In a universe with zero spatial curvature, dark energy cannot exist. One result proves it.

Fernando Salmon Iza

fernandosalmoniza@gmail.com

Abstract

We have obtained a theoretical result that leads us to believe that a universe with zero spatial curvature is incompatible with the results corresponding to the acceleration of our universe over the last 6 billion years, a phenomenon attributed in the ΛCDM model to the existence of dark energy. To this end, we calculated the scale factor of a universe with zero spatial curvature in the FLRW metric and found that this factor is always linear, thus invalidating the possibility of a positive acceleration of the universe such as that attributed to dark energy. We have demonstrated the linearity of the scale factor in spatially flat universes (k=0) through a mathematical development based on the Friedmann equations and conformally flat metrics.

Keywords: dark energy, spatial curvature, general relativity, Friedmann equations

1.- Study of the scale factor in a universe of FLRW metric and zero spatial curvature

Given the Friedmann equations of the FLRW [1]

$$H^2 = \left(\frac{a'}{a}\right)^2 = \frac{8\pi G\rho}{3c^2} - \frac{kc^2}{a^2}$$

$$\left(\frac{a''}{a}\right) = -\frac{4\pi G}{3c^2}(\rho+3p)$$

we are going to show that for universes with FLRW metric and zero spatial curvature, $a'' = 0$ is fulfilled.

Let the FLRW metric be in coordinates (t, x^1, x^2, x^3) [2], where "t" is the commoving time and x^i are the spatial coordinates, (c=1).

$$ds^2 = dt^2 - a(t)^2(g_{\mu\nu}\, dx^\mu dx^\nu)$$

In a spatially flat universe, k= 0, the 3D hypersurface corresponding to each section of cosmic spacetime is the Euclidean space R^3.

We are looking for a coordinate transformation that will convert this metric into a conformal metric. We make the following coordinate change:

$$dt = d\tau.\, a(\tau)$$

$$a = a\,(t(\tau))$$

$$a(\tau) = dt/d\tau$$

The conforming metric will be:

$$ds^2 = a(\tau)^2(d\tau^2 - g_{\mu\nu}\, dx^\mu dx^\nu)$$

where the scale factor $a(\tau) = a(t(\tau))$ is now a function of conformal time. Conformal time is not the proper time of any particular observer, but these coordinates have some advantages, such as clarifying that FLRW metrics with $k = 0$ are a locally flat conformal metrics.

Locally flat conformal metrics are flat conformal metrics valid in a neighborhood of each point, and different at each point. With metrics of this type, and considering that, being flat conformal metrics, their curvature tensors are equal to zero in their domain of definition, we will demonstrate that the scale factor is linear, that is:

$$a(t)'' = 0.$$

According to reference [3] in this metric the term $R_{\tau\tau}$ of the Ricci tensor is given by:

$$R_{rr} =3((a(\tau)''/a(\tau)) - (a(\tau)'/a(\tau))^2)$$

In a conformally flat metric, the curvature tensors are zero.

Therefore, in our case It will be true that:

$$0 = R_{rr} =3((a(\tau)''/a(\tau)) - (a(\tau)'/a(\tau))^2)$$

We show below that: $a''(t) \sim R_{\tau\tau} = 0$;

$$a= a(t(\tau))$$

$$a(\tau)'= da(\tau)/d\tau = (da(t(\tau)) /dt). (dt/d\tau) = a(t)'. a(\tau).$$

$$a(t)'= a(\tau)'/a(\tau)$$

$$a(t)''= d(a(\tau)'/a(\tau)) /dt = (d (a(\tau)'/a(\tau)) /d\tau). (d\tau/dt) =$$

$$= ((a(\tau)''/a(\tau) – a(\tau)'^2) /a(\tau)^2). (1/a(\tau)) = R_{\tau\tau} /(3a(\tau)) = 0$$

Thus, we have shown that for spatially flat universes in the FLRW metric it is true that:

$$a(t)'' = 0$$

Therefore, a(t) is linear.

Thus:

$$a(t)'' = 0$$

$$H(t) = a'/a$$

$$H (t) = 1/t$$

Thus, the Hubble parameter in these linearly expanding universes always decreases over time. Therefore, it will never increase, and the expansion of the universe will never be positively accelerated, as has been happening in our universe for the last 6 billion years due to dark energy. Therefore, dark energy is not possible in a universe with zero spatial curvature.

4. Conclusions

Our theoretical study has concluded with the following result: dark energy, which translates into a cosmological constant in the Friedmann equations, cannot be reconciled with a universe of zero spatial curvature. As we have shown in this work, it has a linear scala factor, implying that the Hubble parameter will always be decreasing, $H = 1/t$. However, in our universe, the Hubble parameter has been increasing for 6 billion years [4], and this constitutes the main cause of the existence of dark energy. This result does not contradict the results of Mission Planck [5], since in this work we have determined a value for the curvature term of the Friedmann equation, $\Omega_k = 0.001 \mp 0.002$, which allows for positive, zero, or negative curvature. Values other than zero are, in principle, compatible with dark energy in the universe.

References

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