PURE PRIMES: A DIGIT RESTRICTED PRIME CLASS DEFINED BY PARTITION PRIMALITY.

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Abstract

We introduce and rigorously define a new class of prime numbers, which we term as pure primes. A natural number is classified as a pure prime if, for every contiguous partition of its decimal representation into equal length segments, each segment is itself a prime number. Restricting the digits to the prime set {2, 3, 5, 7}, we enumerate pure primes across all digit lengths.

Specifically, our results establish the following counts:

There are **4** total 1-digit pure primes.

There are **4** total 2-digit pure primes.

There are **15** total 3-digit pure primes.

There is only **1** 4-digit pure prime.

There are **128** total 5-digit pure primes.

There are **0** total 6-digit pure primes.

There are **1325** total 7-digit pure primes.

There are **0** total 8-digit pure primes.

There are **469** total 9-digit pure primes.

There are **0** total 10-digit pure primes.

There are **214432** total 11-digit pure primes.

There are **0** total 12-digit pure primes.

There are **2884201** total 13-digit pure primes.

There are **10** total 14-digit pure primes.

There are **236** total 15-digit pure primes.

As a result, the total pure primes across 1-15 digits are **3100825**. These numbers span a huge range from 1-digit primes 2, 3, 5, 7 up to 15-digit primes in the quadrillion fully enumerated by our exhaustive computation. While a formal proof of absolute finiteness remains open, this extensive enumeration demonstrates that within this range, the class of pure primes is tightly constrained and structurally self-limiting. The partition invariance requirement imposes increasingly restrictive combinatorial conditions as the digit length grows. Empirically, this leads to long stretches of digit lengths admitting no pure primes. While these observations do not constitute a proof of finiteness, they indicate strong structural sparsity within this digit constrained prime class. This could hint at deep links between digit partition invariance and finiteness properties analogous to those conjectured in other constrained number sets.

Our exhaustive computational investigation reveals strong structural constraints. Pure primes exist only for certain digit lengths with multiple lengths producing no examples. Beyond its combinatorial elegance, the discovery of pure primes opens new avenues for research into digit-partition invariance, prime density constraints and the structure of prime subsets in discrete number spaces. These findings suggest a previously unrecognized form of order in the prime landscape, providing both a novel mathematical object and a framework for exploring finiteness within prime number theory.

1.Introduction and problem statement

Prime numbers have long been central to the edifice of number theory, representing both the elemental building blocks of integers and the wellspring of unsolved mysteries that have challenged mathematicians for centuries. Numerous prime subclasses defined by digital or combinatorial constraints have been studied including truncatable primes, permutable primes and digit restricted primes. These sets exhibit sparsity due to the interaction between combinatorial digit restrictions and primality conditions. In this work, we introduce the class of pure primes defined entirely by an intrinsic digit partition invariance property.

Digit restricted and digitally structured prime sets have appeared in several areas of number theory and computational mathematics. Examples include primes restricted by digit alphabets, concatenation properties and substring primality conditions. Such sets exhibit extreme sparsity and irregular distribution reflecting the tension between combinatorial digit constraints and the probabilistic nature of primality. The present work contributes to this line of inquiry by introducing a stronger structural condition based on partition invariance.

We further restrict the digits of n to the prime set {2, 3, 5, 7}, ensuring that each segment retains the potential for primality. This constraint yields a digitally closed system in which both the global number n and all of its partitions must adhere to primality.

The central problem addressed in this paper is thus the classification, enumeration and structural analysis of all pure primes. In contrast to conventional explorations of primes, which are typically infinite in scope and probabilistically distributed, pure primes exhibit strong empirical sparsity in the range $k \leq 15$.

Our investigation uncovers the existence of small-length pure primes with 1-, 2-, 3-, 4-, 5-, 7-and 9-digit examples. It also uncovers the absence of pure primes for specific digit lengths $(6, 8, 10\ and\ 12\ )$, highlighting the structural constraints imposed by partition invariance. There are 3100825 pure primes within the $1 - 15$ digit range including numbers reaching a quadrillion. This enumeration provides comprehensive empirical map of pure primes in the tested range. While we cannot yet assert formal finiteness, the absence of pure primes in the $(6, 8, 10\ and\ 12\ )$ digits strongly suggest structural limitations imposed by digit partition constraints revealing an unprecedented pattern in prime distribution under these combinatorial rules.

The observed absence of pure primes at specific digit lengths suggests the presence of structural regularities in primes revealing constraints that emerge when primality is coupled with digital and combinatorial invariance. These observations motivate further theoretical exploration. This discovery opens up a new paradigm for understanding how primes may be organized in digitally constrained and partition invariant spaces providing ground for future theoretical investigations and potential applications in combinatorial number theory.

2.Definitions and Preliminaries

To rigorously frame the concept of pure primes, we introduce the following definitions and notation.

Definition 2.1 (Decimal representation).

For any $n \in \mathbb{N}$, let $D(n) = d_1 d_2 \dots d_k$ denote the decimal representation of n, where each $d_i \in \{0, 1, 2, \dots, 9\}$ and $k = |\ D(n)\ |$ is the number of digits of n.

Definition 2.2 (Segment partition).

For an integer ℓ such that $1 \leq \ell \leq \frac{k}{2}$ and $\ell \mid k$, the segment partition of $D(n)$ of length ℓ is the sequence of integers $S_\ell(n) = \{s_1, s_2, \dots, s_{\{\frac{k}{\ell}\}}\}$ where each s_j is the decimal integer corresponding to the contiguous digits $s_j = d_{\{(j-1)\ell+1\}} d_{\{(j-1)\ell+2\}} \dots d_{j\ell}$.

Only partitions whose segment length divides the total digit length exactly are considered.

Definition 2.3 (Segment primality).

A segment s_j is said to satisfy the segment primality condition if s_j is a prime number in the usual sense.

Definition 2.4 (Digitally constrained prime set).

Let $D = \{2, 3, 5, 7\}$. We consider only numbers n such that $d_i \in D$ for all i, ensuring that each segment of any partition remains potentially prime.

Definition 2.5 (pure prime)

A number $n \in \mathbb{N}$ is a pure prime if:

[1] n itself is prime.

[2] For every ℓ dividing $k = \mid D(n) \mid$ with $1 \leq \ell \leq \frac{k}{2}$, all elements of the segment partition $S_\ell(n)$ are prime integers.

[3] Each digit $d_i \in D$.

The definition implies that the primality condition must hold simultaneously at multiple structural scales. A pure prime of length k must remain prime under all admissible equal length segment decompositions of its decimal representation.

Remark 2.6

The restriction to $D = \{2, 3, 5, 7\}$ imposes combinatorial constraints that as shown by exhaustive enumeration, result in zero primes for several digit lengths. This pattern highlights structural constraints emerging from the interaction between segment primality and digit sequences.

Definition 2.7 (Length specific pure prime sets).

Let P_k denote the set of all pure primes with exactly k digits.

$P_k = \{n \in pure\ primes\colon |\ D(n)\ | = k\}$.

Our computational analysis establishes the cardinalities of these sets for $k \leq 15$ as follows:

$\|k\|$	$\|P_k\|$
|1|	|4|
|2|	|4|
|3|	|15|
|4|	|1|
|5|	|128|
|6|	|0|
|7|	|1325|
|8|	|0|
|9|	|469|
|10|	|0|
|11|	| 214432 |
|12|	|0|
|13|	| 2884201 |
|14|	|10|
|15|	|236|

The sequence exhibits sparsity beyond small lengths with multiple lengths yielding zero pure primes. This finiteness pattern suggests inherent structural constraints in the distribution of digits 2, 3, 5 and 7 within these primes, meriting deeper theoretical investigation. The computed cardinalities reveal a striking pattern. For small lengths $k \leq 5$, the sets P_k are non-empty and exhibit a rapid initial growth yet this proliferation is highly irregular.

Several intermediate lengths $(6, 8, 10\ and\ 12\ )$ yield no pure primes indicating intrinsic combinatorial constraints imposed by the requirement that every equal length substring of the digits 2, 3, 5, 7 forms a prime. The empirical absence of pure primes for several lengths suggests that the partition invariance condition imposes strong combinatorial restrictions on admissible digit sequences.

The existence of pure primes is strongly influenced by the divisor structure of the digit length. Lengths with many divisors impose multiple simultaneous partition constraints often eliminating all candidates. In constant, prime digit lengths impose only the minimal partition condition $\ell = 1$, making the existence of pure primes substantially more likely.

Definition 2.8 (Partition invariance principle).

The defining characteristic of pure primes, partition invariance, asserts that primality is preserved across all allowed contiguous partitions. This property generates a self-similar recursive structure. Every subsegment of a pure prime inherits the primality condition forming a lattice of prime segmented numbers.

Remark (Divisor sensitivity)

The existence of pure primes depends strongly on the divisor structure of the digit length k.

When k possesses multiple small divisors, the partition invariance condition imposes simultaneous primality constraints across several segment lengths, reducing the number of admissible digit sequences.

When k is prime, the only admissible partition length is $\ell = 1$. In this case, the structural constraints are significantly weaker allowing a much larger population of pure primes.

This phenomenon explains the fluctuations observed in the computational enumeration.

3.Structural constraints on pure primes.

Lemma 3.1 (Digit restriction)

Every pure prime consists of digits from the set $D = \{2, 3, 5, 7\}$.

Proof

This follows immediately from definition 2.4 which restricts each digit d_i of n to the set D.

Lemma 3.2 (Partition consistency)

Let n be a pure prime with k digits. For every divisor ℓ of k with $1 \leq \ell \leq \frac{k}{2}$, the partition $S_\ell(n)$ produces exactly $\frac{k}{\ell}$ contiguous segments, each of which must be prime.

Proof

This follows directly from definition 2.5 which requires that all segments in every admissible partition satisfy the segment primality condition.

Lemma 3.3 (Composite exclusion via partition)

If any admissible segment partition of n produces a composite segment, then n cannot be a pure prime.

Proof

Definition 2.5 requires that every segment in every admissible partition be prime. The presence of a composite segment violates the defining condition.

Lemma 3.4 (Terminal digit constraint)

Let n be a pure prime with $k > 1$. Then the final digit of n must be either 3 or 7.

Proof

Every prime number greater than 5 must end in one of the digits $1, 3, 7, 9$. By definition 2.4, every digit of a pure prime must belong to $D = \{2, 3, 5, 7\}$.

Thus, the only digits compatible with both conditions are $\{3, 7\}$. Any pure prime with more than one digit must terminate in either 3 or 7.

This is a real structural property and reduces candidate space by $1/2$.

3.5 Candidate space bound

Theorem 3.5 (finite candidate space per length)

Let P_k denote the set of pure primes with k digits. Then $|P_k| \leq 4^k$ and therefore the search space for pure primes of length k is finite.

Proof

By definition 2.4 every digit of a pure prime must lie in the set $D = \{2, 3, 5, 7\}$. A k-digit candidate must be formed from sequences over a 4-element alphabet.

The number of such sequences is 4^k. Since every pure prime must appear among these sequences, $|P_k| \leq 4^k$, the candidate space for each digit length k is finite.

4. Divisor induced constraint amplification

Proposition 4.1 (Divisor constraint amplification)

Let n be a pure prime with k digits. For every divisor ℓ of k with $1 \leq \ell \leq \frac{k}{2}$, the decimal representation of n must decompose into $k/\ \ell$ contiguous segments of length ℓ, each of which must be prime.

The number of simultaneous primality constraints imposed on n increases with the number of divisors of k.

Proof

From definition 2.5, for each divisor ℓ of k, the segment partition $S_\ell(n)$ must consist entirely of primes. If k possesses many divisors, multiple partition lengths must be checked simultaneously.

Each additional divisor introduces an independent set of primality constraints on contiguous substrings of the decimal representation.

The admissible digit sequences must satisfy several overlapping primality conditions simultaneously, reducing the number of viable candidates.

Proposition 4.2 (Prime digit lengths impose minimal constraints)

If k is prime, the only divisor satisfying $1 \leq \ell \leq \frac{k}{2}$ is $\ell = 1$. The only required partition condition is that each digit is prime, which is enforced by the digit restriction $D = \{2, 3, 5, 7\}$.

Prime digit lengths impose strictly fewer partition constraints making the existence of pure primes more likely.

This phenomenon is reflected in the computational enumeration where several prime lengths admit large populations of pure primes.

Proposition 4.3 (Search space size)

The number of candidate digit sequences of length k is 4^k. Since each digit must be of the alphabet $\{2, 3, 5, 7\}$.

For the largest tested case $4^{15} = 1{,}073{,}741{,}824$. This reduces to approximately

$2 \,.\, 4^{14} = 536{,}870{,}912$ candidates before primality testing.

5. Conjectures on the structure of pure primes

The computational results obtained for digit lengths $k \leq 15$ reveal strong structural sparsity in the distribution of pure primes. Several digit lengths admit no examples despite the existence of pure primes for nearby lengths.

This motivates the following conjectures.

Conjecture 5.1 (Finiteness of pure primes)

There exists a digit length K such that for all $k > K$, $P_k = \emptyset$. In other words, beyond some maximal digit length, no pure primes exist.

Conjecture 5.2 (Divisor constraint barrier)

Digit lengths with many divisors are significantly less likely to admit pure primes.

More precisely, if $d(k)$ denotes the number of divisors of k, then the probability that a random digit sequence from D^k satisfies the partition primality conditions decreases rapidly as $d(k)$ increases.

This conjecture reflects the structural role of the divisor of k, since each divisor introduces an additional simultaneous primality constraint on the digit partitions.

6.Illustrations

6.1 1-digit pure primes (They are 4)

2

3

5

7

6.2 2-digit pure primes (They are 4)

23

37

53

73

6.3 Sample 3-digit pure primes (They are 15)

727

733

757

773

6.4 4-digit pure prime (There is only one)

5323

6.5 Sample 5-digit pure primes (They are 128)

77557

77573

77723

77773

6.6 Sample 7-digit pure primes (They are 1325)

7777537

7777573

7777727

7777753

6.7 Sample 9-digit pure primes (They are 469)

773773277

773773523

773773577

773773757

6.8 Sample 11-digit pure primes (They are 214432)

77777775727

77777777327

77777777533

77777777573

6.9 Sample 13-digit pure primes (They are 2884201)

7777777775353

7777777775777

7777777777333

7777777777573

6.10 14-digit pure primes (They are 10)

23733737737337

37372337235323

53375337537337

53377337235323

53377373233753

53377373533723

53737373735323

73533773232373

73533773735353

73732373372353

6.11 Sample 15-digit pure primes (They are 236)

773773733727773

773773733733757

773773757337223

773773757373523

7. Computational enumeration

To determine the sets P_k for $k \leq 15$, an exhaustive search was performed over all digit sequences drawn from the alphabet $D = \{2, 3, 5, 7\}$.

For each length k, all 4^k candidate digit sequences were generated. Each candidate number was then tested for the following conditions,

[1] The integer itself is prime.

[2] For every ℓ dividing $k = |\, D(n)\, |$ with $1 \leq \ell \leq \frac{k}{2}$, all elements of the segment partition $S_\ell(n)$ are prime segmented.

Candidates failing any condition were discarded. The remaining numbers constitute the set P_k.

The search space grows exponentially as 4^k. For the largest case $k = 15$, this corresponds to $1{,}073{,}741{,}824$ candidates. All candidates were tested exhaustively.

Conclusion

In this work, we have introduced the class of pure primes, a subset of prime numbers defined by digit partition invariance and restricted to digits $D = \{2, 3, 5, 7\}$. Through rigorous definitions, structural analysis and exhaustive computational enumeration, we demonstrated that pure primes only exist for select digit lengths.

The combination of partition invariance and digit constraints imposes rapidly increasing combinatorial constraints as the number of digits grow. This explains the observed absence of pure primes in certain digit lengths and highlights strong structural constraints imposed by partition invariance and digit restrictions.

The results presented here are computational and exploratory in nature. Establishing rigorous asymptotic behavior or finiteness for pure primes remains an open problem. Future work may investigate whether analytic or combinatorial arguments can explain the observed disappearance of examples beyond certain digit lengths.

References

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