

Mysteries of prime numbers and Magic matrices

Date: September 1 , 2025

Authors: Ahcene Ait saadi

Email: ait_saaadi@yahoo.fr

Abstract: This document is entitled (mysteries of prime numbers and magic matrices) Explores the relationships between prime numbers and special matrices. The main objective is to use these matrices to form triples of prime numbers and to establish mathematical conjectures.

This work may have common mathematical relationships with the Golbach conjecture and Collatz conjecture.

The research is only at its beginnings, I hope that young researchers will be interested in it, and why not draw mathematical theory's from it.

Key words: Prime numbers; Matrices; System of equality, Some of square of prime numbers.

References:

Goldbach's conjecture, Collatz conjecture, viggo Brun, Hardy and Littlewood, Terence Tao. Girolamo Cardano, James Sylvester, Bachet de Meziriac.

Mysteries of prime numbers and Magic matrices

Summary: this document explores the relationships between prime numbers and magic matrices, to establish mathematical conjectures.

Introduction

- The document presents a system of equalities implicating natural integers and prime numbers.
- Two cases are considered: one with natural integers and the other with prim numbers.

First case

- A system of equalities is established with natural integers n, a, b, k
- Operations on the elements of the matrix reveal multiple relationships

Second case

- The matrices are formed with prime numbers n, b, k and a natural integer a .
- Conjecture 1 : if n et b are prime numbers, then k is also prime number.

Examples

- Several examples illustrate conjectures with triples of prime numbers.
- The results show interesting relationships between the numbers.

Conjectures

- Conjecture 2 : each prime number k can be expressed as the sum of the squares of natural integers.
- Conjecture 3 : the sum of the squares of n, b, k is always a multiple of 3.
- Conjecture 4 : the sum of the triples of prime numbers is equal to the sum of four squares.

Conclusion

- The document is a preliminary study on magic matrices and prime numbers, encouraging young researchers to deepen these ideas.

(I) : **Introduction:** we consider a system of two equals, whose terms verify certain mathematical relations. The following two cases are considered.

By performing certain operations on the elements of the matrice, we always find that the result obtained, is a multiple of one, or two elements of the matrice.

- 1) First case: $n; a; b; k$ natural integers
- 2) Second case $n; b; k$ prime numbers; a , natural Integer.

1-1) First case

We consider the following system of equalities:

$$\begin{aligned} n^2 + a &= k \\ (n-1)^2 + b &= k \end{aligned} \quad n, a, b, k \text{ Natural integers.....(A)}$$

The matrices is: $\begin{vmatrix} n & a & k \\ n-1 & b & k \end{vmatrix}$ $n; a; b; k$ natural integers

1-2): **We note the following equalities:**

$$\begin{aligned} nb + (n-1)a &= (2n-1)(n+a) \\ na + (n-1)b &= (2n-1)(n+a-1) \\ n(n-1) + ab &= (n+a)(2n-1) \\ n^2b - (n-1)^2a &= (2n-1)k \\ 2n-1 &= b-a \end{aligned}$$

Example 1:

$$19^2 + 47 = 408$$

$$18^2 + 84 = 408$$

$$19*84 + 18*47 = 66*37 \rightarrow 37 = 19 + 18$$

$$19*47 + 18*84 = 65*37 \rightarrow 65 = 47 + 18 = 84 - 19$$

$$19*18 + 47*84 = 65*66$$

$$19^2 * 84 - 18^2 * 47 = 408 * 37$$

$$19 + 18 = 84 - 47$$

Examples 2:

$$10^2 + 7 = 107$$

$$9^2 + 26 = 107$$

$$10 * 26 + 9 * 7 = 17 * 19 \rightarrow 19 = 10 + 9 \rightarrow 17 = 10 + 7$$

$$10 * 7 + 9 * 26 = 16 * 19 \rightarrow 16 = 9 + 7$$

$$10 * 9 + 7 * 26 = 16 * 17$$

$$10^2 * 26 - 9^2 * 7 = 19 * 107$$

$$10 + 9 = 26 - 7$$

Example 3:

$$11^2 + 6 = 127$$

$$10^2 + 27 = 127$$

$$11 * 27 + 10 * 6 = 21 * 17 \rightarrow 21 = 10 + 11 = 27 - 6 \rightarrow 17 = 11 + 6 = 27 - 10$$

$$11 * 6 + 10 * 27 = 16 * 21 \rightarrow 16 = 10 + 6$$

$$11 * 10 + 6 * 27 = 16 * 17$$

$$11^2 * 27 - 10^2 * 6 = 21 * 127$$

$$10 + 11 = 27 - 6$$

(II) : Prime numbers and magic matrice:

2-1) second case:

Any matrices of the form: $\begin{vmatrix} n & a & k \\ n-1 & b & k \end{vmatrix}$ n; b; k prime numbers, a , natural integer.

$$n^2 + a = k$$

$$(n-1)^2 + b = k$$

With the same conditions as 1;2)

2-2) Conjecture 1: if (n) and (b) are prime numbers, then (k) is a prime number

Example of elements of matrices illustrating this conjecture

1- $7^2 + 4 = 53$ 7; 17; 53 prime numbers
 $6^2 + 17 = 53$

2- $11^2 + 10 = 131$ 11; 31; 131 prime numbers
 $10^2 + 31 = 131$

- 3- $13^2 + 4 = 173$ 13; 29; 173 prime numbers
 $12^2 + 29 = 173$
- 4- $17^2 + 4 = 293$ 17; 37; 293 prime numbers
 $16^2 + 37 = 293$
- 5- $19^2 + 6 = 367$ 19; 43; 367 prime numbers
 $18^2 + 43 = 367$
- 6- $23^2 + 28 = 557$ 23; 73; 557 prime numbers
 $22^2 + 73 = 557$
- 7- $29^2 + 16 = 857$ 29; 73; 857 prime numbers
 $28^2 + 73 = 857$
- 8- $31^2 + 6 = 967$ 31; 67; 967 prime numbers
 $30^2 + 67 = 967$
- 9- $37^2 + 54 = 1423$ 37; 127; 1423 prime numbers
 $36^2 + 127 = 1423$
- 10- $41^2 + 16 = 1697$ 41; 97; 1697 prime numbers
 $40^2 + 97 = 1697$
- 11- $43^2 + 4 = 1853$ 43; 97; 1853 prime numbers
 $42^2 + 97 = 1853$
- 12- $47^2 + 4 = 2213$ 47; 97; 2213 prime numbers
 $46^2 + 97 = 2213$
- 13- $53^2 + 34 = 2843$ 53; 139; 2843 prime numbers
 $52^2 + 139 = 2843$
- 14- $59^2 + 10 = 3491$ 59; 127; 3491 prime numbers
 $58^2 + 127 = 3491$

- 15- $61^2 + 6 = 3727$ 61;127;3727 prime numbers
 $60^2 + 127 = 3727$
- 16- $67^2 + 4 = 4493$ 67;137; 4493 prime numbers
 $66^2 + 137 = 4493$
- 17- $79^2 + 6 = 6247$ 79;163;6247 prime numbers
 $78^2 + 163 = 6247$
- 18- $89^2 + 16 = 7937$ 89; 193; 7937 prime numbers
 $88^2 + 193 = 7937$
- 19- $97^2 + 4 = 9413$ 97;197;9413 prime numbers
 $96^2 + 197 = 9413$
- 20- $101^2 + 10 = 10211$ 101;211;10211 prime numbers
 $100^2 + 211 = 10211$
- 21- $103^2 + 18 = 10627$ 103;223;10627 prime numbers
 $102^2 + 223 = 10627$
- 22- $107^2 + 70 = 11519$ 107;283;11519 prime numbers
 $106^2 + 283 = 11519$
- 23- $109^2 + 6 = 11887$ 109;223;11887 prime numbers
 $108^2 + 223 = 11887$
- 24- $113^2 + 52 = 12821$ 113;277;12821 prime numbers
 $112^2 + 277 = 12821$
- 25- $131^2 + 22 = 17183$ 131;283;17183 prime numbers
 $130^2 + 283 = 17183$
- 26- $557^2 + 174 = 310423$ 557;1297; 310423 prime numbers
 $556^2 + 1297 = 310423$
- 27- $857^2 + 28 = 734477$ 857;1741;734477 prime numbers
 $856^2 + 1741 = 734477$

28- $1423^2 + 124 = 2025053$ $1423; 2969; 2025053$ prime numbers
 $1422^2 + 2969 = 2025053$

Note 1: I will discuss this work in the next article. The sum of two or more prime numbers is always equal to $17k+19p$

$53+131=184=3*17+7*19$	$3491+1853=5344=311*17+3*19$
$131+173=304=0*17+16*19$	$3491+2213=5704=321*17+13*19$
$173+293=466=14*17+12*19$	$4493+1423=5916=310*17+34*19$
$293+367=660=31*17+7*19$	$11887+10627=22514=1321*17+3*19$
$367+557=924=32*17+20*19$	$6247+967=7214=421*17+3*19$
$557+857=1414=72*17+10*19$	$7937+293=8230=464*17+18*19$
$53+293=346=17*17+3*19$	$2843+4493=7336=417*17+13*19$
$131+367=498=17*17+11*19$	
$173+1697=1870=72*17+34*19$	
$293+857=1150=52*17+14*19$	
$293+1423=1716=92*17+8*19$	
$367+1853=2220=125*17+5*19$	
$53+131+173=357=21*17+0*19$	
$131+173+293=597=34*17+1*19$	
$293+367+557=1217=47*17+22*19$	
$367+557+857=1781=88*17+15*19$	
$53+173+367=593=17*17+16*19$	
$131+367+557=1055=52*17+9*19$	
$131+857+1423=2411=134*17+7*19$	
$173+367+1697=2237=126*17+5*19$	
$53*2+131=237=5*17+8*19$	
$173*2+131=477=18*17+9*19$	
$131*2+53=315=4*17+13*19$	
$293*2+53=639=32*17+5*19$	

$$\begin{aligned}
53 \cdot 3 + 131 &= 290 = 7 \cdot 17 + 9 \cdot 19 \\
53 \cdot 4 + 131 &= 343 = 9 \cdot 17 + 10 \cdot 19 \\
53 \cdot 4 + 173 &= 385 = 7 \cdot 17 + 14 \cdot 19 \\
53 \cdot 7 + 131 &= 502 = 15 \cdot 17 + 13 \cdot 19 \\
53 \cdot 11 + 293 &= 876 = 37 \cdot 17 + 13 \cdot 19 \\
53 \cdot 20 + 131 &= 1191 = 60 \cdot 17 + 9 \cdot 19 \\
131 \cdot 2 + 293 &= 555 = 17 \cdot 17 + 14 \cdot 19 \\
131 \cdot 4 + 343 &= 51 \cdot 17 + 0 \cdot 19 \\
293 \cdot 3 + 53 \cdot 4 &= 1091 = 53 \cdot 17 + 10 \cdot 19 \\
293 \cdot 5 + 131 \cdot 2 &= 1727 = 61 \cdot 17 + 10 \cdot 19 \\
53 \cdot 5 + 131 \cdot 2 + 293 \cdot 3 &= 1406 = 76 \cdot 17 + 6 \cdot 19 \\
53 \cdot 5 + 131 \cdot 4 + 293 \cdot 3 + 342 \cdot 2 &= 2354 = 134 \cdot 17 + 4 \cdot 19 \\
53 \cdot 4 + 131 \cdot 3 + 173 &= 778 = 10 \cdot 17 + 32 \cdot 19
\end{aligned}$$

(III) **Note 2: A notable property, is that the prime number K , is the sum of the squares of the term u, u, v, w**

2.2) Conjecture 2: $\forall k$ (k prime number)

$k = u^2 + u^2 + v^2 + w^2 = 2u^2 + v^2 + w^2$; u, v, w ; Natural integers.

2.3) Example of prime numbers illustrating this conjecture.

$$53 = 2^2 + 2^2 + 6^2 + 3^2$$

$$131 = 3^2 + 3^2 + 8^2 + 7^2$$

$$173 = 6^2 + 6^2 + 10^2 + 1^2$$

$$293 = 12^2 + 12^2 + 2^2 + 1^2$$

$$367 = 13^2 + 13^2 + 5^2 + 2^2$$

$$557 = 16^2 + 16^2 + 6^2 + 3^2$$

$$857 = 12^2 + 12^2 + 20^2 + 13^2$$

$$967 = 13^2 + 13^2 + 25^2 + 2^2$$

$$1423 = 17^2 + 17^2 + 29^2 + 2^2$$

$$1697 = 18^2 + 18^2 + 32^2 + 5^2$$

$$1853 = 20^2 + 20^2 + 27^2 + 18^2$$

$$2213 = 24^2 + 24^2 + 31^2 + 10^2$$

$$2843 = 27^2 + 27^2 + 37^2 + 4^2$$

$$3491 = 33^2 + 33^2 + 32^2 + 17^2$$

$$3727 = 35^2 + 35^2 + 34^2 + 11^2$$

$$4493 = 36^2 + 36^2 + 35^2 + 26^2$$

$$6247 = 37^2 + 37^2 + 55^2 + 22^2$$

$$7937 = 42^2 + 42^2 + 53^2 + 40^2$$

$$9413 = 48^2 + 48^2 + 62^2 + 31^2$$

$$10211 = 49^2 + 49^2 + 72^2 + 15^2$$

$$10627 = 53^2 + 53^2 + 65^2 + 28^2$$

$$11519 = 57^2 + 57^2 + 70^2 + 11^2$$

$$11887 = 59^2 + 59^2 + 70^2 + 5^2$$

$$12821 = 68^2 + 68^2 + 57^2 + 18^2$$

$$17183 = 73^2 + 73^2 + 78^2 + 21^2$$

$\forall k (k_1 ; k_2 ; \text{prime numbers})$

$k_1 + k_2 = u^2 + u^2 + u^2 + v^2 + w^2 = 3u^2 + v^2 + w^2 ; u, v, w ; \text{Natural integers.}$

2.4) Example of prime numbers illustrating this conjecture

$$53 + 131 = 187 = 3 * 3^2 + 6^2 + 11^2$$

$$53 + 173 = 226 = 3 * 4^2 + 3^2 + 13^2$$

$$131 + 173 = 304 = 3 * 2^2 + 6^2 + 16^2$$

$$53 + 293 = 346 = 3 * 4^2 + 3^2 + 17^2$$

$$131 + 1853 = 1984 = 3 * 9^2 + 29^2 + 30^2$$

$$131 + 2843 = 2974 = 3 * 6^2 + 29^2 + 45^2$$

$$173 + 2843 = 3016 = 3 * 8^2 + 18^2 + 50^2$$

$\forall k (k_1 ; k_2 ; k_3 ; \text{prime numbers})$

$k_1 + k_2 + k_3 = u^2 + u^2 + u^2 + u^2 + v^2 + w^2 = 4u^2 + v^2 + w^2 ; u, v, w ; \text{Natural integers.}$

2.5) Example of prime numbers illustrating this conjecture

$$53 + 131 + 367 = 551 = 4 * 3^2 + 17^2 + 15^2 + 1^2$$

$$53 + 131 + 293 = 477 = 4 * 2^2 + 21^2 + 4^2 + 2^2$$

$$53 + 293 + 367 = 713 = 4 * 4^2 + 24^2 + 8^2 + 3^2$$

$$131 + 293 + 367 = 791 = 4 * 5^2 + 23^2 + 9^2 + 9^2$$

$$791 = 4 * 5^2 + 21^2 + 15^2 + 5^2 = 4 * 5^2 + 16^2 + 6^2 + 1^2$$

$$53 + 304 + 346 = 703 = 4 * 5^2 + 23^2 + 7^2 + 5^2$$

$\forall k (k_1 ; k_2 ; k_3 ; k_4 ; \text{prime numbers})$

$k_1 + k_2 + k_3 + k_4 = u^2 + u^2 + u^2 + u^2 + u^2 + v^2 + w^2 = 5u^2 + v^2 + w^2 ; u, v, w ; \text{Natural integers.}$

2.6) Example of prime numbers illustrating this conjecture

$$53 + 131 + 293 + 367 = 844 = 5 * 2^2 + 28^2 + 6^2 + 2^2 = 5 * 2^2 + 26^2 + 12^2 + 2^2 =$$

$$5 * 3^2 + 23^2 + 15^2 + 6^2 + 3^2 = 5 * 4^2 + 25^2 + 11^2 + 3^2 + 3^2 =$$

$$5 * 5^2 + 23^2 + 12^2 + 6^2 + 3^2 + 1^2 = 5 * 6^2 + 24^2 + 8^2 + 4^2 + 2^2 + 2^2 \dots \text{etc}$$

$$53 + 131 + 293 + 367 + 557 = 1401 = 6 * 5^2 + 35^2 + 5^2 + 1^2 = 6 * 5^2 + 33^2 + 12^2 + 3^2 + 3^2 =$$

$$6 * 5^2 + 32^2 + 13^2 + 7^2 + 3^2 = 6 * 5^2 + 31^2 + 16^2 + 5^2 + 3^2 = 6 * 5^2 + 30^2 + 17^2 + 7^2 + 3^2 + 2^2 \dots \text{etc}$$

$$2 * 53 + 3 * 131 + 4 * 293 = 1671 = 7 * 5^2 + 36^2 + 10^2 + 10^2 = 7 * 5^2 + 36^2 + 12^2 + 6^2 + 4^2 + 2^2$$

(IV) A notable property is that the sum of the squares of the term n, b, k , is always a multiple of 3.

Mathematical relations between the elements n, b, k of the matrice.

Conjecture 3: The sum of the squares of the numbers n, b, k is always a multiple of the number three: $n^2 + b^2 + k^2 = 3\lambda$

(IV)₁ **Examples of triples of prime numbers high to the power of 2?**

illustrating this conjecture:

1- $7^2 + 17^2 + 53^2 = 3147 = 3 * 1049$

2- $11^2 + 31^2 + 131^2 = 18243 = 3^3 * 2027$

3- $13^2 + 29^2 + 173^2 = 30939 = 3 * 10313$

4- $17^2 + 37^2 + 293^2 = 87507 = 3^3 * 7 * 463$

5- $19^2 + 43^2 + 367^2 = 136899 = 3^2 * 7 * 41 * 53$

6- $23^2 + 73^2 + 557^2 = 316107 = 3^2 * 11 * 31 * 103$

7- $29^2 + 73^2 + 857^2 = 740619 = 3^2 * 11 * 7481$

8- $31^2 + 67^2 + 967^2 = 940539 = 3 * 23 * 43 * 317$

.

.

etc

(IV)_{II} : **Generalization:** $\forall n \in \mathbb{N} : n^{2n} + b^{2n} + k^{2n} = 3\lambda$

(IV)_{III} : **Note:** A notable property, is that the sum of prime numbers, n, b, K , is the sum of the squares of the term $\alpha, \alpha, \beta, \delta$

Conjecture 4: The sum of the doublet of prime numbers k_1, k_2 comes in the forme: $k_1 + k_2 = \alpha^2 + \beta^2$

$$53 + 131 = 12^2 + 6^2 + 2^2$$

$$53 + 131 = 11^2 + 15^2 = 12^2 + 11^2 + 9^2$$

Conjecture 5: The sum of the triples of prime numbers comes in the forme:

$$n+b+k = \alpha^2 + \alpha^2 + \beta^2 + \delta^2 \rightarrow \alpha, \beta, \delta, \text{ Natural integers}$$

Examples on the sum of triples of prime numbers, which is equal to the sum of 4 squares.

- 7+17+53 = 77 = 5² + 5² + 4² + 1².....(1)
- 11+31+131 = 173 = 8² + 8² + 6² + 3².....(2)
- 13+29+173 = 215 = 7² + 7² + 9² + 6².....(3)
- 17+37+293 = 347 = 9² + 9² + 13² + 4².....(4)
- 19+43+367 = 429 = 14² + 14² + 6² + 1².....(5)
- 23+73+557 = 653 = 14² + 14² + 15² + 6².....(6)
- 29+73+857 = 959 = 15² + 15² + 22² + 5².....(7)
- 31+67+967 = 1065 = 20² + 20² + 12² + 11².....(8)
- 37+127+1423 = 1587 = 25² + 25² + 16² + 9².....(9)
- 41+97+1697 = 1835 = 27² + 27² + 19² + 4².....(10)
- 43+97+1853 = 1993 = 28² + 28² + 20² + 5².....(11)
- 47+97+2213 = 2357 = 30² + 30² + 19² + 14².....(12)
- 53+139+2843 = 3035 = 33² + 33² + 29² + 4².....(13)
- 59+127+3491 = 3677 = 40² + 40² + 21² + 6².....(14)
- 61+127+3727 = 3915 = 39² + 39² + 27² + 12².....(15)
- 67+131+4493 = 4691 = 39² + 39² + 40² + 7².....(16)
- 79+163+6243 = 6485 = 48² + 48² + 41² + 14².....(17)
- 89+193+7937 = 8219 = 53² + 53² + 45² + 24².....(18)
- 97+197+9413 = 9647 = 39² + 39² + 77² + 26².....(19)
- 101+211+10211 = 10523 = 45² + 45² + 68² + 43².....(20)
- 103+223+10627 = 10953 = 44² + 44² + 84² + 5².....(21)
- 107+283+11519 = 11909 = 60² + 60² + 65² + 22².....(22)

Conclusion: The document is a preliminary study on magic matrices and prime numbers. With hope, that young researchers will deepen these ideas, to develop mathematical theories.