Twin Paradox with Symmetric Acceleration: An Unresolved Inconsistency

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Abstract
Conventional treatments of the twin paradox in special relativity typically employ an asymmetric setup: one twin at rest in an inertial frame, the other twin undergoing accelerated space travel. The standard resolution attributes the younger age of the traveling twin to the breaking of inertial symmetry via acceleration, often invoking general relativity for a complete account. This paper presents a modified thought experiment in which both twins execute symmetric, oppositely directed relativistic journeys with identical acceleration profiles. We show that the paradox persists even under full general–relativistic treatment, because acceleration effects remain symmetric. Furthermore, we introduce a gedanken experiment involving rod–clocks with lengths perpendicular to the direction of relative motion, linking time dilation to transverse length observation. This configuration yields an independent, logically irreconcilable contradiction: each observer must simultaneously judge the other's clock as longer and capable of leaving a physical scratch, creating an unambiguous inconsistency that cannot be eliminated by frame transformations.
Keywords: twin paradox; time dilation; special relativity; length measurement; inertial symmetry; Gedanken experiment

Introduction

The twin paradox stands as a foundational conceptual test of special relativity, exposing apparent tensions between inertial symmetry and time dilation. In the classical formulation, one twin remains on Earth while the other travels at relativistic speed and returns. Special relativity predicts time dilation: the traveling twin ages less. Yet from the traveler's inertial frames during coasting phases, the Earth–bound twin appears to move, implying reciprocal youthfulness. This apparent symmetry underlies the paradox [1–15].
The widely accepted resolution emphasizes that the traveling twin experiences non–inertial motion: acceleration, deceleration, and direction reversal break the equivalence of reference frames. Within general relativity, this asymmetry produces a net difference in proper time, ensuring the traveler is younger upon reunion. Importantly, time dilation is experimentally confirmed by muon decay, atomic–clock tests, and particle–accelerator measurements, so the paradox is regarded as apparent rather than fatal to relativity [8–15].

Nevertheless, nearly all analyses rely on asymmetric motion. This work investigates whether the paradox can be reintroduced under perfectly symmetric acceleration. We show that symmetric twin travel preserves the paradox at a conceptual level and reveals a second contradiction involving perpendicular length and clock rates.

Symmetric Twin–Travel Gedanken Experiment

We propose a fully symmetric configuration that departs from all standard setups:

- Both twins depart Earth simultaneously.
- They travel along antiparallel trajectories at identical speeds and durations.
- They execute identical acceleration, turn–around, and deceleration profiles.
- Both return to Earth to compare ages.

In this arrangement, both twins follow identical acceleration and deceleration profiles along symmetric trajectories, ensuring that all acceleration-related relativistic effects are exactly equivalent for both. A paradox therefore arises: in the inertial segments of their journeys, each twin observes the other to be in relative motion and thus infers that the other should age more slowly. Upon their return to a common rest frame, this leads to an irresolvable contradiction: which twin is actually younger?

We also consider two closely related configurations:

1. Twin travel along symmetric large–angle arcs, returning to the origin.
2. Symmetric approach from distant points, meeting at the midpoint.

In all cases, inertial symmetry is preserved during relative motion, and acceleration effects remain balanced. The core paradox persists.

Discussions

In the classic twin paradox, the standard resolution relies on the asymmetry of the twins' trajectories: only the traveling twin undergoes acceleration, breaking the symmetry between their reference frames and yielding a unique, physically consistent prediction of differential aging. In this work, we examine two symmetric variants of the paradox in which both twins experience identical acceleration profiles, restoring full symmetry between their journeys. These configurations demonstrate that the paradox persists even when general relativistic effects of acceleration are explicitly accounted for, leaving no unambiguous resolution.

Two symmetric setups are analyzed:

1. **Symmetric Arcing Trajectories**: Both twins depart simultaneously in opposite directions, follow large circular arcs through space with identical speed and acceleration profiles, and return to their common starting point.
2. **Symmetric Head-On Motion**: The twins begin at rest at equal distances from a common midpoint, then accelerate toward each other at identical speeds, meeting at the midpoint after symmetric journeys.
3.

In both cases, the twins' motion is perfectly symmetric, ensuring that all general relativistic effects of acceleration are identical for both. Yet, during their inertial coasting phases, each twin observes the other to be in relative motion, leading each to predict that the other's clock will run slow due to time dilation. This reciprocal prediction creates an irresolvable contradiction: upon reunion, only one outcome can be physically realized (the twins are either the same age or one is younger), but the symmetry of the setup forbids a consistent asymmetric prediction.

Beyond the twin paradox, we introduce a new, independent contradiction that arises from combining time dilation with transverse length measurements. Consider a specialized "stick clock": a slender rod oriented perpendicular to the direction of motion, whose length decreases linearly with proper time, analogous to a burning incense stick or a stack of identical blocks with one removed per unit time. The remaining length of the rod directly encodes the elapsed proper time, making it a physical record of clock rate.

Now consider two such identical, synchronized stick clocks, initially placed at rest at a large distance from each other (Fig. 1). The clocks are then set into symmetric, head-on motion toward each other at identical constant speeds. As they pass each other at the midpoint, each observer measures the other's clock to be running slow due to time dilation. Because the stick's length is proportional to elapsed time, a slower clock implies a longer remaining rod.
From the frame of the left-moving clock (Clock-L), the right-moving clock (Clock-R) is observed to run slow, so its rod appears longer (Fig. 2). Conversely, from the frame of Clock-R, Clock-L is observed to run slow, so its rod appears longer (Fig. 3). If the setup is designed so that the shorter rod will scratch the longer one as they pass, each observer predicts that their own shorter rod will leave a permanent mark on the other's longer rod. However, the physical outcome must be unambiguous: either one rod is scratched, both are scratched, or neither. The requirement that both observers must simultaneously see their own rod as shorter than the others creates a logical contradiction that cannot be resolved by frame transformations, synchronization adjustments, or Lorentz contraction (which acts only along the direction of motion, leaving transverse lengths unchanged). This result challenges the internal consistency of reciprocal time dilation under direct physical comparison.

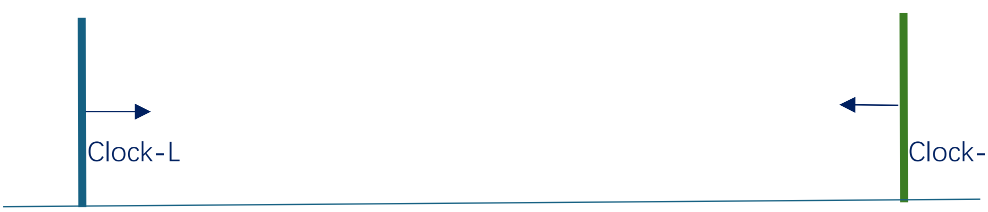

Fig.1. Two identical stick-clocks, initially positioned at remote locations, are set into reciprocal motion toward each other at identical speeds.

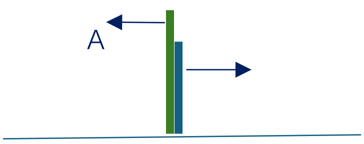

Fig.2. Upon meeting and passing each other at the midpoint, from the frame of clock-L, the stick-clock-R is observed to be taller than stick-clock-L.

Fig.3. Upon meeting and passing each other at the midpoint, from the frame of clock-R, the stick-clock-L is observed to be taller than stick-clock-R.

References

[1]. Ø Grøn 2006 Eur. J. Phys. 27(2006) 885, DOI 10.1088/0143-0807/27/4/019

[2]. Ying-Qiu Gu, Advances in Applied Clifford Algebras. March 2011.
DOI: 10.1007/s00006-010-0244-6 · Source: arXiv

[3]. Robert Cacioppo and Asim Gangopadhyaya, Physics Education. March 2012
DOI: 10.1088/0031-9120/47/5/563 · Source: arXiv

[4]. Robert Shuler, Journal of Modern Physics, May 2014.DOI: 10.4236/jmp.2014.512108

[5]. Gruber, R. and Price, R.: Zero time dilation in an accelerating rocket. American Journal of Physics, 65 (10), 979-980(1997). http://dx.doi.org/10.1119/1.18700

[6]. Lasky, R.: Time and the Twin Paradox. Scientific American, Special Editions 16 (1s), 2023 (February 2006). doi:10.1038/scienti camerican0206-20sp

[7]. Frye, R. and Brigham, V.: Paradox of the Twins. American Journal of Physics, 25 (8), 553-555(1957). http://dx.doi.org/10.1119/1.1934547

[8]. Einstein, A.: The Berlin Years: Writings, 1918-1921 (English Translation of Selected Texts). The Collected Papers of Albert Einstein, 7, Princeton University Press, Princeton, (2002).

[9]. Iorio, L.:. Found. Phys. Lett. 18 (3), 119(2005). DOI.org/10.1007/s10702005-2466-8

[10]. GrØn, Ø.: The twin paradox and the principle of relativity. Physica Scripta, 87 (3), 8-21(2013). DOI: 10.1088/0031-8949/87/03/035004

[11]. Good, R.: Uniformly accelerated reference frame and twin paradox. American Journal of Physics, 50 (3), 232-238(1982). http://dx.doi.org/10.1119/1.12862

[12]. Nikolic, H.: The role of acceleration and locality in the twin paradox. Found. Phys. Lett. 13 (12), 595-601(2000). https://doi.org/10.1023/A:1007866532278

[13]. Minguzzi, E.: Differential aging from acceleration: An explicit formula. American Journal of Physics, 73 (9), 876-880(2005). http://dx.doi.org/10.1119/1.1924490

[14]. Styer, D.: How do two moving clocks fall out of sync? A tale of trucks, threads, and twins. American Journal of Physics, 75 (9), 805-814(2007). http://dx.doi.org/10.1119/1.2733691

[15]. Gamboa, J.: F. Mendez, M.B. Paranjape, The twin paradox.: the role of acceleration. Canadian Journal of Physics, 97 (10), 1-17(2019). https://doi.org/10.1139/cjp-2018-0788