

## **Galaxy Rotation Curves: Dark Matter as a Solution**

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## **Abstract**

This paper investigates the galaxy rotation curve problem while validating dark matter as the primary solution. Standard Newtonian Dynamics show that stellar speeds decrease with radial distance from the central area of galaxies. Since the early 20th century, physicists have found flat rotation curves during observations, which demonstrate unidentified mass in galaxies that they have termed dark matter. The primary focus of this investigation delves into how well dark matter functions as an answer to resolve the galaxy rotation curve issue. The study evaluates the accuracy of dark matter models, particularly the NFW and Burkert profiles, by comparing mathematical model-generated rotation curves with experimental data obtained from the Gaia and APOGEE databases. The first outcome shows that standard baryonic matter explanations do not duplicate observed rotation patterns, particularly in the Milky Way. The combination of NFW and Burkert profiles produces significant improvements in the match while analyzing older data points. Recently acquired data with elevated accuracy shows departures from current dark matter theory to describe galaxy rotation curves. Current data requires evaluating alternative scientific models, including Modified Newtonian Dynamics (MOND) and revised models of dark matter distributions. This paper indicates that dark matter remains the primary theory for unexplained observations, yet details that high-quality new datasets demand improved theoretical models. The research advances the ongoing astrophysical discussions about how galaxies function and shape dark matter patterns and potential modern physics theories beyond existing theoretical boundaries.

## **Introduction**

The area of galaxy rotation curves in astrophysics is receiving increasing attention from the physics community, especially regarding its implications for dark matter and gravitational forces. Before diving into this topic, defining specific key terms would be helpful. Galaxy Rotation Curves are the central problem that this paper wishes to address. To make a rotation curve, one calculates the rotational velocity of celestial objects along the length of a galaxy by measuring their Doppler shifts, then plots this quantity versus their respective distance away from the center. Explaining Doppler shifts is best done using light waves; if one approaches a light source, one sees shorter wavelengths (a blue shift), while if you are moving away, one sees longer wavelengths (a red shift). The issue with galaxy rotation curves is quite complicated. When studying galaxies, it is invariably found that the stellar rotational velocity remains constant, or "flat," with increasing distance away from the galactic center. This result is highly counterintuitive since, based on Newton's law of gravity, the rotational velocity would steadily decrease for stars further away from the galactic center. The only way for these values to remain flat would be if extra mass in the galaxy were not visible. Thus, dark matter was proposed as a solution for this problem. It is generally accepted that the dark matter must be located in a massive, roughly spherical halo enshrouding each galaxy. Dark matter is an invisible and hypothetical matter that does not interact with light or electromagnetic radiation. It was first hypothesized as a way for astronomers to explain the motion of celestial bodies at the edge of galaxies, as their orbits were too fast for the amount of matter present in the galaxy. With this brief description out of the way, the problem can now be addressed.

This paper's research question is, "To what extent is dark matter a viable solution to the galaxy rotation curve problem?". New data regarding galaxy rotation curves has cast doubt on this problem. While a majority of galaxies still follow this flat rotation curve, a significant portion of galaxies do not, making this investigation worthwhile. This paper uses computer simulations and analytical methods to analyze this new data and provide a comprehensive view of the galaxy rotation curve problem. This may lead to discussions on dark matter and its role as the solution to this problem, as declining galaxy rotation curves would mean the necessity of dark matter as a solution would decrease. The status of galaxy rotation curves as a problem could also be questioned, as declining curves detected in the Milky Way would be a different result than expected. Ultimately, while this is a fascinating field of research, its complexity makes it challenging to analyze and provide an accurate hypothesis comprehensively.

## **Literature Review**

The discovery of galaxy rotation curves starts with the discovery of M31 (Andromeda) and how, using a spectrogram, physicists also found that the galaxy was indeed rotating. Using this data, several authors also calculated the angular velocity of Andromeda's central region and the galaxy's mass distribution. Horace Babcock (Bertone & Hooper, 2016) then presented the rotation curve of M31 and found high values for the orbital velocity at large radii, which were high enough to be at odds with modern measurements. Approximating M31 as a sphere surrounded by a flattened ellipsoid, he calculated the mass distribution of the galaxy, recognizing that the observed rising rotation curve at large radii implied the existence of large amounts of mass in the outer parts of the galaxy (Bertone & Hooper, 2016). With technological advances in

the 1960's, physicists could make more conclusive predictions about galactic rotation curves and mass distributions. Ken Freeman (Bertone & Hooper, 2016) then compared the radius at which the rotation curve was predicted to peak and found that for M33, the observed rotation curves peaked at larger radii than predicted, which prompted him to conclude that if the data was correct, the galaxies must have additional matter that had been undetected. Then, in 1978, Albert Bosma (Bertone & Hooper, 2016) published the results of his PhD thesis, which convincingly proved that most of these objects had flat rotation curves out to the largest observed radius, which again exceeded the optical size of the galaxies, therefore demonstrating that their mass continued to grow beyond the region occupied by the stars and gas.

Typically, based on visible matter, rotational velocities should decrease with distance from a galaxy's center, but observations show flat rotation curves, suggesting unseen mass (dark matter) extends beyond visible regions (Alexeev, 2017). The dark matter likely forms a large, spherical halo around galaxies. Studies of galaxy clusters and superclusters also support this, as large mass-to-light ratios indicate significant dark matter across cosmic structures. When studying other galaxies, it is invariably found that the stellar rotational velocity remains constant or "flat" with increasing distance away from the galactic center. This result is highly counterintuitive since, based on Newton's law of gravity, the rotational velocity would steadily decrease for stars further away from the galactic center. Analogously, inner planets within the Solar System travel more quickly about the Sun than the outer planets. Fritz Zwicky (White, n.d.) of Caltech discovered the presence of dark matter in a galaxy cluster and found that nearly 10 times as

much mass as observed in visible light was needed to keep the individual galaxies within the cluster gravitationally bound. It was clear to Zwicky, as it had been to Oort, that a large sum of mass was extant, which was not visible. This missing mass accounted for the flat portion of the curve since the orbital velocity would only stay constant if there were additional mass in these galaxies.

New literature has questioned this paradigm of dark matter being a solution to the galaxy rotation curve problem, with scientists from both Case Western Reserve University and MIT (Ou et al., 2024) providing data that suggests the answer is not as clear cut as previously thought. This is the gap I seek to address, and with conflicting theories presenting varied explanations, data analysis will be required to answer this question.

## **Methods**

In order to effectively analyze galaxy rotation curves and provide evidence for the necessity of dark matter as a solution to this problem, the method needs to be a two-pronged approach. First, galaxy rotation curves will be generated using a mathematical relationship. This will be done by using only spiral galaxies for simplicity and approximating these galaxies as cylindrical to use the proper mass equation (Sofue, n.d.). The mass equation is:

$$\int_0^R \rho(r) 2\pi r h * dr = M(r)$$

Where  $\rho(r)$  is the density of the stars in the galaxy,  $r$  is the radius, and  $h$  is the height of the cylinder approximation. This equation is primarily helpful in this analysis because it helps us calculate the velocities of celestial objects in the specific galaxy we are studying. With the mass of the galaxy now identified, we can plug this value into the Keplerian equation for velocity:

$$V(R) = \sqrt{\frac{GM(r)}{R}}$$

Here  $R = r$ . These equations combined can then be used to generate a galaxy rotation curve for a specific galaxy. This would be a predicted rotation curve, which can then be used to see if a discrepancy exists between an actual and a predicted one. This predicted curve will be generated through data analysis through code, and a graph will be generated from the equations.

Researchers M. Persic and P. Salucci, in their report titled "Rotation curves of 967 spiral galaxies," lay out the groundwork for multiple galaxy rotation curves and generate them for a variety of different galaxies (Persic & Salucci, 1995). This repository of information will be used as a source of confirmation to ensure that the galaxy rotation curves generated are approximately accurate compared to the researcher's model. This comparison is helpful since the researchers are able to use more accurate methods and measurements to generate their curves.

The actual rotation curve will come from astrophysical data that is available for public use. The databases will be peer-reviewed and published either by major astrophysical organizations or research groups and will be of relative recency (*Home | NASA/IPAC Extragalactic Database*, 2018). This will be done by collecting, analyzing, and graphing data regarding radial distance

from the galaxy's center and orbital velocities of specific celestial objects in those galaxies. This will be done through finding data in datasets and plotting radial distance vs orbital velocities.

This plot will be done either through code or specifically with Keplerian models. This approach is used to compare two models and see the discrepancies that arise because of them (Sofue, 1996). The data will be analyzed by graphing radial distance vs orbital velocity using code specifically in Python. This will then be compared to predicted curves and will overlap to showcase the discrepancy. It will be interpreted depending on the discrepancy itself. If it is significant, dark matter will play a role and be analyzed accordingly. If it is not significant, another explanation will have to be found, and dark matter would not be the primary driver.

The dark matter portion of this problem will come from dark matter haloes that are present in galaxies that provide the additional mass the galaxy requires to have a flat galaxy rotation curve. The object of our analysis will be to prove this idea with enough solid evidence. First, to define a dark matter halo, it is a basic unit of cosmological structure. It is a hypothetical region that has decoupled from cosmic expansion and contains gravitationally bound matter (Wechsler & Tinker, 2018). They usually envelop the disk and extend beyond the galaxy's edge. For the purpose of our analysis to be simple, we will use something called a "nonsingular isothermal" profile of dark matter (Freeman, 1970). It is a model where we assume that the dark matter in the center of the galaxy is finite and remains relatively constant throughout a particular core region. Using this assumption allows us to use equation (Battaner & Florido, 2024)

$$\rho = \frac{\rho_0}{1 + \left(\frac{R}{R_0}\right)^2}$$

Where  $R$  is the galactocentric radius (distance of the object from the center of its galaxy) and is the dark matter density distribution. Using this density distribution, we can then use this model to address the apparent lack of mass in the galaxy. The initial conclusion could be schematized by considering the rotation curve to be high and flat. If it is high, the dark halo should be massive; if it is flat, it should be vast. Indeed, the flatness of the rotation curves was explained "too" well because if the disk and halo had such different distributions, meticulous matching was required between the falling disk rotation curve and the rising halo one. (Bahcall and Casertano, 1985, van Albada and Sancisi, 1986).

Then, to obtain its distribution, there are several different techniques. One of the most widely used is the "maximum disk hypothesis" (Begeman, Broeils, and Sanders, 1991). Here, the mass-to-light  $M/L$  ratio is fixed for the different visible components, with values higher than about 10 that are difficult to assign to a stellar population. Then, the innermost regions are adjusted so that the disk can produce the observed rotation curve without a halo. The disk  $M/L$  obtained is then kept constant at all radii, and the circular velocity of the halo is obtained for higher radii. Another possibility is the so-called "best fit" technique. Adopting a halo profile defined with several adjustable parameters is necessary in this case. These models provide a way to measure dark matter distribution in these specific galaxies and then use them to explain the discrepancy between the curves if they exist. These models will be added to the Keplerian

velocity equation, and the mass will be changed, generating a rotation curve that is different from the original predicted curve. This analysis supports the study's goals as it generates the object of analysis, that being the galaxy rotation curves themselves, and it compares them to predicted curves, showcasing how dark matter plays a role in the discrepancy between them. All information and data used in this paper are ethical and do not violate ethical considerations.

## Findings

This section will start by showcasing the problem using the Milky Way's circular velocity and radius data (Eilers et al., 2019). Then, we will look at the two most prominent models for dark matter haloes, NFW and Burkert profiles. NFW, or the Navarro-Frenk-White profile (Navarro et al., 1996), is one of the most commonly used model profiles for dark matter halos where the density is given by:

$$\rho(r) = \frac{\rho_0}{\frac{r}{R_s} \left(1 + \frac{r}{R_s}\right)^2}$$

Where  $\rho_0$  = “scale radius” and  $R_s$  are parameters that vary from halo to halo. The rotational velocity is given by

$$v_{nfw}(r) = v_h \sqrt{\frac{r_0}{r} \ln\left(\frac{r+r_0}{r_0}\right) - \frac{r_0}{r+r_0}}$$

Where  $v_h = (4\pi\rho_0 r_0^2 G)^{1/2}$ . The dark matter density profiles of massive galaxy clusters can be measured directly by gravitational lensing and agree well with the NFW profiles predicted for

cosmologies with the parameters inferred from other data. Another common profile is the Burkert profile (Burkert, 1995), where the density is given by:

$$\rho(r) = \frac{\rho_0 r_0^3}{(r_0+r)(r^2+r_0^2)}$$

And the rotational velocity is given by:

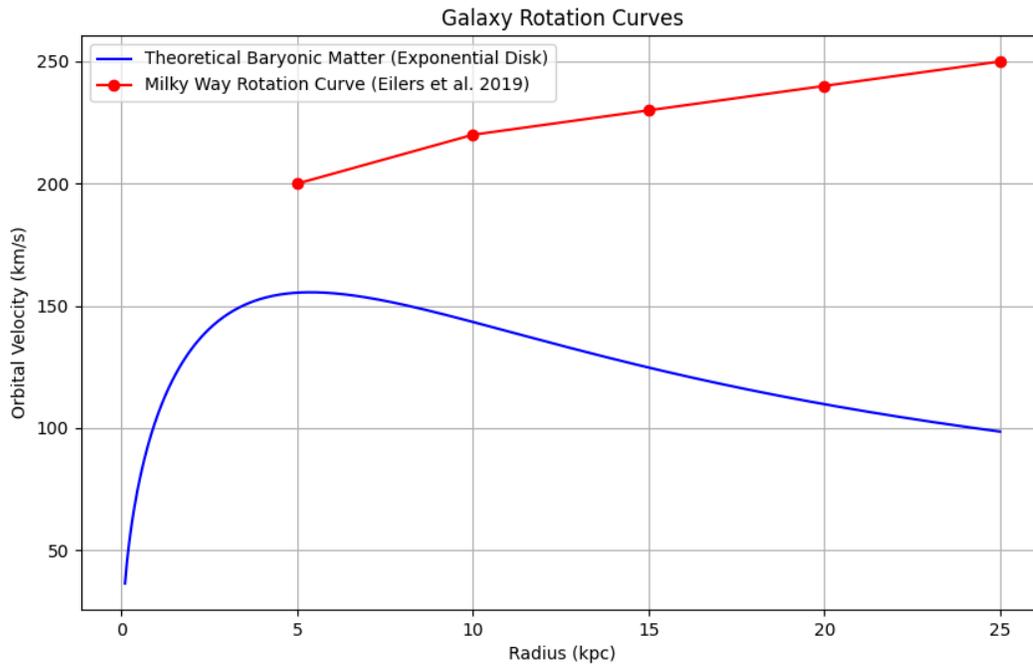
$$v_{bur}(r) = v_h \sqrt{\frac{r_0}{4r} \left[ \ln\left(\frac{(r_0+r)(r^2+r_0^2)}{r_0^4}\right) - 2 * \arctan\left(\frac{r}{r_0}\right) \right]}$$

Together, these relationships help us understand the NFW and Burkert profile models we will be using for dark matter (Lin & Li, 2019). We will primarily focus on using the NFW model, as there is substantial evidence to back this model up. After this, we will focus our attention on old Milky Way data, which confirms the dark matter model relatively accurately with little deviation. Then, we will shift our focus to the new Milky Way data from more accurate instruments, which shows that the predicted dark matter in the Milky Way might differ from the actual dark matter in the galaxy. Our first result clearly indicates that there is indeed a discrepancy between theoretical rotation curves predicted using baryonic matter models and actual rotation curves. The NFW and Burkert dark matter models also show apparent differences in their respective rotation curves and apparent discrepancies compared to baryonic matter curves. The old Milky Way data shows a clear trend toward rotation curves predicted when dark matter halo density profiles are added to traditional baryonic models. The curve generated by using this model is accurate, with little discrepancy between the actual values and the curve itself. The new data (Ou et al., 2024) doesn't agree with the dark matter density profile curve and, in fact, agrees more with the traditional baryonic matter curve, indicating that the amount of dark matter present in the galaxy may not be

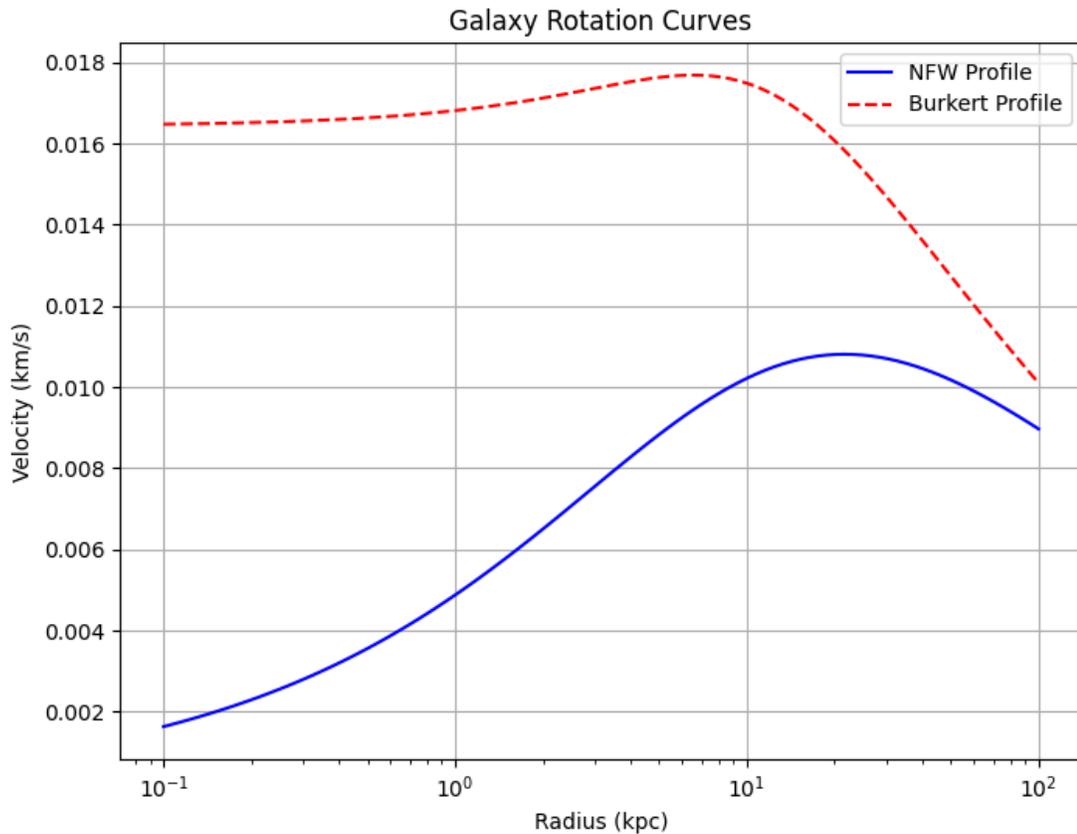
calculated accurately and might need changes to reflect these changes better. These results indicate that more work needs to be done to fully understand the new data and its implications regarding galaxy rotation curves and dark matter haloes present in these galaxies.

## **Analysis**

The first graph contrasts the theoretical baryonic matter distribution, which is described as an exponential disk, with the known Milky Way rotation curve (data from Eilers et al. 2019). According to Newtonian predictions, the baryonic matter curve (blue) peaks at a few kiloparsecs and then declines. However, rather than declining at increasing radii, the observed rotation curve (red) stays relatively flat, suggesting the presence of extra mass that cannot be explained by visible matter alone. One of the key pieces of evidence supporting dark matter is this disparity. Figure 1 shows this discrepancy and the two rotation curves, which were generated using models that were available online. This discrepancy can be explained by the lack of additional mass in the galaxy, with baryonic matter not accounting for dark matter which makes up a majority of the galaxy's mass, almost 85% (Eleftherios Papantonopoulos, 2007).

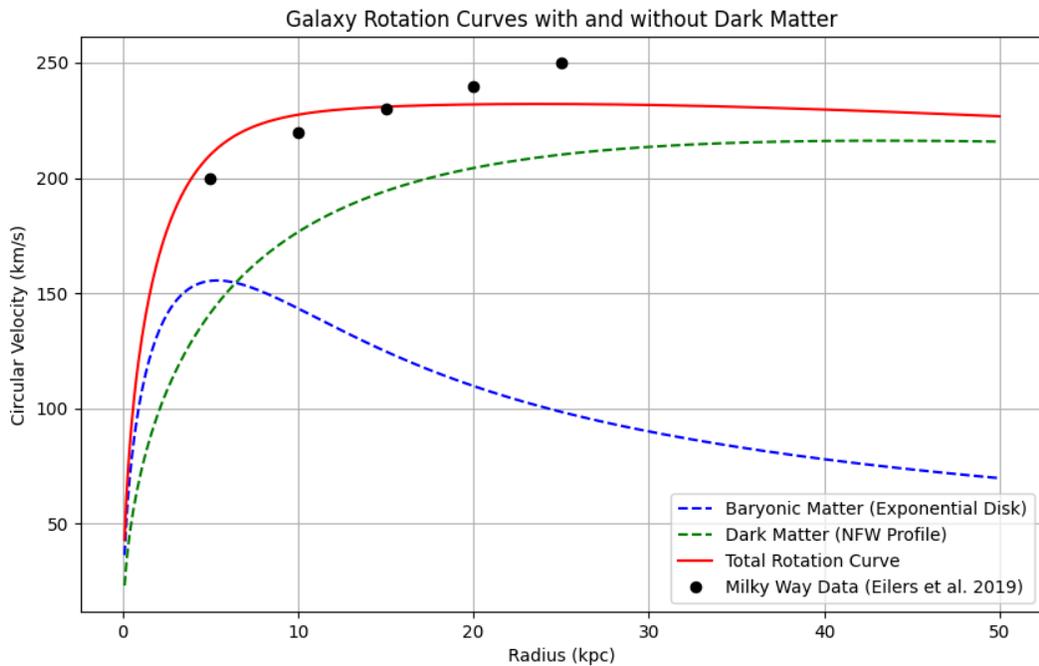


The second graph introduces two standard dark matter density profiles: the Navarro-Frenk-White (NFW) and Burkert profiles. The NFW profile, derived from cold dark matter simulations, predicts a central density, whereas the Burkert profile suggests a cored distribution with a central region of nearly constant density. The graph shows the NFW curve (blue) increasing until it hits a maximum and starts decreasing, and it shows the same trend for the Burkert profile (red), albeit it starts at a higher velocity and increases at a much slower rate before it reaches a maximum. From this, we can conclude that adding dark matter density profiles into the traditional baryonic matter models will change the rotation curves significantly.



The baryonic matter curve, the NFW dark matter curve, and the total rotation curve obtained from both components are synthesized in the third graph. The overlay of the raw data from Eilers et al. (2019) shows how the observed velocities match the rotation curve. The commonly held belief that galaxies exist in extensive dark matter halos is supported by this graph, which emphasizes the need to consider dark matter considerations to match observable data. The graph shows this clearly, as the NFW profile added to the baryonic matter model gives us a curve that fits the data (Eilers et al., 2019). The dataset is used from this specific source, and the curves are generated using the profiles mentioned earlier in the findings section. While the curve doesn't exactly overlap with the data, it represents the data well enough to make an

assumption of its correctness. The cylindrical approximation used in this paper has also contributed to the slight deviation witnessed.



The baryonic matter curve, NFW profile, and best-fit total rotation curve are shown in the final graph, together with extra data from Xiaowei Ou et al. This graph improves the dark matter model by maximizing parameter selections to attain the best match to observational data. While acknowledging some deviations that might point to novel physics, such as changes to gravity or other dark matter distributions, the improved fit emphasizes the predictive ability of dark matter models. The best-fit line on the graph (purple) indicates the new data gained by MIT researchers from the new Gaia and APOGEE measurements. Gaia is an orbiting space telescope that tracks the precise location, distance, and motion of more than 1 billion celestial objects throughout the Milky Way galaxy, while APOGEE is a ground-based survey. The discrepancy observed between the predicted total galaxy rotation curve using dark matter considerations and the actual values

for the stars in the Milky Way suggests that dark matter might not be the most optimal solution or that the theories for dark matter density profiles and halos need to be reworked. The data follows a baryonic matter curve, indicating that dark matter may not be as abundant in these galaxies as previously thought.

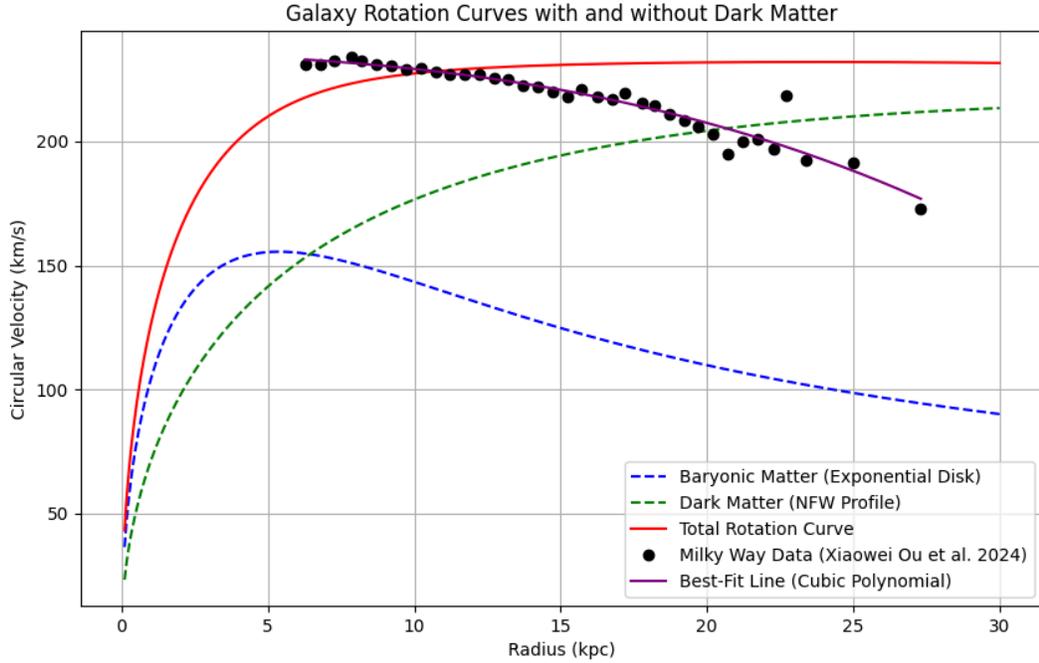


Table 1: Circular Velocity for the Milky Way

$R$	$v_c$	$\sigma_{v_c}^+$	$\sigma_{v_c}^-$	$N_{\text{star}}$
[kpc]	[km/s]	[km/s]	[km/s]	
6.27	231.07	1.28	1.00	764
6.78	230.93	0.97	0.97	676

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7.28	232.87	1.13	0.79	812
7.86	234.14	0.63	0.62	1631
8.19	232.83	0.56	0.55	2270
8.71	231.27	0.54	0.66	1445
9.23	230.47	0.44	0.46	2179
9.72	229.16	0.54	0.43	2505
10.24	229.37	0.25	0.50	2560
10.74	227.95	0.53	0.34	2528
11.23	227.09	0.44	0.46	2692
11.73	227.15	0.35	0.45	2419
12.23	226.90	0.39	0.41	2285
12.73	225.61	0.64	0.53	1994
13.22	224.95	0.66	0.69	1665
13.72	222.79	0.68	0.53	1308
14.22	222.13	1.04	0.64	938
14.73	220.08	0.65	0.89	641

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15.24	218.25	1.14	0.79	449
15.72	221.16	1.06	1.24	322
16.24	218.30	2.17	1.88	243
16.77	217.07	1.39	1.35	164
17.21	219.56	1.80	1.69	150
17.77	215.49	1.98	2.08	114
18.23	214.62	2.24	1.59	102
18.73	210.89	1.42	1.32	94
19.22	208.48	2.20	1.65	71
19.71	205.97	1.20	1.55	70
20.22	202.97	1.55	2.25	65
20.72	195.16	2.60	1.97	46
21.22	200.20	2.84	1.45	38
21.72	201.11	2.72	3.81	30
22.27	196.79	4.82	6.25	14
22.71	218.65	14.93	17.54	11

23.40	192.49	4.25	4.8	22
25.02	191.48	6.41	9.61	11
27.31	172.98	15.82	17.07	7

While the NFW and Burkert profiles provide a good fit, alternative explanations such as Modified Newtonian Dynamics (MOND) propose adjustments to gravity rather than invoking dark matter. This solution could explain the discrepancy without using non-baryonic matter; however, more research is needed to flesh out this solution fully. The assumption that dark matter is cold and collisionless is built into these models. Alternative frameworks like self-interacting or warm dark matter, might lead to different conclusions. These frameworks, however, show that the discrepancy is significant enough to cause further investigation into the subject and the problem of the rotation curves themselves. The best-fit line is a cubic function that is significantly different from either of the profiles we used to construct the curves themselves. With the best-fit line decreasing like baryonic matter curves, the necessity of dark matter as a solution to the problem becomes a question that needs to be answered. The data indicates that the Milky Way indeed follows the Keplerian decrease expected of galaxies, albeit to a lesser extent and not as pronounced. However, given that these deviations are slight and the complexity of the problem makes the analysis hard, we can conclude that dark matter is still the best solution for galaxy rotation curves.

One of the most reliable methods for determining the presence of dark matter is still the analysis of galaxy rotation curves. When comparing theoretical models to empirical evidence, we discover that baryonic matter cannot account for the observed rotational velocities. Though differences still call for more investigation, adding dark matter models, such as the NFW profile, significantly increases agreement with evidence. Our knowledge of dark matter and its function in galaxy formation and development will be improved by future research employing high-resolution simulations and novel observational methods (such as gravitational lensing).

## **Conclusion**

The study of galaxy rotation curves has always been of great interest to the scientific community, shaping our understanding of dark matter and galaxies as we know them. Analyzing newly available data from Gaia and APOGEE (Chu, 2024) suggests that dark matter, as currently modeled by the NFW and Burkert profiles, does not fully account for observed galactic dynamics. However, the discrepancies identified in this study are relatively small, leaving the possibility that dark matter remains the best available explanation. This research highlights the need for refining existing models or considering alternative frameworks that better reconcile observational data with theoretical predictions. The debate surrounding the validity of dark matter as an explanation for galaxy rotation curves hinges on models' ability to describe observational data accurately. The NFW and Burkert profiles have traditionally been used to fit rotation curves with varying degrees of success. Introducing new data from Gaia and APOGEE

provides unprecedented precision in stellar motion measurements, allowing for a more critical assessment of these models. The findings of this study suggest that the traditional profiles may require modification or reconsideration in light of the new data. This research connects to broader astrophysical discussions regarding the nature of dark matter and alternative theories such as Modified Newtonian Dynamics (MOND) or emergent gravity models. It also has implications for large-scale cosmological simulations that rely on dark matter assumptions. The study's findings encourage further scrutiny of existing models and their alignment with high-resolution observational datasets. Through this investigation, it became evident that dark matter remains a compelling explanation for galaxy rotation curves; discrepancies in the data suggest that current models are incomplete. The study underscores the importance of high-precision astronomical observations in testing theoretical frameworks and refining our understanding of galactic dynamics.

The research findings indicate a slight but meaningful deviation from the expected dark matter distributions that NFW and Burkert profiles predicted. This deviation necessitates further investigation into whether adjustments to dark matter models can reconcile these differences or if alternative explanations should be considered. One advantage of this study is its reliance on high-precision data from Gaia and APOGEE, which enhances the reliability of the results. However, a limitation is the scope of the dataset—while extensive, it may not yet fully represent the diversity of galaxies present in the universe. The GAIA instrument primarily consists of data from the Milky Way and even then only represents a mere 1% of all celestial bodies in the

galaxy. While precise, it is only a small sample of one galaxy. Another potential disadvantage is the reliance on existing theoretical profiles, which have not been proven and may not capture the full complexity of dark matter interactions or alternative gravitational effects. Another limitation is the complexity of the problem itself, which contains concepts outside the scope of a high school curriculum. The primary rationale for this study was to test the robustness of dark matter models against the latest observational data. Given the minor but noticeable inconsistencies observed, it is crucial to investigate whether they result from measurement limitations, modeling assumptions, or fundamentally new physics. Future research should expand the dataset by incorporating additional observations from upcoming missions such as the Vera C. Rubin Observatory. Moreover, alternative dark matter models, such as self-interacting or fuzzy dark matter, should be explored to assess their compatibility with the new findings. Investigating alternative gravitational theories also provides additional insight into the observed anomalies. If the discrepancies observed persist with future datasets, it could signal a need for the reevaluation of galaxy rotation curves. This could lead to revisions in cosmological simulations, influence the search for direct dark matter detection, or provide support for alternative theories of gravity. Ultimately, the findings of this study contribute to an ongoing refinement of astrophysical models and the pursuit of a more comprehensive understanding of the universe.

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