REVIEW

Why Goldbach's strong conjecture is hard to prove by amateurs and renowned mathematicians ?

Bahbouhi Bouchaib

Independent researcher in Mathematics.

Nantes, France

1. **Abstract**

This essay addresses the often-debated question of whether an independent mathematician—one outside the framework of traditional academic institutions—can resolve one of the most enduring open problems in number theory: the strong Goldbach Conjecture. We examine historical, institutional, and methodological considerations and confront assumptions held within the mathematical community. The analysis suggests that while the challenge is formidable, advances in computational tools, access to academic literature, and creative heuristics have opened new opportunities, regardless of one's affiliation. But the hardest point in Gldbach's strong conjecture is to prove it to infinity. Even if a new theorem demonstrates Goldbach's strong conjecture, there must be no counterexample to infinity. Till now, none can map prime numbers to infinity without using primeness tests that become probabilistic when numbers tend to infinity.

2. Can an Independent Mathematician Prove the Goldbach Conjecture?

The Goldbach Conjecture, positing that every even integer greater than 2 is the sum of two prime numbers, remains one of the most famous unsolved problems in number theory. While generations of professional mathematicians have attempted to solve it using powerful tools from analytic number theory [1-6], it is still unresolved more than 280 years after it was first proposed.

This naturally raises the question: could an independent mathematician, working outside formal academia, prove the Goldbach Conjecture? While many scholars would be skeptical, mathematical history offers a more nuanced view.

Academic skepticism arises for several valid reasons. The conjecture is entangled with highly complex structures: the distribution of prime numbers, the Riemann zeta function, sieve theory, and deep analytic techniques [2-6]. Proving the conjecture in full likely requires either a breakthrough in these areas or a radically new approach. Moreover, most current efforts rely on heavy machinery like the circle method [1], Fourier analysis, and zero-density estimates of L-functions—topics not easily mastered without formal training.

Nevertheless, mathematics is rich with counterexamples to the idea that only institutional affiliation leads to discovery. Fermat, Ramanujan, and Yitang Zhang each contributed fundamentally to mathematics outside traditional structures. Zhang's landmark result on bounded prime gaps, for instance, came while he worked as an independent scholar.

Thus, the capacity to solve a major conjecture like Goldbach's may not lie in credentials but in methodology. An independent mathematician could make headway by reframing the problem—perhaps by modeling the distribution of primes geometrically, probabilistically, or through computational insights. New heuristic models or predictive patterns might eventually suggest a path to a formal proof.

One key challenge lies in the dual nature of the conjecture: it is easily verified computationally for vast ranges, but this verification doesn't count as a proof. Still, such large-scale verifications may offer empirical data to inform or validate new theoretical frameworks. If a predictive model consistently identifies valid Goldbach pairs, it could be the seed of a new paradigm.

Ultimately, mathematics values rigor above all. But it also values creativity. The Goldbach Conjecture is simple to state yet profoundly difficult to prove, and as such, may be amenable to unexpected insights. Whether those insights arise from a tenured professor or a solitary thinker is immaterial, as long as they illuminate the truth.

In conclusion, while the professional mathematical community may lean toward skepticism, history shows that true advances often emerge from unorthodox sources. The decisive factor is not institutional endorsement but the depth, coherence, and verifiability of the idea. It is mathematically plausible—if not common—that a non-academic mind could someday prove the Goldbach Conjecture.

3. Known and unknown fields in mathematics.

In modern mathematics, we can define two fields: the known and the unknown. However, the golden rule in mathematics is the formal proof or demonstrable theorem that is verified in both the known and the unknown fields. When we use computing or the power of modern computers, especially those with very high processing and memory capacity, we can push the limits of the known field but we cannot claim to hold a formal mathematical truth [2]. In addition, the known field is visible depending on the means available to the researcher. It's as if you wanted to explore an island. On foot, you will spend an enormous amount of time and you will accumulate immense data that is impossible to analyze, whereas with drones or sophisticated equipment such as GPS, you will very quickly succeed in drawing the map of the island. This is what makes the big difference between amateurs with their limited means and experienced researchers within institutions and academies who have all the most sophisticated means to analyze a question. So in computing, an amateur has very little chance unless by some miracle he finds a new formula or a new, original hypothesis, which is like looking for a needle in a haystack. Goldbach's strg conjecture is really hard to prove although it is easier to verify with numbers or with acceptable arguments in the known fields.

4. A truth in the known field must be verified in the unknown fields to be acceptable. In fact, the vast majority of articles on the strong conjecture of Goldbach claim its solution, but their authors are frustrated and sad to see that no peer-reviewed academic journal is interested in it. ***Indeed, the reason is that these articles, which are often based on acceptable arguments, are the product of the known field but cannot be valid in the unknown field***. For example, the current articles on the strong conjecture of Goldbach were possible to make because we have lists of known prime numbers and primality softwares (these two are part of the known field), but without knowing them, none of these articles would have been possible. This work of archiving known prime numbers and creating factorization and primality sites allows amateur researchers to test their arguments very quickly. Unfortunately, the argument must also be verified in the unknown field, for example with unknown prime numbers of millions or billions of digits that only academic researchers and institutions can barely study closely. Even if an amateur brandishes a formula that verifies Goldbach's strong conjecture at infinity, a sort of historical theorem, he must verify it at infinity, which requires a very high cutoff, knowing that determining the primality of a prime number with millions of digits can crash a very high-capacity computer. Let's say there is a critical and historical threshold beyond which Goldbach's strong conjecture can be formally verified. This threshold is unknown. The known field must push its limits to this threshold before this conjecture can be resolved. The big problem in mathematics is that nothing is actually true or absolutely true, and we can always look for a counterexample. For example, Phytagoras' theorem appears absolutely true in all right triangles, but we can always have fun looking for a right triangle that escapes this rule and that will constitute a counterexample. The theorem remains true as long as it cannot be contradicted. Imagine that Goldbach's strong conjecture must remain true for even astronomical even numbers, which will require searching for new prime numbers that exceed all the capabilities of known primality software, which will either be probabilistic or they will simply crash.

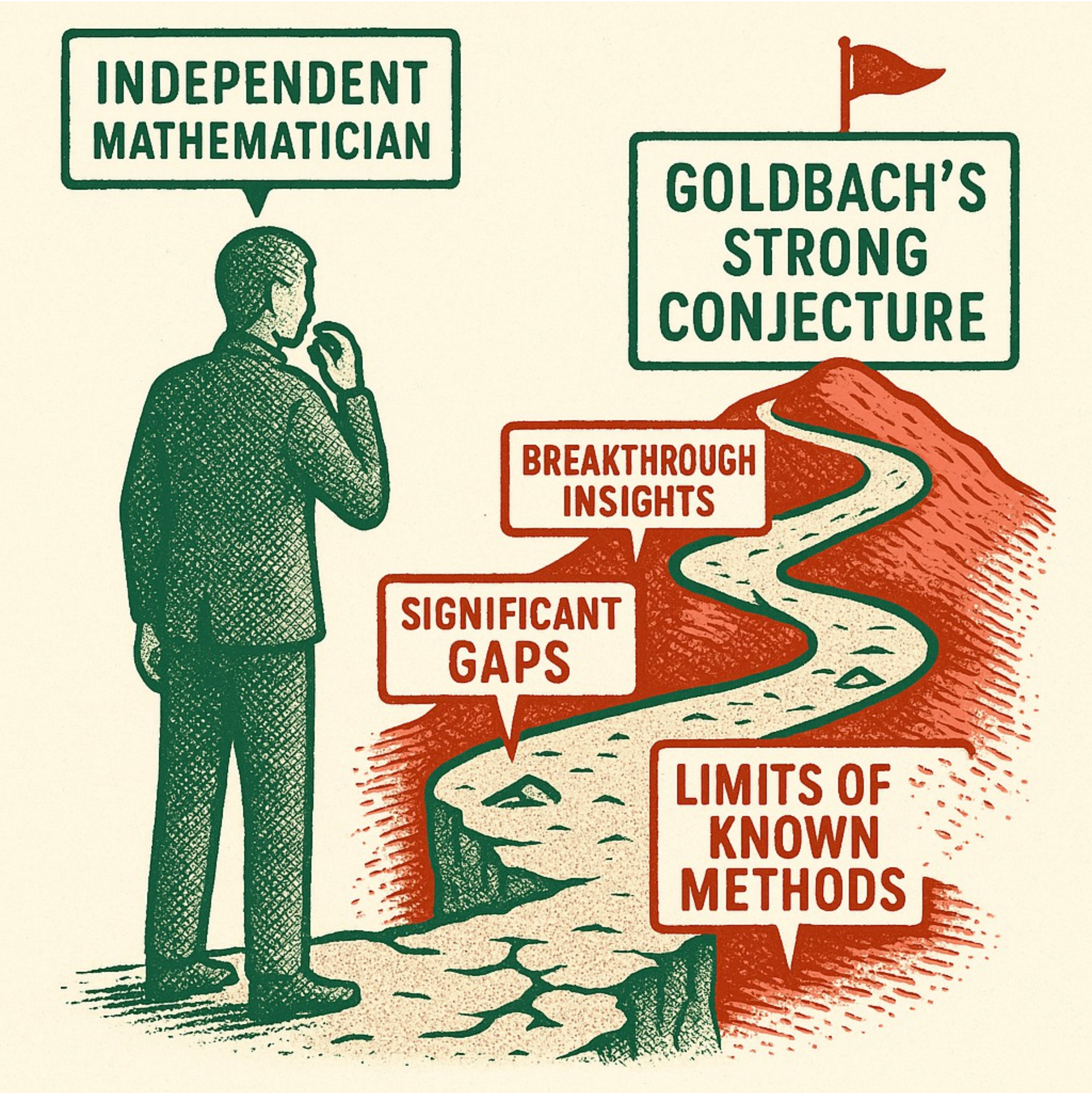
So a theorem on strog conjecture of must be verified at infinity. **We can find formulas that work well in the known field, but the doubt persists because we do not know if a counterexample exists in the unknown field**. The problem is not the fact that a proposition is true or false i mathematics, but if it is true, it must be verifiable at infinity, which is impossible when we touch prime numbers. Even if new theorems on the length of prime gaps or infinity of twin primes might appear in near future, Goldbach's strong conjecture will not be proved unless no shadow of doubt remains to infinity (or at least be able to verify it to infinity). This is the hardest part in pure mathematics, when ***A*** is true, ***A*** must be true with no one counterexample to infinity. That is the main reason why Goldbach's strong conjecture is not proved till now and that is also the reason why works of amateurs (despite their good quality) are ignored by renowned mathematicians.

5. Future Perspective

Looking forward, it is likely that progress on the Goldbach Conjecture will continue to be multi-pronged—balancing heuristic models, computational experimentation, and partial results. Independent researchers may contribute significantly by exploring unorthodox methods or by pushing computational verification to new limits. Improved access to collaborative platforms and peer feedback mechanisms may also increase the impact of non-institutional efforts. Ultimately, the resolution of the conjecture may result from a synthesis of ideas across disciplinary and institutional boundaries. ***It is possible that solving Goldbach's strong conjecture will require a revolution in mathematics, namely the emergence of a new number theory. Or even that its resolution will go beyond the formal mathematical system we know today. Its resolution will certainly revolutionize mathematics and will only be possible if the mystery of infinitely prime numbers is finally clarified***.

6. Visual Representation of the Goldbach Conjecture Challenge

This illustration offers a symbolic representation of the pursuit to resolve the strong Goldbach Conjecture. The green path symbolizes the predictive models and harmonic structures discovered in recent research. The red regions represent the unknowns, obstacles, and increasing gaps that mathematicians encounter when approaching larger even numbers. Together, they illustrate the interplay between order and complexity in the distribution of prime numbers.



References

1. [1] Hardy, G.H., and Littlewood, J.E. 'Some Problems of 'Partitio Numerorum'; III: On the expression of a number as a sum of primes.' Acta Mathematica 44 (1923): 1–70.
2. [2] Oliveira e Silva, T. 'Goldbach Conjecture Verification Results.' http://sweet.ua.pt/tos/goldbach.html
3. [3] Tao, T. 'The parity problem in sieve theory.' arXiv:1401.0846 (2014).
4. [4] Ramaré, O. 'On Šnirel'man's constant.' Annales de l'Institut Fourier, 41(4): 921–930, 1991.
5. [5] Granville, A. 'Harald Cramér and the distribution of prime numbers.' Scandinavian Actuarial Journal 1 (1995): 12–28.
6. [6] Helfgott, H.A. 'The ternary Goldbach conjecture is true.' Annals of Mathematics, 190(1): 1–100, 2019.