a value that lies at the intersection of π^{20} s and $(\pi/2)^{50}$ s. That is,

$$t_{\rm CPR} \approx \pi^{20} \, {\rm s} \tag{34}$$

and

$$t_{\rm CPR} \approx (\pi/2)^{50} \, \mathrm{s} \tag{35}$$

The value of the intersection is defined as the mean value of π^{20} s and $(\pi/2)^{50}$ s, which is of value $\pi^{19.87}$ s and $(\pi/2)^{50.38}$ s. Cognition has given rise to the values $t_{CPR} = \pi^{19.89}$ s and $(\pi/2)^{50.41}$ s.

The age of the universe (13.797 Gyr, i.e., 4.354×10^{17} s) has been judged to be of a value that lies at the intersection of $\alpha \pi^{40}$ s and $\alpha (\pi/2)^{100}$ s. That is,

$$t_{\rm universe} \approx \alpha \pi^{40} \, {\rm s}$$
 (36)

and

$$t_{\text{universe}} \approx \alpha (\pi/2)^{100} \text{ s}$$
 (37)

The value of the intersection is defined as the mean value of $\alpha \pi^{40}$ s and $\alpha (\pi/2)^{100}$ s, which is of value $\alpha \pi^{39.77}$ s and $\alpha (\pi/2)^{100.81}$ s. Cognition has given rise to the values $t_{\text{univ}} = \alpha \pi^{39.78}$ s and $\alpha (\pi/2)^{100.83}$ s.

6. Discussion

A magnitude may be cognised in any unit of the appropriate dimensionality. At first, most magnitudes were cognised in Planck units. Later, the same magnitudes, and others, were cognised in SI units. A system of knowledge was then constructed, in which the magnitudes in SI units of quantities that are judged particularly significant by this cogniser assume notable values in terms of the cogniser's conceptual rules. Thoughts of consistency, symmetry, elegance and physics all influence the value adopted by a magnitude in cognition. Included in the system of knowledge are magnitudes derived from the directly cognised magnitudes according to relationships known from physics. The numerical values of the magnitudes in SI units of many particularly significant physical quantities may then be expressed as integer powers of $(\pi/2)^{25}$, sometimes in combination with an intelligible multiplicative factor.

The magnitude need not be denominate – any number can be cognised by the procedure described in the introduction – and the formula need not have a physical basis. Although a magnitude is fixed for all on first cognition, its subsequent cognition in different units is possible, as we have seen for magnitudes cognised in both Planck and SI units. However,

since the magnitudes of the many particularly significant quantities included here have assumed their notable values specifically in terms of this cogniser's mathematical rules, it is clear that they had not previously been cognised by another cogniser. It is only in the knowledge of the procedure necessary for the cognition of a magnitude that cognition can be acquired, that is, in the synthesis of a sensible intuition and a mathematical concept based on rational and other intelligible numbers.

7. Conclusions

- 1. A mathematical procedure has been invented to cognise the magnitude of a quantity.
- 2. Cognition is gained in the synthesis of sensible intuition and concept.
- 3. A cognised magnitude is intelligible.
- 4. The magnitude of a particularly significant physical quantity is judged to be of special value.
- 5. The value of a magnitude is fixed on first cognition, and is universally objective.
- 6. A logically coherent system of knowledge has been established, of the magnitudes in SI units of particularly significant physical quantities.

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