Plank’s constant

Kim Miri[1]

[1]no affiliation, skfekd@gmail.com

Multivectors change the paradigm of electromagnetic theory. Multivectors derive not only Maxwell's equations but also energy density, Poynting vectors, Joule heat and electromagnetic force solely through calculation without the aid of experimental laws. Moreover, this process can be repeated three more times by changing the dimension of multivector in 4-dim. And these four Maxwell equations tell us what charge, spin, quarks, and mass are. The 2-vector rotor and booster transform the coordinate system and derive the ladder operator without quantum mechanics. The commutative relation of the angular momentum operator, the uncertainty principle, and the reason photons can be counted in the photoelectric effect are explained by the infinitesimal rotor and booster. These three phenomena, including Planck's constant, indicate that the infinitesimal rotor and booster are always operating. The universe is vibrating, and the current magnitude of cosmic background vibration is Planck's constant.

1 Curl and divergence in multivectors

Curl and divergence can be defined as the multibases of any curvilinear coordinate system.

1.1. *Basis and reciprocal basis*

In n-dim curvilinear coordinates system, A point is $X(p^1, p^2, \dots, p^n)$, which means the vector from the origin O to X. The basis u_i at X is defined as follows.

$$u_i = \frac{\partial X}{\partial p^i}$$

When $u^i \cdot u_j = \delta^i_j$, $u^1, u^2 \dots u^n$ are reciprocal basis at X. Assuming matrices g^{ij} and g_{jk},

$$u^i = g^{ij} u_j, \qquad u^i \cdot u^k = g^{ij} u_j \cdot u^k = g^{ik}, \qquad u_i = g_{ij} u^j, \qquad u_i \cdot u_k = g_{ij} u^j \cdot u_k = g_{ik}$$

g_{jk} and g^{ij} are inverse matrices relationships.

$$u^i \cdot u_k = g^{ij} u_j \cdot u_k = g^{ij} g_{jk} = \delta^i_k$$

Therefore, find the basis at point X, find $g_{ij} = u_i \cdot u_j$, find the inverse matrix g^{ij}, and find the u^i

1.2. *Multibasis and dot product*

If $u_1, \dots, u_r$ are bases, then $(u_1 \wedge u_2)$ is a 2-basis and $(u_1 \wedge u_2 \wedge u_3)$ is a 3-basis. Multibasis is anti-commutative in order.

$$(u_1 \wedge u_2) = -(u_2 \wedge u_1), \qquad (u_1 \wedge u_1) = 0$$

$$(\dots \wedge u_{i-1} \wedge u_i \wedge \dots) = -(\dots \wedge u_i \wedge u_{i-1} \wedge \dots)$$

The dot product of a multibasis and a reciprocal multibasis is defined as follows.

$$(u^{\sigma_1} \wedge \dots \wedge u^{\sigma_r}) \cdot (u_{s_1} \wedge \dots \wedge u_{s_k})$$
$$= (u^{\sigma_2} \wedge \dots \wedge u^{\sigma_r}) \cdot \{u^{\sigma_1} \cdot (u_{s_1} \wedge \dots \wedge u_{s_k})\} = (u^{\sigma_3} \wedge \dots \wedge u^{\sigma_r}) \cdot [u^{\sigma_2} \cdot \{u^{\sigma_1} \cdot (u_{s_1} \wedge \dots \wedge u_{s_k})\}] = \cdots$$

$$u^{\sigma_1} \cdot (u_{s_1} \wedge \dots \wedge u_{s_k}) = (-1)^{i-1} (u^{\sigma_1} \cdot u_{s_i})(u_{s_1} \wedge \dots \wedge \hat{u}_{s_i} \wedge \dots \wedge u_{s_r}), \qquad \hat{u}_{s_i} : u_{s_i}\ skip$$

If $s_i = 1,\ s_k = 2,\ (i < k),$ then

$$(u^1 \wedge u^2) \cdot (u_{s_1} \wedge \dots \wedge u_{s_k}) = (-1)^{i-1}(-1)^{k-2}(u_1 \wedge \dots \wedge \hat{u}_{s_i} \wedge \dots \wedge \hat{u}_{s_k} \wedge \dots \wedge u_r)$$
$$= -(u^2 \wedge u^1) \cdot (u_{s_1} \wedge \dots \wedge u_{s_k}) = -(-1)^{k-1}(-1)^{i-1}(u_1 \wedge \dots \wedge \hat{u}_{s_i} \wedge \dots \wedge \hat{u}_{s_k} \wedge \dots \wedge u_r)$$

1.3. *Infinitesimal volume $[dV]_r$ and its boundary $\partial[dV]_r$*

$[dV]_r$ is an r-vector. It means something like a very small r-dimensional parallelepiped. The reference vertex is X. The r edges are $(u_{\sigma_1} dp^{\sigma_1}), (u_{\sigma_2} dp^{\sigma_2}), \dots, (u_{\sigma_r} dp^{\sigma_r})$.

$$[dV]_r = (u_{\sigma_1} \wedge u_{\sigma_2} \wedge \dots \wedge u_{\sigma_r}) dp^{\sigma_1} dp^{\sigma_2} \dots dp^{\sigma_r}, \qquad \{\sigma_1, \sigma_2, \dots, \sigma_r\} \subset \{1,2,\dots,n\}, \qquad (\sigma_1 < \cdots < \sigma_r)$$

ex) $[dV]_1 = u_2 dp^2,\ [dV]_2 = (u_1 \wedge u_2) dp^1 dp^2,\ [dV]_3 = (u_1 \wedge u_3 \wedge u_4) dp^1 dp^3 dp^4$

The 2r (r-1)-vector boundaries on the surface of $[dV]_r$ is $\partial[dV]_r$.

$$\partial[dV]_r = \frac{\pm u^i}{dp^i} \cdot [dV]_r = \begin{cases} \dfrac{+u^i}{dp^i} \cdot [dV]_r \quad at\ P(\dots, p^i + dp^i, \dots) \\ \dfrac{-u^i}{dp^i} \cdot [dV]_r \quad at\ P(\dots, p^i, \dots) \end{cases}$$

ex) $[dV]_1 = u_2 dp^2$

$$\partial[dV]_1 = \frac{\pm u^i}{dp^i} \cdot [dV]_1 = \begin{cases} \dfrac{+u^2}{dp^2} \cdot u_2 dp^2 = 1 \quad at\ P(\dots, p^2 + dp^2, \dots) \\ \dfrac{-u^2}{dp^2} \cdot u_2 dp^2 = -1 \quad at\ P(\dots, p^2, \dots) \end{cases}$$

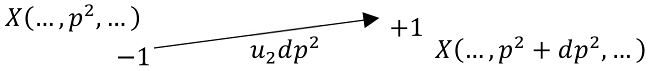

Fig.1 $\partial[dV]_1$ is ± 1. It has opposite signs to adjacent $\partial[dV]_1$

ex) $[dV]_2 = (u_1 \wedge u_2) dp^1 dp^2$

$$[dV]_2 = \frac{\pm u^i}{dp^i} \cdot [dV]_2 = \begin{cases} \dfrac{+u^1}{dp^1} \cdot (u_1 \wedge u_2) dp^1 dp^2 = +u_2 dp^2 \quad at\ P(\dots, p^1 + dp^1, \dots) \\ \dfrac{-u^1}{dp^1} \cdot (u_1 \wedge u_2) dp^1 dp^2 = -u_2 dp^2 \quad at\ P(\dots, p^1, \dots) \\ \dfrac{+u^2}{dp^2} \cdot (u_1 \wedge u_2) dp^1 dp^2 = -u_1 dp^1 \quad at\ P(\dots, p^2 + dp^2, \dots) \\ \dfrac{-u^2}{dp^2} \cdot (u_1 \wedge u_2) dp^1 dp^2 = +u_1 dp^1 \quad at\ P(\dots, p^2, \dots) \end{cases}$$

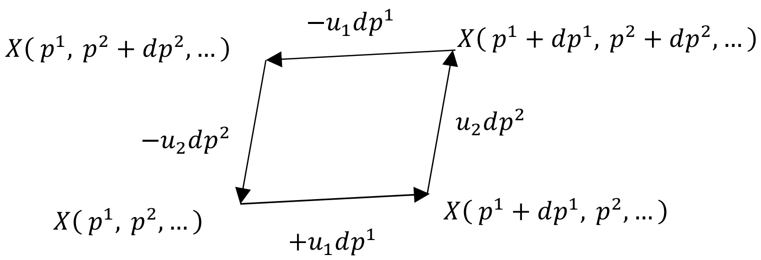

Fig.2 $\partial[dV]_2$ rotates in one direction, curl. It rotates in the opposite direction to adjacent $\partial[dV]_2$

1.4. *Curl*

r-vector field A in n-dim.

$$A = A_{\sigma_1 \dots \sigma_r} (u^{\sigma_1} \wedge \dots \wedge u^{\sigma_r}), \qquad (\sigma_1 < \dots < \sigma_r), \qquad \sigma_1, \dots, \sigma_r, \dots \in \{1,2, \dots n\}$$

The definition of $curl(A)$ is from the dot product of a (r-1)-vector field A and the boundary vector $\partial[dV]_r$.

$$A \cdot \partial[dV]_r = A \cdot \left(\frac{\pm u^i}{dp^i} \cdot [dV]_r \right) = \left(\frac{\pm u^i}{dp^i} \wedge A \right) \cdot [dV]_r = \left(u^i \wedge \frac{\pm A}{dp^i} \right) \cdot [dV]_r$$

$$= \left(u^i \wedge \frac{\partial A}{\partial p^i} \right) \cdot [dV]_r = curl(A) \cdot [dV]_r, \qquad curl(A) = \left(u^i \wedge \frac{\partial A}{\partial p^i} \right)$$

The definition itself is Stokes' theorem. This does not mean that a new vector field $curl(A)$ is created, but rather that it is a view of A from one dimension higher. A and $curl(A)$ are one entity and both are densities because they possess physical quantities proportional to $\partial[dV]_r$ and $[dV]_r$.

$$\oint A \cdot \partial[dV]_r = \oint curl(A) \cdot [dV]_r$$

In calculus, a vector field must be represented by reciprocal bases so that the bases cancel out with dot product.

And differentiate only the coefficients of A to find $curl(A)$.

$$curl^2(A) = curl\{curl(A)\} = \left(u^j \wedge u^i \wedge \frac{\partial^2 A}{\partial p^j \partial p^i}\right) = \left(u^i \wedge u^j \wedge \frac{\partial^2 A}{\partial p^i \partial p^j}\right) = 0$$

That is, only A and $curl(A)$ are one entity.

1.5. *Comparison to conventional curl*

$A = A_r \mathbb{e}_r + A_\theta \mathbb{e}_\theta + A_\emptyset \mathbb{e}_\emptyset$ in spherical coordinates.

$$u_r = \frac{\partial X}{\partial r} = \mathbb{e}_r, \qquad u_\theta = \frac{\partial X}{\partial \theta} = r\mathbb{e}_\theta, \qquad u_\emptyset = \frac{\partial X}{\partial \emptyset} = r\,sin\,\theta\;\mathbb{e}_\emptyset$$

$$u^i \cdot u_k = \delta^i_k, \qquad u^r = \mathbb{e}_r, \qquad u^\theta = \frac{\mathbb{e}_\theta}{r}, \qquad u^\emptyset = \frac{\mathbb{e}_\emptyset}{r\,sin\,\theta}$$

$$A = A_r \mathbb{e}_r + A_\theta \mathbb{e}_\theta + A_\emptyset \mathbb{e}_\emptyset = A_r u^r + A_\theta r u^\theta + A_\emptyset r\,sin\,\theta\, u^\emptyset = A_r u_r + \frac{A_\theta}{r} u_\theta + \frac{A_\emptyset}{r\,sin\,\theta} u_\emptyset$$

$(u^\theta \wedge u^\emptyset)$ term in $curl(A)$.

$$\left(u^\theta \wedge \frac{\partial(A_\emptyset r\,sin\,\theta)}{\partial\theta} u^\emptyset\right) + \left(u^\emptyset \wedge \frac{\partial(A_\theta r)}{\partial\emptyset} u^\theta\right) = \left\{\frac{\partial(A_\emptyset r\,sin\,\theta)}{\partial\theta} - \frac{\partial(A_\theta r)}{\partial\emptyset}\right\}(u^\theta \wedge u^\emptyset)$$

$$= \frac{1}{r^2\,sin\,\theta}\left\{\frac{\partial(A_\emptyset r\,sin\,\theta)}{\partial\theta} - \frac{\partial(A_\theta r)}{\partial\emptyset}\right\}(\mathbb{e}_\theta \wedge \mathbb{e}_\emptyset)$$

Compared to the conventional results, the coefficient is the same. However, it is not the $\mathbb{e}_r$ term, but the $(\mathbb{e}_\theta \wedge \mathbb{e}_\emptyset)$ term. They are dual basis each other.

1.6. *Pseudoscalar and dual basis*

In n-dimension, Cartesian coordinates basis $\mathbb{e}_1, \mathbb{e}_2, \dots, \mathbb{e}_n$, unit pseudoscalar I.

$$I = (\mathbb{e}_1 \wedge \dots \wedge \mathbb{e}_n)$$

For example

$$I = (\mathbb{e}_1 \wedge \mathbb{e}_2 \wedge \mathbb{e}_3) = (\mathbb{e}_r \wedge \mathbb{e}_\theta \wedge \mathbb{e}_\emptyset) = r^2\,sin\,\theta\,(u^r \wedge u^\theta \wedge u^\emptyset)$$

dual basis

$$\mathbb{e}^d_r = \mathbb{e}_r \cdot I = \mathbb{e}_r \cdot (\mathbb{e}_r \wedge \mathbb{e}_\theta \wedge \mathbb{e}_\emptyset) = (\mathbb{e}_\theta \wedge \mathbb{e}_\emptyset)$$

1.7. *Divergence*

$divergence(A)$ is $curl$ with respect to A^d, dual vector field of A.

$$div(A) = curl(A^d), \qquad A^d = A \cdot I$$

$$A^d = A \cdot I = \left(A_r u_r + \frac{A_\theta}{r} u_\theta + \frac{A_\emptyset}{r\,sin\,\theta} u_\emptyset\right) \cdot r^2\,sin\,\theta\,(u^r \wedge u^\theta \wedge u^\emptyset)$$

$$= A_r r^2\,sin\,\theta\,(u^\theta \wedge u^\emptyset) - A_\theta r\,sin\,\theta\,(u^r \wedge u^\emptyset) + A_\emptyset r(u^r \wedge u^\theta)$$

$$div(A) = curl(A^d) = \left\{\frac{\partial(A_r r^2\,sin\,\theta)}{\partial r} + \frac{\partial(A_\theta r\,sin\,\theta)}{\partial\theta} + \frac{\partial(A_\emptyset r)}{\partial\emptyset}\right\}(u^r \wedge u^\theta \wedge u^\emptyset)$$

$$= \frac{1}{r^2\,sin\,\theta}\left\{\frac{\partial(A_r r^2\,sin\,\theta)}{\partial r} + \frac{\partial(A_\theta r\,sin\,\theta)}{\partial\theta} + \frac{\partial(A_\emptyset r)}{\partial\emptyset}\right\}(\mathbb{e}_r \wedge \mathbb{e}_\theta \wedge \mathbb{e}_\emptyset)$$

same coefficient, dual basis.

1.8. *Continuity equation, $div(A) = 0$*

$$div(A) \cdot [dV]_r = curl(A^d) \cdot [dV]_r = A^d \cdot \partial[dV]_r = (A \cdot I) \cdot \partial[dV]_r = (A \wedge \partial[dV]_r) \cdot I$$

$(A \wedge \partial[dV]_r) \cdot I$ is a physical quantity that moves in or out at speed A through the boundary $\partial[dV]_r$ of $[dV]_r$. $(A \wedge \partial[dV]_r)$ is the product $\partial[dV]_r$ and only the components of A perpendicular to $\partial[dV]_r$. The parallel components cancel out in the wedge product. If $div(A) = 0$ at all points, this means that a physical quantity flows without accumulation or leakage, then A is said to be continuous.

2. Maxwell's equations

The following propositions are Maxwell's equations.

1) If vector potential A exists then $curl(A)$ exists. They are one entity.

2) Then, $curl^d(A)$ and $curl\{curl^d(A)\} = J$ also exist. It's duality.

$curl^d(A) = curl(A) \cdot I, \qquad curl\{curl^d(A)\} = div\{curl(A)\} = Laplacian(A) = J$

This is Gauss's law and Ampere-Maxwell's law.

3) $curl^2(A) = 0$ and $curl(J) = curl^2\{curl^d(A)\} = 0$.

These are Gauss's law for magnetism, and Faraday's law, and charge conservative law.

4) After enough time, at equilibrium, A becomes continuous. $div(A) = 0$. Lorentz condition.

Then A and J formulate the wave equation.

The Cartesian coordinate system is convenient for comparison with existing laws.

$$\mathbb{e}^t \cdot \mathbb{e}_t = 1, \qquad \mathbb{e}_t \cdot \mathbb{e}_t = -1, \qquad \mathbb{e}^t = -\mathbb{e}_t$$

$$\mathbb{e}^i \cdot \mathbb{e}_i = 1, \qquad \mathbb{e}_i \cdot \mathbb{e}_i = 1, \qquad \mathbb{e}^i = \mathbb{e}_i, \qquad (i = 1,2,3)$$

$$I = (\mathbb{e}_1 \wedge \mathbb{e}_2 \wedge \mathbb{e}_3 \wedge \mathbb{e}_t) = -(\mathbb{e}^1 \wedge \mathbb{e}^2 \wedge \mathbb{e}^3 \wedge \mathbb{e}^t)$$

$$\partial_\mu = \frac{\partial}{\partial x^\mu}, \qquad \partial_{\mu\nu} = \frac{\partial^2}{\partial x^\mu \partial x^\nu}, \qquad (\mu, \nu = 1,2,3,t), \qquad \square = \partial_{tt} - \partial_{11} - \partial_{22} - \partial_{33}$$

1) $A = A_1\mathbb{e}^1 + A_2\mathbb{e}^2 + A_3\mathbb{e}^3 - \phi\mathbb{e}^t = A_1\mathbb{e}_1 + A_2\mathbb{e}_2 + A_3\mathbb{e}_3 + \phi\mathbb{e}_t$

$$curl(A) = (\mathbb{e}^\mu \wedge \partial_\mu A) = B + E$$

$$= (\partial_2 A_3 - \partial_3 A_2)(\mathbb{e}^2 \wedge \mathbb{e}^3) + (\partial_3 A_1 - \partial_1 A_3)(\mathbb{e}^3 \wedge \mathbb{e}^1) + (\partial_1 A_2 - \partial_2 A_1)(\mathbb{e}^1 \wedge \mathbb{e}^2)$$

$$-(\partial_1\phi + \partial_t A_1)(\mathbb{e}^1 \wedge \mathbb{e}^t) - (\partial_2\phi + \partial_t A_2)(\mathbb{e}^2 \wedge \mathbb{e}^t) - (\partial_3\phi + \partial_t A_3)(\mathbb{e}^3 \wedge \mathbb{e}^t)$$

$$= B_1(\mathbb{e}^2 \wedge \mathbb{e}^3) + B_2(\mathbb{e}^3 \wedge \mathbb{e}^1) + B_3(\mathbb{e}^1 \wedge \mathbb{e}^2) + E_1(\mathbb{e}^1 \wedge \mathbb{e}^t) + E_2(\mathbb{e}^2 \wedge \mathbb{e}^t) + E_3(\mathbb{e}^3 \wedge \mathbb{e}^t)$$

Conventional coefficient representation.

$$B = \nabla \times A, \qquad E = -\nabla\phi - \partial_t A$$

2) $curl^d(A) = E_1(\mathbb{e}^2 \wedge \mathbb{e}^3) + E_2(\mathbb{e}^3 \wedge \mathbb{e}^1) + E_3(\mathbb{e}^1 \wedge \mathbb{e}^2) - B_1(\mathbb{e}^1 \wedge \mathbb{e}^t) - B_2(\mathbb{e}^2 \wedge \mathbb{e}^t) - B_3(\mathbb{e}^3 \wedge \mathbb{e}^t)$

$$J = curl(curl^d(A)) = \rho(\mathbb{e}^1 \wedge \mathbb{e}^2 \wedge \mathbb{e}^3) - J_1(\mathbb{e}^2 \wedge \mathbb{e}^3 \wedge \mathbb{e}^t) - J_2(\mathbb{e}^3 \wedge \mathbb{e}^1 \wedge \mathbb{e}^t) - J_3(\mathbb{e}^1 \wedge \mathbb{e}^2 \wedge \mathbb{e}^t)$$

$$= (\partial_1 E_1 + \partial_2 E_2 + \partial_3 E_3)(\mathbb{e}^1 \wedge \mathbb{e}^2 \wedge \mathbb{e}^3) - (\partial_2 B_3 - \partial_3 B_2 - \partial_t E_1)(\mathbb{e}^2 \wedge \mathbb{e}^3 \wedge \mathbb{e}^t)$$

$$-(\partial_3 B_1 - \partial_1 B_3 - \partial_t E_2)(\mathbb{e}^3 \wedge \mathbb{e}^1 \wedge \mathbb{e}^t) - (\partial_1 B_2 - \partial_2 B_1 - \partial_t E_3)(\mathbb{e}^1 \wedge \mathbb{e}^2 \wedge \mathbb{e}^t)$$

ρ: charge density, J: charge current density.

$$\nabla \cdot E = \rho, \qquad \nabla \times B = J + \partial_t E$$

3) $curl^2(A) = 0$

$$= (\partial_1 B_1 + \partial_2 B_2 + \partial_3 B_3)(\mathbb{e}^1 \wedge \mathbb{e}^2 \wedge \mathbb{e}^3) + (\partial_2 E_3 - \partial_3 E_2 + \partial_t B_1)(\mathbb{e}^2 \wedge \mathbb{e}^3 \wedge \mathbb{e}^t)$$

$$+(\partial_3 E_1 - \partial_1 E_3 + \partial_t B_2)(\mathbb{e}^3 \wedge \mathbb{e}^1 \wedge \mathbb{e}^t) + (\partial_1 E_2 - \partial_2 E_1 + \partial_t B_3)(\mathbb{e}^1 \wedge \mathbb{e}^2 \wedge \mathbb{e}^t)$$

$$\nabla \cdot B = 0, \qquad \nabla \times E + \partial_t B = 0$$

$$curl(J) = 0 = (\partial_1 J_1 + \partial_2 J_2 + \partial_3 J_3 + \partial_t \rho)I$$

$$\nabla \cdot J + \partial_t \rho = 0$$

4) $A^d = -A_1(\mathbb{e}^2 \wedge \mathbb{e}^3 \wedge \mathbb{e}^t) - A_2(\mathbb{e}^3 \wedge \mathbb{e}^1 \wedge \mathbb{e}^t) - A_1(\mathbb{e}^1 \wedge \mathbb{e}^2 \wedge \mathbb{e}^t) + \phi(\mathbb{e}^1 \wedge \mathbb{e}^2 \wedge \mathbb{e}^3)$

$$div(A) = curl\{A^d\} = (\partial_1 A_1 + \partial_2 A_2 + \partial_3 A_3 + \partial_t \phi)I = 0$$

$$\nabla \cdot A + \partial_t \phi = 0$$

Substituting A_i, ϕ for E_i, B_j in the 2).

$$\rho = \partial_1 E_1 + \partial_2 E_2 + \partial_3 E_3 = -\partial_{11}\phi - \partial_{1t}A_1 - \partial_{22}\phi - \partial_{2t}A_2 - \partial_{33}\phi - \partial_{3t}A_3$$

$$= -\partial_{11}\phi - \partial_{22}\phi - \partial_{33}\phi + \partial_{tt}\phi - \partial_t(\partial_1 A_1 + \partial_2 A_2 + \partial_3 A_3 + \partial_t \phi)$$

$$= -\partial_{11}\phi - \partial_{22}\phi - \partial_{33}\phi + \partial_{tt}\phi = \square\phi$$

$$J_1 = \partial_2 B_3 - \partial_3 B_2 - \partial_t E_1 = \partial_{21}A_2 - \partial_{22}A_1 - \partial_{33}A_1 + \partial_{31}A_3 + \partial_{tt}A_1 + \partial_{t1}\phi$$

$$= \partial_{tt}A_1 - \partial_{11}A_1 - \partial_{22}A_1 - \partial_{33}A_1 + \partial_1(\partial_1 A_1 + \partial_2 A_2 + \partial_3 A_3 + \partial_t \phi) = \Box A_1$$
$$\rho = \Box\phi, \qquad J_i = \Box A_i$$

The results for vector fields of other dimensions are summarized as follows. 2-vector field

$$A = A_1(\mathbb{e}^2 \wedge \mathbb{e}^3) + A_2(\mathbb{e}^3 \wedge \mathbb{e}^1) + A_3(\mathbb{e}^1 \wedge \mathbb{e}^2) - \phi_1(\mathbb{e}^1 \wedge \mathbb{e}^t) - \phi_2(\mathbb{e}^2 \wedge \mathbb{e}^t) - \phi_3(\mathbb{e}^3 \wedge \mathbb{e}^t)$$
$$curl(A) = B_\phi(\mathbb{e}^1 \wedge \mathbb{e}^2 \wedge \mathbb{e}^3) - E_1(\mathbb{e}^2 \wedge \mathbb{e}^3 \wedge \mathbb{e}^t) - E_2(\mathbb{e}^3 \wedge \mathbb{e}^1 \wedge \mathbb{e}^t) - E_3(\mathbb{e}^1 \wedge \mathbb{e}^2 \wedge \mathbb{e}^t)$$
$$B_\phi = \nabla \cdot A, \qquad E = \nabla \times \phi - \partial_t A$$
$$curl^d(A) = E_1\mathbb{e}^1 + E_2\mathbb{e}^2 + E_3\mathbb{e}^3 - B_\phi\mathbb{e}^t$$
$$J = \rho_1^s(\mathbb{e}^2 \wedge \mathbb{e}^3) + \rho_2^s(\mathbb{e}^3 \wedge \mathbb{e}^1) + \rho_3^s(\mathbb{e}^1 \wedge \mathbb{e}^2) + J_1^s(\mathbb{e}^1 \wedge \mathbb{e}^t) + J_2^s(\mathbb{e}^2 \wedge \mathbb{e}^t) + J_3^s(\mathbb{e}^3 \wedge \mathbb{e}^t)$$

ρ^s: spin density, J^s: spin current density.

$$\rho^s = \nabla \times E, \qquad J^s = -\nabla B_\phi - \partial_t E$$
$$\nabla \cdot E + \partial_t B_\phi = 0, \qquad \nabla \cdot \rho^s = 0, \qquad \nabla \times J^s + \partial_t \rho^s = 0$$
$$\nabla \cdot \phi = 0, \qquad \nabla \times A + \partial_t \phi = 0, \qquad \rho_i^s = \Box\phi_i, \qquad J_i^s = \Box A_i$$

3-vector field

$$A = -A_1(\mathbb{e}^2 \wedge \mathbb{e}^3 \wedge \mathbb{e}^t) - A_2(\mathbb{e}^3 \wedge \mathbb{e}^1 \wedge \mathbb{e}^t) - A_3(\mathbb{e}^1 \wedge \mathbb{e}^2 \wedge \mathbb{e}^t) + \phi(\mathbb{e}^1 \wedge \mathbb{e}^2 \wedge \mathbb{e}^3)$$
$$curl(A) = B = B_\phi(-I), \qquad B_\phi = -\nabla \cdot A - \partial_t \phi$$
$$curl^d(A) = B_\phi$$
$$J = \rho_1^q\mathbb{e}^1 + \rho_2^q\mathbb{e}^2 + \rho_3^q\mathbb{e}^3 - J^q\mathbb{e}^t$$

ρ^q: quark density, J^q: quark current density(Higgs).

$$\rho^q = \nabla B_\phi, \qquad J^q = -\partial_t B_\phi, \qquad \nabla \times \rho^q = 0, \qquad \nabla J^q + \partial_t \rho^q = 0$$
$$\nabla \times A = 0, \qquad \partial_t A + \nabla\phi = 0, \qquad \rho_i^q = \Box A_i, \qquad J^q = \Box\phi$$

scalar field

$$A = \phi$$
$$curl(A) = E_1\mathbb{e}^1 + E_2\mathbb{e}^2 + E_3\mathbb{e}^3 - B_\phi\mathbb{e}^t, \qquad E = \nabla\phi, \qquad -B_\phi = \partial_t\phi$$
$$curl^d(A) = B_\phi(\mathbb{e}^1 \wedge \mathbb{e}^2 \wedge \mathbb{e}^3) - E_1(\mathbb{e}^2 \wedge \mathbb{e}^3 \wedge \mathbb{e}^t) - E_2(\mathbb{e}^3 \wedge \mathbb{e}^1 \wedge \mathbb{e}^t) - E_3(\mathbb{e}^1 \wedge \mathbb{e}^2 \wedge \mathbb{e}^t)$$
$$J = \left(\partial_t B_\phi + \partial_1 E_1 + \partial_2 E_2 + \partial_3 E_3\right)I = \rho^m(-I)$$

ρ^m: mass density.

$$\rho^m = -\partial_t B_\phi - \nabla \cdot E$$
$$\nabla \times E = 0, \qquad \nabla B_\phi + \partial_t E = 0, \qquad \rho^m = \Box\phi$$

To summarize, charge density $\rho(\mathbb{e}^1 \wedge \mathbb{e}^2 \wedge \mathbb{e}^3)$ is like a particle with volume, spin density $\rho^s(\mathbb{e}^i \wedge \mathbb{e}^j)$ is like a membrane with area, quark density $\rho^q\mathbb{e}^i$ is a string with length. And quark current density $J^q\mathbb{e}^t$ is a time particle Higgs, it is necessary for mass density $\rho^m(\mathbb{e}^1 \wedge \mathbb{e}^2 \wedge \mathbb{e}^3 \wedge \mathbb{e}^t)$. charge current density $J_1(\mathbb{e}^2 \wedge \mathbb{e}^3 \wedge \mathbb{e}^t)$, superconductor is in a state where spins are aligned.

3. Physical Quantity

Physical quantities are expressed differently depending on the coordinate system, but they are one entity. If vector field A is a physical quantity, then $curl(A)$, $curl^d(A)$, J, energy density eng, Poynting vector S, and force F are also physical quantities.

I is obviously the same in all coordinate system. $|m_{ij}|$ is determinant of matrix m_{ij}

$$X(p^1, \dots, p^n) = X(q^1, \dots, q^n), \qquad u_i = \frac{\partial X}{\partial p^i}, \qquad v_j = \frac{\partial X}{\partial q^j}$$
$$u_i = \alpha_i^j v_j = (u_i \cdot v^j)v_j, \qquad u_i \cdot v^k = (a_i^j v_j) \cdot v^k = \alpha_i^k$$
$$u_i = \frac{\partial X}{\partial p^i} = \frac{\partial q^j}{\partial p^i}\frac{\partial X}{\partial q^j} = \frac{\partial q^j}{\partial p^i}v_j = (u_i \cdot v^j)v_j, \qquad (u_i \cdot v^j) = \frac{\partial q^j}{\partial p^i}, \qquad \left(v_j \cdot u^i\right) = \frac{\partial p^i}{\partial q^j}$$

$$u^i = (u^i \cdot v_j)v^j = \frac{\partial p^i}{\partial q^j} v^j$$
$$u_i \cdot v^j = (u_i \cdot \mathbb{e}^k)\mathbb{e}_k \cdot \mathbb{e}^l(\mathbb{e}_l \cdot v^j) = (u_i \cdot \mathbb{e}^k)(\mathbb{e}_k \cdot v^j)$$
$$v_i \cdot v^j = \delta_i^j = (v_i \cdot \mathbb{e}^k)(\mathbb{e}_k \cdot v^j), \qquad |v_i \cdot \mathbb{e}^j||\mathbb{e}_i \cdot v^j| = 1$$
$$(u_1 \wedge \dots \wedge u_n) = ((u_1 \cdot \mathbb{e}^{\sigma_1})\mathbb{e}_{\sigma_1} \wedge \dots \wedge (u_n \cdot \mathbb{e}^{\sigma_n})\mathbb{e}_{\sigma_n}) = \left|(u_i \cdot \mathbb{e}^j)\right|(\mathbb{e}_1 \wedge \dots \wedge \mathbb{e}_n) = \left|(u_i \cdot \mathbb{e}^j)\right| I$$
$$(u_1 \wedge \dots \wedge u_n) = |u_i \cdot v^j|(v_1 \wedge \dots \wedge v_n) = |(u_i \cdot \mathbb{e}^j)||(\mathbb{e}_i \cdot v^j)|(v_1 \wedge \dots \wedge v_n)$$
$$I(p) = (u_1 \wedge \dots \wedge u_n)/|(u_i \cdot \mathbb{e}^j)| = (v_1 \wedge \dots \wedge v_n)/|(v_i \cdot \mathbb{e}^j)| = I(q)$$

$A(p) = A(q)$ then
$$curl(A(p)) = \left(u^i \wedge \frac{\partial A(p)}{\partial p^i}\right) = \left(\frac{\partial p^i}{\partial q^j} v^j \wedge \frac{\partial A(p)}{\partial p^i}\right) = \left(v^j \wedge \frac{\partial p^i}{\partial q^j}\frac{\partial A(q)}{\partial p^i}\right) = \left(v^j \wedge \frac{\partial A(q)}{\partial q^j}\right) = curl(A(q))$$

$$curl^d(A(p)) = curl(A(p)) \cdot I = curl(A(q)) \cdot I = curl^d(A(q))$$
$$J(p) = curl(curl^d(A(p))) = curl(curl^d(A(q))) = J(q)$$

To describe the pseudoscalar eng and the 2-vector field S, multivector multiplication is required. See 3.1. Example of a 1-vector field.
$$(curl(A)curl^d(A))/2 = eng(A) + S(A)$$
$$curl(A) = B_1(\mathbb{e}^2 \wedge \mathbb{e}^3) + B_2(\mathbb{e}^3 \wedge \mathbb{e}^1) + B_3(\mathbb{e}^1 \wedge \mathbb{e}^2) + E_1(\mathbb{e}^1 \wedge \mathbb{e}^t) + E_2(\mathbb{e}^2 \wedge \mathbb{e}^t) + E_3(\mathbb{e}^3 \wedge \mathbb{e}^t)$$
$$curl^d(A) = -B_1(\mathbb{e}^1 \wedge \mathbb{e}^t) - B_2(\mathbb{e}^2 \wedge \mathbb{e}^t) - B_3(\mathbb{e}^3 \wedge \mathbb{e}^t) + E_1(\mathbb{e}^2 \wedge \mathbb{e}^3) + E_2(\mathbb{e}^3 \wedge \mathbb{e}^1) + E_3(\mathbb{e}^1 \wedge \mathbb{e}^2)$$
$$eng(A) = (curl(A) \wedge curl^d(A))/2 = ((B_i)^2 - (E_i)^2)I/2$$
$$S(A) = \varepsilon_{ijk}E_iB_j(\mathbb{e}^i \wedge \mathbb{e}^j), \qquad (i,j,k = 1,2,3)$$
Conventional expression.
$$eng(A) = (B^2 - E^2)I/2, \quad S(A) = E \times B$$

The fatal error of the conventional energy density $(E^2 + B^2)/2$ is that a moving observer in the Lorentz transformation changes the energy density of the entire background space and breaks energy conservation. 4.1. Electric, and magnetic energy density cannot be separated. They are generated by one vector field A.

$$eng^d(A) = eng(A) \cdot I = (curl^d(A) \cdot (curl(A) \cdot I))/2 = (curl^d(A) \cdot curl^d(A))/2 = (E^2 - B^2)/2$$
$$F = div(eng(A)) = curl(eng^d(A)) = curl(curl^d(A) \cdot curl^d(A))/2 = J \cdot curl^d(A)$$
$$J = \rho(\mathbb{e}^1 \wedge \mathbb{e}^2 \wedge \mathbb{e}^3) - J_1(\mathbb{e}^2 \wedge \mathbb{e}^3 \wedge \mathbb{e}^t) - J_2(\mathbb{e}^3 \wedge \mathbb{e}^1 \wedge \mathbb{e}^t) - J_3(\mathbb{e}^1 \wedge \mathbb{e}^2 \wedge \mathbb{e}^t)$$
$$F_1\mathbb{e}^1 = (\rho E_1 + J_2B_3 - J_3B_2)\mathbb{e}^1, \qquad F_t\mathbb{e}^t = (-J_1E_1 - J_2E_2 - J_3E_3)\mathbb{e}^t$$
Conventional expression.
$$F = \rho E + J \times B, \qquad F_t = -J \cdot E\ (Joule\ heat)$$

The Poynting vector is itself an energy flow and pressure. The same process can be repeated for vector fields of other dimensions.

$$eng(A(p)) = eng(A(q)), \qquad S(A(p)) = S(A(q)), \qquad F(A(p)) = F(A(q))$$

3.1. *Multibases multiplication*

Multiplication of multibases in Cartesian coordinate system. The common basis is omitted. There is no difficulty in converting to a curvilinear coordinate system.
$$(\mathbb{e}^{\sigma_1} \wedge \dots \wedge \mathbb{e}^{\sigma_r})(\mathbb{e}^{s_1} \wedge \dots \wedge \mathbb{e}^{s_k}) = (\mathbb{e}^{\sigma_1} \wedge \dots \wedge \mathbb{e}^{\sigma_{r-1}})\{\mathbb{e}^{\sigma_r}(\mathbb{e}^{s_1} \wedge \dots \wedge \mathbb{e}^{s_k})\}$$
$$= (\mathbb{e}^{\sigma_1} \wedge \dots \wedge \mathbb{e}^{\sigma_{r-2}})[\mathbb{e}^{\sigma_{r-1}}\{\mathbb{e}^{\sigma_r}(\mathbb{e}^{s_1} \wedge \dots \wedge \mathbb{e}^{s_k})\}] = \cdots$$

$$\mathbb{e}^{\sigma_r}(\mathbb{e}^{s_1} \wedge \dots \wedge \mathbb{e}^{s_k}) = \begin{cases} (-1)^{i-1}(\mathbb{e}^{s_1} \wedge \dots \wedge \widehat{\mathbb{e}^{s_i}} \wedge \dots \wedge \mathbb{e}^{s_k}) & (\sigma_r = s_i) \\ (\mathbb{e}^{\sigma_r} \wedge \mathbb{e}^{s_1} \wedge \dots \wedge \mathbb{e}^{s_k}) & else \end{cases}$$

$\mathbb{e}^t\mathbb{e}^t = 1, \qquad \mathbb{e}^2\mathbb{e}^3 = (\mathbb{e}^2 \wedge \mathbb{e}^3), \qquad (\mathbb{e}^1 \wedge \mathbb{e}^t)(\mathbb{e}^1 \wedge \mathbb{e}^t) = \mathbb{e}^1\{\mathbb{e}^t(\mathbb{e}^1 \wedge \mathbb{e}^t)\} = \mathbb{e}^1\{(-1)^{2-1}\mathbb{e}^1\} = -1$

$(\mathbb{e}^2 \wedge \mathbb{e}^t)(\mathbb{e}^1 \wedge \mathbb{e}^2 \wedge \mathbb{e}^t) = \mathbb{e}^2(\mathbb{e}^1 \wedge \mathbb{e}^2) = -\mathbb{e}^1$

4. Rotor and booster

In the Cartesian coordinate system, the rotor and booster R transform the bases. The notation is as follows.

$$\mathbb{e}_{\mu'} = \mathbb{e}_\mu|R = \mathbb{e}_\mu \cdot R, \qquad \mathbb{e}_\mu|1 = \mathbb{e}_\mu$$

An infinitesimal rotor R_i

$$R_i = 1 + \Delta\mathbb{s}_i, \qquad (\Delta \ll 1), \qquad \mathbb{s}_1 = (\mathbb{e}_2 \wedge \mathbb{e}_3), \qquad \mathbb{s}_2 = (\mathbb{e}_3 \wedge \mathbb{e}_1), \qquad \mathbb{s}_3 = (\mathbb{e}_1 \wedge \mathbb{e}_2)$$

$$\mathbb{e}_1|R_3 = \mathbb{e}_1|(1 + \Delta\mathbb{s}_3) = \mathbb{e}_1|1 + \mathbb{e}_1|\Delta\mathbb{s}_3 = \mathbb{e}_1 + \mathbb{e}_1 \cdot \Delta\mathbb{s}_3 = \mathbb{e}_1 + \Delta\mathbb{e}_2$$

$$\mathbb{e}_2|R_3 = \mathbb{e}_2|1 + \mathbb{e}_2|\Delta\mathbb{s}_3 = \mathbb{e}_2 - \Delta\mathbb{e}_1, \qquad \mathbb{e}_3|R_3 = \mathbb{e}_3, \qquad \mathbb{e}_t|R_3 = \mathbb{e}_t$$

$$\mathbb{e}_\nu|\mathbb{s}_3|\mathbb{s}_3 = (\mathbb{e}_\nu \cdot \mathbb{s}_3) \cdot \mathbb{s}_3$$

$$(\mathbb{e}_1 \cdot \mathbb{s}_3) \cdot \mathbb{s}_3 = -\mathbb{e}_1, \qquad (\mathbb{e}_2 \cdot \mathbb{s}_3) \cdot \mathbb{s}_3 = -\mathbb{e}_2, \qquad (\mathbb{e}_3 \cdot \mathbb{s}_3) \cdot \mathbb{s}_3 = 0, \qquad (\mathbb{e}_t \cdot \mathbb{s}_3) \cdot \mathbb{s}_3 = 0,$$

$$\mathbb{s}_3|\mathbb{s}_3 = \begin{cases} -1 \\ 0 \end{cases}$$

Infinite series of infinitesimal plane rotors. $\Delta_1 + \Delta_2 + \Delta_3 \ldots = \theta$

$$R_3(\theta) = (1 + \Delta_1\mathbb{s}_3)|(1 + \Delta_2\mathbb{s}_3)| \ldots = \lim_{\Delta\to 0}(1 + \Delta\mathbb{s}_3|)^{\theta/\Delta} = e^{\mathbb{s}_3|\theta}$$

$$= 1 + \theta\mathbb{s}_3 + \frac{1}{2!}\theta^2\mathbb{s}_3|\mathbb{s}_3 + \frac{1}{3!}\theta^3\mathbb{s}_3|\mathbb{s}_3|\mathbb{s}_3 + \cdots = \begin{cases} \cos\theta + \sin\theta\,\mathbb{s}_3 \\ 1 + \theta\mathbb{s}_3 \end{cases}$$

An infinitesimal booster R_i^d

$$R_i^d = 1 + \Delta\mathbb{s}_i^d, \qquad \mathbb{s}_1^d = \mathbb{s}_1 \cdot I = (\mathbb{e}_1 \wedge \mathbb{e}_t), \qquad \mathbb{s}_2^d = (\mathbb{e}_2 \wedge \mathbb{e}_t), \qquad \mathbb{s}_3^d = (\mathbb{e}_3 \wedge \mathbb{e}_t),$$

$$\mathbb{e}_1|R_1^d = \mathbb{e}_1 + \Delta\mathbb{e}_t, \qquad \mathbb{e}_t|R_1^d = \mathbb{e}_t + \Delta\mathbb{e}_1, \qquad \mathbb{e}_2|R_1^d = \mathbb{e}_2, \qquad \mathbb{e}_3|R_1^d = \mathbb{e}_3$$

$$\mathbb{e}_\nu|\mathbb{s}_1^d|\mathbb{s}_1^d = (\mathbb{e}_\nu \cdot \mathbb{s}_1^d) \cdot \mathbb{s}_1^d$$

$$(\mathbb{e}_1 \cdot \mathbb{s}_1^d) \cdot \mathbb{s}_1^d = \mathbb{e}_1, \qquad (\mathbb{e}_2 \cdot \mathbb{s}_1^d) \cdot \mathbb{s}_1^d = 0, \qquad (\mathbb{e}_3 \cdot \mathbb{s}_1^d) \cdot \mathbb{s}_1^d = 0, \qquad (\mathbb{e}_t \cdot \mathbb{s}_1^d) \cdot \mathbb{s}_1^d = \mathbb{e}_t$$

$$\mathbb{s}_1^d|\mathbb{s}_1^d = \begin{cases} 1 \\ 0 \end{cases}$$

Infinite series of infinitesimal booster. $\Delta_1 + \Delta_2 + \Delta_3 \ldots = \theta$

$$R_1^d(\theta) = (1 + \Delta_1\mathbb{s}_1^d)|(1 + \Delta_2\mathbb{s}_1^d) \ldots = \lim_{\Delta\to 0}(1 + \Delta\mathbb{s}_1^d|)^{\theta/\Delta} = e^{\mathbb{s}_1^d|\theta}$$

$$= 1 + \theta\mathbb{s}_1^d + \frac{1}{2!}\theta^2\mathbb{s}_1^d|\mathbb{s}_1^d + \frac{1}{3!}\theta^3\mathbb{s}_1^d|\mathbb{s}_1^d|\mathbb{s}_1^d + \cdots = \begin{cases} \cosh\theta + \sinh\theta\,\mathbb{s}_1^d \\ 1 + \theta\mathbb{s}_1^d \end{cases}$$

4.1. *Multibasis transformations*

The multibasis transformations is as follows. Pseudoscalar and scalar are not changed by the rotor and booster.

$$(\mathbb{e}_\mu \wedge \mathbb{e}_\nu)|R = ((\mathbb{e}_\mu|R) \wedge (\mathbb{e}_\nu|R))$$

$$(\mathbb{e}_\mu \wedge \mathbb{e}_\nu \wedge \mathbb{e}_\lambda)|R = ((\mathbb{e}_\mu|R) \wedge (\mathbb{e}_\nu|R) \wedge (\mathbb{e}_\lambda|R))$$

$$I|R = I, \qquad (\mathbb{e}_\mu \cdot \mathbb{e}_\nu) = (\mathbb{e}_\mu \cdot \mathbb{e}_\nu)|R = ((\mathbb{e}_\mu|R) \cdot (\mathbb{e}_\nu|R))$$

For examples

$$\mathbb{e}^1|R_1^d(\theta) = \cosh\theta\,\mathbb{e}^1 - \sinh\theta\,\mathbb{e}^t, \qquad \mathbb{e}^2|R_1^d(\theta) = \mathbb{e}^2, \qquad \mathbb{e}^t|R_1^d(\theta) = \cosh\theta\,\mathbb{e}^t - \sinh\theta\,\mathbb{e}^1$$

$$(\mathbb{e}^1 \wedge \mathbb{e}^2)|R_1^d(\theta) = ((\cosh\theta\,\mathbb{e}^1 - \sinh\theta\,\mathbb{e}^t) \wedge \mathbb{e}^2) = \cosh\theta\,(\mathbb{e}^1 \wedge \mathbb{e}^2) + \sinh\theta\,(\mathbb{e}^2 \wedge \mathbb{e}^t)$$

$$(\mathbb{e}^1 \wedge \mathbb{e}^t)|R_1^d(\theta) = ((\cosh\theta\,\mathbb{e}^1 - \sinh\theta\,\mathbb{e}^t) \wedge (\cosh\theta\,\mathbb{e}^t - \sinh\theta\,\mathbb{e}^1)) = (\mathbb{e}^1 \wedge \mathbb{e}^t)$$

$$I|R_1^d(\theta) = -(\mathbb{e}^1 \wedge \mathbb{e}^2 \wedge \mathbb{e}^3 \wedge \mathbb{e}^t)|R_1^d(\theta) = -(\mathbb{e}^1 \wedge \mathbb{e}^2 \wedge \mathbb{e}^3 \wedge \mathbb{e}^t) = I$$

$$(\mathbb{e}_1 \cdot \mathbb{e}_1) = ((\mathbb{e}_1|R_1^d(\theta)) \cdot (\mathbb{e}_1|R_1^d(\theta))) = ((\cosh\theta\, \mathbb{e}_1 + \sinh\theta\, \mathbb{e}_t) \cdot (\cosh\theta\, \mathbb{e}_1 + \sinh\theta\, \mathbb{e}_t)) = 1$$
$$((\mathbb{e}^1 \wedge \mathbb{e}^2) \cdot (\mathbb{e}^1 \wedge \mathbb{e}^2))|R_1^d(\theta) = (\mathbb{e}^1 \wedge \mathbb{e}^2)|R_1^d(\theta) \cdot (\mathbb{e}^1 \wedge \mathbb{e}^2)|R_1^d(\theta) = 1$$

For example, electromagnetic field in Lorenz boost $R_1^d(\theta)$

$$curl(A(p)) = curl(A(q))$$
$$= B_1(\mathbb{e}^2 \wedge \mathbb{e}^3) + B_2(\mathbb{e}^3 \wedge \mathbb{e}^1) + B_3(\mathbb{e}^1 \wedge \mathbb{e}^2) + E_1(\mathbb{e}^1 \wedge \mathbb{e}^t) + E_2(\mathbb{e}^2 \wedge \mathbb{e}^t) + E_3(\mathbb{e}^3 \wedge \mathbb{e}^t)$$
$$= B_1'(\mathbb{e}^{2'} \wedge \mathbb{e}^{3'}) + B_1'(\mathbb{e}^{3'} \wedge \mathbb{e}^{1'}) + B_1'(\mathbb{e}^{1'} \wedge \mathbb{e}^{2'}) + E_1'(\mathbb{e}^{1'} \wedge \mathbb{e}^{t'}) + E_1'(\mathbb{e}^{2'} \wedge \mathbb{e}^{t'}) + E_1'(\mathbb{e}^{3'} \wedge \mathbb{e}^{t'})$$

Obtain the coefficient relationship by substituting $(\mathbb{e}^{\mu'} \wedge \mathbb{e}^{\nu'})$ with $(\mathbb{e}^{\mu} \wedge \mathbb{e}^{\nu})$.

$$B_1 = B_1', \qquad B_2 = \cosh\theta\, B_2' - \sinh\theta\, E_3', \qquad B_3 = \cosh\theta\, B_3' + \sinh\theta\, E_2'$$
$$E_1 = E_1', \qquad E_2 = \sinh\theta\, B_3' + \cosh\theta\, E_2', \qquad E_3 = -\sinh\theta\, B_2' + \cosh\theta\, E_3'$$

$$eng(A(p)) = ((E_i)^2 - (B_i)^2)I/2 = ((E_i')^2 - (B_i')^2)I/2 = eng(A(q))$$

The energy density, I field is the same for a moving observer and a rest frame observer.

$$J(p) = -\rho(\mathbb{e}^1 \wedge \mathbb{e}^2 \wedge \mathbb{e}^3) - J_1(\mathbb{e}^2 \wedge \mathbb{e}^3 \wedge \mathbb{e}^t) - J_2(\mathbb{e}^3 \wedge \mathbb{e}^1 \wedge \mathbb{e}^t) - J_3(\mathbb{e}^1 \wedge \mathbb{e}^2 \wedge \mathbb{e}^t)$$
$$= J(q) - \rho'(\mathbb{e}^{1'} \wedge \mathbb{e}^{2'} \wedge \mathbb{e}^{3'}) - J_1'(\mathbb{e}^{2'} \wedge \mathbb{e}^{3'} \wedge \mathbb{e}^{t'}) - J_2'(\mathbb{e}^{3'} \wedge \mathbb{e}^{1'} \wedge \mathbb{e}^{t'}) - J_3'(\mathbb{e}^{1'} \wedge \mathbb{e}^{2'} \wedge \mathbb{e}^{t'})$$
$$\rho = \cosh\theta\, \rho' - \sinh\theta\, J_1', \qquad J_1 = \cosh\theta\, J_1' - \sinh\theta\, J_t', \qquad J_2 = J_2', \qquad J_3 = J_3'$$

$$F(p) = F_1\mathbb{e}^1 + F_2\mathbb{e}^2 + F_3\mathbb{e}^3 + F_t\mathbb{e}^t = F_1'\mathbb{e}^{1'} + F_2'\mathbb{e}^{2'} + F_3'\mathbb{e}^{3'} + F_t'\mathbb{e}^{t'} = F(q)$$
$$F_t = \cosh\theta\, F_t' - \sinh\theta\, F_1', \qquad F_1 = \cosh\theta\, F_1' - \sinh\theta\, F_t', \qquad F_2 = F_2', \qquad F_3 = F_3'$$

Check in another way

$$F_1 = (-J_tE_1 + J_2B_3 - J_3B_2)$$
$$= -(\cosh\theta\, \rho' - \sinh\theta\, J_1')E_1' + J_2'(\cosh\theta\, B_3' + \sinh\theta\, E_2') - J_3'(\cosh\theta\, B_2' - \sinh\theta\, E_3')$$
$$= \cosh\theta\,(-\rho'E_{1'} + J_2'B_3' - J_3'B_2') - \sinh\theta\,(-J_1'E_1' - J_2'E_2' - J_3'E_3') = \cosh\theta\, F_t' - \sinh\theta\, F_1'$$

5. Quantization

The two rotors $(R_A|R_B)$ and $(R_B|R_A)$ are different. This difference causes quantization. R^* is the Hermitian rotor with respect to R.

$$\mathbb{e}_\mu' = \mathbb{e}_\mu|R, \qquad \mathbb{e}_\mu = \mathbb{e}_\mu'|R^*$$
$$\mathbb{e}_\mu = \mathbb{e}_\mu|(R|R^*), \qquad \mathbb{e}_\mu' = \mathbb{e}_\mu'|(R^*|R), \qquad (R|R^*) = (R^*|R) = 1$$
$$\mathbb{e}_\mu \cdot (\mathbb{e}_\nu|R) = (\mathbb{e}_\mu \cdot (\mathbb{e}_\nu|R))|R^* = (\mathbb{e}_\mu|R^*) \cdot (\mathbb{e}_\nu|(R|R^*)) = (\mathbb{e}_\mu|R^*) \cdot \mathbb{e}_\nu$$

For examples

$$R_3(\theta) = \cos\theta + \sin\theta\, \mathbb{s}_3, \qquad R_3^*(\theta) = \cos\theta - \sin\theta\, \mathbb{s}_3 = R_3(-\theta)$$
$$R_1^d(\theta) = \cosh\theta + \sinh\theta\, \mathbb{s}_1^d, \qquad R_1^{d*}(\theta) = R_i^*(-\theta)$$

5.1. *Commutator*

$$[R_A, R_B] = (R_A|R_B) - (R_B|R_A)$$
$$R_A(\theta), R_B(\theta) \in \{(\cos\theta + \sin\theta\, \mathbb{s}_i), (1 + \theta\mathbb{s}_j), (\cosh\theta + \sinh\theta\, \mathbb{s}_k^d), (1 + \theta\mathbb{s}_l^d)\}$$
$$[R_A(\theta_1), R_B(\theta_2)] = [R_A(-\theta_1), R_B(-\theta_2)] = [R_A^*(\theta_1), R_B^*(\theta_2)]$$

The following equation must be satisfied.

$$\mathbb{e}_\mu \cdot (\mathbb{e}_\nu|(R_A|R_B)) = (\mathbb{e}_\mu|R_B^*) \cdot (\mathbb{e}_\nu|(R_A)) = (\mathbb{e}_\mu|(R_B^*|R_A^*)) \cdot \mathbb{e}_\nu$$
$$\mathbb{e}_\mu \cdot (\mathbb{e}_\nu|[R_A, R_B]) = (\mathbb{e}_\mu|[R_B^*, R_A^*]) \cdot \mathbb{e}_\nu = -(\mathbb{e}_\mu|[R_A, R_B]) \cdot \mathbb{e}_\nu$$

By substituting various cases and omitting the coefficients, the following relationship satisfying the above conditions is obtained. And this relationship holds even in the case of multibases. for example.

$$[\mathbb{s}_1, \mathbb{s}_2] = -\mathbb{s}_3$$

$$\mathbb{e}_1|[\mathbb{s}_1, \mathbb{s}_2] = (\mathbb{e}_1 \cdot \mathbb{s}_1) \cdot \mathbb{s}_2 - (\mathbb{e}_1 \cdot \mathbb{s}_2) \cdot \mathbb{s}_1 = -\mathbb{e}_2 = \mathbb{e}_1 \cdot (-\mathbb{s}_3)$$
$$\mathbb{e}_2|[\mathbb{s}_1, \mathbb{s}_2] = \mathbb{e}_1 = \mathbb{e}_2 \cdot (-\mathbb{s}_3), \qquad \mathbb{e}_3 \cdot [\mathbb{s}_1, \mathbb{s}_2] = 0 = \mathbb{e}_3 \cdot (-\mathbb{s}_3), \qquad \mathbb{e}_t \cdot [\mathbb{s}_1, \mathbb{s}_2] = 0 = \mathbb{e}_t \cdot (-\mathbb{s}_3)$$
$$\mathbb{e}_1 \cdot (\mathbb{e}_2|[\mathbb{s}_1, \mathbb{s}_2]) = -(\mathbb{e}_1|[\mathbb{s}_1, \mathbb{s}_2]) \cdot \mathbb{e}_2, \qquad \ldots$$
$$[\mathbb{s}_i, \mathbb{s}_j] = -\varepsilon_{ijk}\mathbb{s}_k, \qquad [\mathbb{s}_i^d, \mathbb{s}_j^d] = \varepsilon_{ijk}\mathbb{s}_k, \qquad [\mathbb{s}_i, \mathbb{s}_j^d] = [\mathbb{s}_i^d, \mathbb{s}_j] = -\varepsilon_{ijk}\mathbb{s}_k^d$$

5.2. *ladder*

The ladder is obtained by adding and subtracting the two equations above.

$$[\mathbb{s}_k, \mathbb{s}_j^d] = -\varepsilon_{kji}\mathbb{s}_i^d = \varepsilon_{ijk}\mathbb{s}_i^d, \qquad [\mathbb{s}_i^d, \mathbb{s}_j^d] = \varepsilon_{ijk}\mathbb{s}_k$$
$$[(\mathbb{s}_k \pm \mathbb{s}_i^d), \mathbb{s}_j^d] = \pm\varepsilon_{ijk}(\mathbb{s}_k \pm \mathbb{s}_i^d), \qquad [\mathbb{s}_{j\pm}, \mathbb{s}_j^d] = \pm\varepsilon_{ijk}\mathbb{s}_{j\pm}, \qquad \mathbb{s}_{j\pm} = \mathbb{s}_k \pm \mathbb{s}_i^d$$
$$\mathbb{s}_{j\pm}|\mathbb{s}_j^d = \mathbb{s}_j^d|\mathbb{s}_{j\pm} \pm \mathbb{s}_{j\pm}$$

For example, if 1-basis field V^0 exists, then $V^1, V^2, V^3, \ldots$ satisfying the ladder may also exist.

$$V^0 = \mathbb{e}^2 - \mathbb{e}^t, \qquad V^0|\mathbb{s}_2^d = V^0 \cdot (\mathbb{e}_2 \wedge \mathbb{e}_t) = \mathbb{e}^2 - \mathbb{e}^t = V^0$$
$$\mathbb{s}_{2+}|\mathbb{s}_2^d = \mathbb{s}_2^d|\mathbb{s}_{2+} + \mathbb{s}_{2+} = (\mathbb{s}_2^d + 1)|\mathbb{s}_{2+}, \qquad \mathbb{s}_{2+} = \mathbb{s}_3 + \mathbb{s}_1^d$$

$$V^0|(\mathbb{s}_2^d + 1)|\mathbb{s}_{2+} = (V^0 + V^0)|\mathbb{s}_{2+} = 2V^1 = V^0|(\mathbb{s}_{2+}|\mathbb{s}_2^d) = V^1|\mathbb{s}_2^d, \qquad V^0|\mathbb{s}_{2+} = V^1$$
$$V^1|(\mathbb{s}_2^d + 1)|\mathbb{s}_{2+} = (2V^1 + V^1)|\mathbb{s}_{2+} = 3V^2 = V^1|(\mathbb{s}_{2+}|\mathbb{s}_2^d) = V^2|\mathbb{s}_2^d, \qquad V^1|\mathbb{s}_{2+} = V^2$$
$$\ldots$$
$$V^m|\mathbb{s}_2^d = (m+1)V^m, \qquad V^{m-1}|\mathbb{s}_{2+} = V^m$$

6. Planck's constant

Three proofs that the universe vibrates. There is no rest frame. Only the ratio of the expected value in rest frame to the measured value in measuring frame can be known.

First, vibration $(1 + \Delta\mathbb{s}_i)$ create commutation relations of angular momentum operators. 5.1.

$$R_A = 1 + \hbar\mathbb{s}_i, \qquad R_B = 1 + \hbar\mathbb{s}_j$$
$$[\hbar\mathbb{s}_i, \hbar\mathbb{s}_j] = -\varepsilon_{ijk}\hbar(\hbar\mathbb{s}_k), \qquad [L_i, L_j] = -\varepsilon_{ijk}\hbar L_k, \qquad L_i = \hbar\mathbb{s}_i$$

Second, vibration $(1 + \Delta\mathbb{s}_i^d)$ create uncertainty principle. If the location is measured accurately, the time cannot be measured accurately.

$$X = x^1\mathbb{e}_1 + t\mathbb{e}_t = x^{1\prime}\mathbb{e}_1' + t'\mathbb{e}_t' = x^{1\prime}(\mathbb{e}_1 + \Delta\mathbb{e}_t) + t'(\mathbb{e}_t + \Delta\mathbb{e}_1) = (x^{1\prime} + \Delta t')\mathbb{e}_1 + (t' + \Delta x^{1\prime})\mathbb{e}_t$$
$$x^1 = x^{1\prime} + \Delta t', \qquad t = t' + \Delta x^{1\prime}$$
$$\left(\frac{x^1}{x^{1\prime}}\right)\left(\frac{t}{t'}\right) = \left(1 + \Delta\frac{t'}{x^{1\prime}}\right)\left(1 + \Delta\frac{x^{1\prime}}{t'}\right) \geq 1 + 2\Delta = 1 + 2\hbar$$

Third, photons are beat waves created by vibration $\left(1 + \Delta\mathbb{s}_i^d\right)$. Then, we can count photons.

$$x^1 - t = (x^{1\prime} - t')(1 - \Delta), \qquad e^{i(x^1 - t)} = e^{i(x^{1\prime} - t')}e^{-i\Delta(x^{1\prime} - t')}$$
$$\psi = \frac{e^{i(x^1 - t)}}{e^{i(x^{1\prime} - t')}} = e^{-i\Delta(x^{1\prime} - t')} = e^{-i\hbar(x^{1\prime} - t')}$$

Here, x^1 is a value in units reduced wavelength $(\overline{\lambda} = \lambda/2\pi)$, and t is a value in units of reduced period $(\overline{T} = T/2\pi)$. The frequency ν is due to wave $e^{i(x^{1\prime} - t')}$, and the energy $h\nu = \hbar\omega$ is likely due to the beat wave $e^{-i\hbar(x^{1\prime} - t')}$. Schrödinger's wave is also a beat wave. $\hbar$ is not a constant coefficient of the equation.