<u>Quantum Vacuum as Relic of Primordial Spacetime</u>

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Abstract

The vacuum of Quantum Field Theory (QFT) is a state containing an infinite amount of quantum entanglement between different regions of spacetime. This nonlocal behavior reflects a fundamental property of quantum fields: vacuum fluctuations are *globally correlated* and cannot be localized to finite regions of spacetime. Here we point out that this property echoes the behavior of primordial spacetime undergoing a sustained transition to classical chaos. Following the entropy framework of [6], we interpret vacuum entanglement as *relic of primordial spacetime.* Moreover, building off our previous contributions [7-10], results suggest that gravitational physics, the QFT vacuum and the complex behavior of ultraviolet phenomena share a common root in the dynamics of primordial spacetime.

Key words: QFT vacuum, global entanglement, Reeh-Schlieder theorem, classical chaos, primordial spacetime, Kolmogorov-Sinai entropy.

1. Introduction

The vacuum state of relativistic Quantum Field Theory (QFT) is characterized by a *highly nontrivial entanglement structure*. Even in free theories, the vacuum exhibits divergent entanglement entropy across disjoint spacetime regions and long-range correlations implied by the Reeh–Schlieder theorem [1–4]. These properties are usually understood as kinematic consequences of locality, relativistic covariance, and the existence of infinitely many degrees of freedom inherent in QFT [5].

While this explanation is internally consistent, it leaves open a broader conceptual question: *is vacuum entanglement an intrinsic, irreducible feature of QFT, or can it be interpreted as the residual effect of an earlier dynamical process?* The present work explores the latter possibility within a framework based on a minimal set of established inputs.

Recent progress in complex dynamics and entropy theory has established a direct connection between classical chaos and quantum entanglement. It has been shown that for unstable bosonic Hamiltonian systems, the rate of entanglement entropy production is bounded from above by the Kolmogorov–Sinai (KS) entropy rate, defined as the sum of positive Lyapunov exponents of the corresponding classical system [6]. This result provides a rigorous link between dynamical instability and entanglement and applies to quadratic Hamiltonians and Gaussian states, which are central to free quantum field theories.

Independently, several contributions suggest that spacetime near the Planck scale may exhibit effective fractal or multifractal structure, with scale-dependent dimensionality [7–10]. Such systems are generically non-integrable and exhibit classical chaotic dynamics characterized by positive Lyapunov spectra and extensive KS entropy [11,12]. At the same time, the vacuum of QFT—defined as the ground state of a quadratic Hamiltonian—can be expressed as a squeezed state generated by Bogoliubov transformations [13–15], a structure known to produce large entanglement entropy.

Motivated by these observations, we offer here an interpretation in which the QFT vacuum is viewed as a *frozen relic of chaotic dynamics describing primordial spacetime*. As the universe transitions from a highly unstable, fractal regime to an effectively smooth and a classically stable geometry, entanglement generated during the chaotic phase becomes encoded in the vacuum state of low-energy quantum fields.

Crucially, this interpretation *does not* modify standard QFT, *does not* introduce new infrared degrees of freedom, and *does not* require observable violations of Lorentz invariance or locality. It is therefore compatible with effective field theory and current experimental constraints [16–18]. The goal of this work is not to advance yet another model of Quantum Gravity, but to provide a unifying conceptual framework linking vacuum entanglement, classical chaos, and early Universe dynamics.

2. Entanglement and the Kolmogorov–Sinai Bound

Consider a bosonic Hamiltonian system with unstable modes expressed in terms of generalized coordinates and momenta *(q,p)* [6],

$$H = \frac{1}{2}\sum_{ij}\left(p_i G_{ij} p_j + q_i V_{ij} q_j\right),$$

where the classical dynamics admits positive Lyapunov exponents $\lambda_i > 0$. A key result of [6] is that:

$$\frac{dS_A}{dt} \le h_{\mathrm{KS}} = \sum_{\lambda_i > 0} \lambda_i\,,$$

where:

- S_A is the entanglement entropy of a subsystem,
- h_{KS} is the Kolmogorov–Sinai entropy rate.

This result shows that entanglement evolution is dynamically controlled by classical chaos. Note that,

- The result is independent of any semiclassical approximations,

- It applies to Gaussian states and quadratic Hamiltonians—which are the typical setting of nearly-free QFT and Bogoliubov transformations.

3. Bosonization of Fermion Pairs and Vacuum Instability

In relativistic QFT, the vacuum is defined relative to a choice of mode decomposition. For fermions, Bogoliubov transformations mix creation and annihilation operators as in:

$$\alpha_k = u_k a_k + v_k b_{-k}^{\dagger}.$$

This has two key consequences:

1. Fermion pairs behave as *effective bosonic modes*, whose occupation numbers grow exponentially when the Bogoliubov coefficients satisfy instability conditions.
2. The vacuum becomes a *squeezed state*, with entanglement entropy scaling logarithmically with squeezing parameters.

Such squeezed vacua are those states for which:

$$S \sim \sum_{k} \log \ | v_k |^2,$$

mirroring the entropy production in unstable classical systems. In summary, fermionic QFT vacua inherit bosonic chaotic features through pair bosonization.

4. Global Vacuum Entanglement and Reeh–Schlieder Theorem

The *Reeh–Schlieder theorem* states that local operators acting on the vacuum in a specific, bounded spacetime region can generate any state in the entire Hilbert space. It demonstrates the onset of a strong vacuum entanglement, showing that local measurements can create, with high probability, states (e.g., quantum particles) at arbitrary distances, without violating causality. This means that:

- The vacuum contains correlations at all length scales.
- Arbitrarily small perturbations can approximate any global state.

- The entanglement entropy across any spatial domain diverges in the continuum limit.

From an entropic perspective, this is the signature of a system that has undergone an extensive entanglement process, consistent with saturation of a KS-type bound at early Universe times.

5. Fractal Spacetime and the Onset of Chaos

The onset of complex dynamics near the Planck scale strongly suggest that [7-10]:

- Spacetime exhibits *scale-dependent effective dimensions*,
- Geodesic flow becomes nonintegrable,
- Classical trajectories exhibit sensitivity to initial conditions.

In fractal or multifractal spacetimes:

$$d_{\text{eff}}(L) = \frac{d\log\, V(L)}{d\log\, L} < 4,$$

where $V(L)$ is the spacetime volume (or the spatial volume) of a geodesic ball or radius L. Specifically,

- Choose a point x in spacetime (or on a spatial slice),
- Construct a geodesic ball $B_L(x)$ as the set of all points whose geodesic distance from x is less than L,
- $V(L)$ is the measure of this ball with respect to the underlying spacetime (or spatial) measure.

For a smooth D-dimensional Riemannian (or Lorentzian, spatial slice) manifold,

$$V(L) \sim C_D \, L^D \quad (L \to 0),$$

so that

$$d_{\mathrm{eff}}(L) = D.$$

which recovers the usual notion of dimension. In a fractal or multifractal geometry, the volume scaling is anomalous:

$$V(L) \sim L^{d_H(L)},$$

where:

- $d_H(L)$ is the Hausdorff (or effective) dimension at scale L,
- the exponent may or it may not depend on scale.

The formula above leads to:

- anomalous dispersion relations,
- fractional Laplacians,
- nonlocal mode coupling.

These are the conditions under which:

- classical chaos emerges,
- Lyapunov spectra become dense,
- KS Entropy becomes extensive.

Near the Planck scale, primordial spacetime dynamics naturally support large KS entropy rates.

6. QFT Vacuum as a Relic of Primordial Spacetime

We now consolidate the line of arguments as follows:

1. Primordial fractal spacetime induces non-integrable dynamics with large KS entropy.
2. Bosonic (and bosonized fermionic) modes experience exponential instabilities.
3. Entanglement entropy grows at a rate bounded by KS entropy, rapidly saturating.
4. As the universe cools and spacetime evolves towards its classical state, dynamics freeze into an effectively stable QFT.
5. The resulting vacuum is a highly entangled squeezed state, indistinguishable from the standard QFT vacuum at low energies.

7. Consistency with Effective Field Theory and Observations

A. Compatibility with Effective Field Theory

At energies well below the Planck scale, quantum fields are accurately described by local relativistic effective field theories (EFT) [16,17]. In the present framework:

- Chaotic dynamics responsible for entanglement generation occurs at or near the Planck scale.
- Once spacetime turns classical, unstable modes decouple and the Hamiltonian becomes effectively quadratic and stable.
- The vacuum state reached after this transition is indistinguishable from the standard QFT vacuum.

This is consistent with the Decoupling Theorem and standard EFT reasoning, which ensure that ultraviolet dynamics do not induce observable infrared violations provided symmetries are preserved [17,18].

B. Lorentz Invariance and Causality

In our interpretation:

- Fractal behavior is confined to the primordial spacetime regime.
- Lorentz-invariant Wightman functions describe observable vacuum correlators [5].
- No preferred frame or modified dispersion relation survives at accessible energies.

This is consistent with stringent experimental bounds on Lorentz violation [19].

C. Vacuum Entanglement and Area Laws

Entanglement entropy in QFT obeys an area law with ultraviolet divergences regulated by a short-distance cutoff [20–22]. These divergences are universal and independent of the microscopic interpretation of the cutoff.

In our framework, the cutoff scale is *reinterpreted*, not *modified*: it corresponds to the scale at which chaotic dynamics cease to contribute to entanglement growth. The resulting entanglement structure is identical to that of standard QFT, and no modification to entropy scaling is required or predicted.

D. Cosmological Observables

Our interpretation does not introduce new sources of primordial non-Gaussianity, isocurvature perturbations, or deviations from standard inflationary predictions [23–25]. Since the vacuum state relevant for inflationary perturbations remains the standard Bunch–Davies vacuum, observable correlators are unchanged.

Any chaotic phase is assumed to precede or be dynamically decoupled from inflationary modes, consistent with standard cosmological EFT reasoning [26].

8. Conclusions

We have introduced an interpretation of the QFT vacuum as relic of chaotic dynamics associated with primordial spacetime structure. The argument relies on three well-established results:

(1) entanglement entropy production in unstable quantum systems is bounded by the Kolmogorov–Sinai entropy rate [6];

(2) QFT vacua are naturally expressed as squeezed states generated by Bogoliubov transformations [13–15];

(3) QFT vacuum is highly entangled, as formalized by the Reeh–Schlieder theorem [1,2].

Within this framework, vacuum entanglement is not viewed as a consequence of locality, but as the endpoint of an early-time dynamical process. As chaotic instabilities shut off during the transition to classical spacetime, entanglement generated at short scales becomes permanently encoded in the vacuum.

Note that this perspective *does not compete* with standard QFT or Quantum Gravity programs. Rather, it sheds new light onto the bridge linking classical chaos, QFT vacuum and early Universe dynamics. Similar chaos–entanglement connections have been discussed in semiclassical gravity, Black Hole physics, and open quantum systems [30–33], suggesting that the interpretation proposed here may have a broader range of implications.

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