

The Non-Linear Dynamic Architecture of the Hydrogen Atom: Overcoming the Paradoxes of Linear Quantum Mechanics via Quaternion Attractors

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Abstract

This paper introduces an alternative, deterministic paradigm for the micro-world by modeling the hydrogen atom as an open, non-linear dissipative system embedded within the hydrodynamic substratum of Wheeler’s quantum foam. By replacing the abstract complex-valued wave function of standard quantum mechanics with a modified three-dimensional Van der Pol system formulated via Hamilton’s quaternions ($\mathbb{H}$), we resolve the fundamental paradoxes of stationarity, instantaneous quantum jumps, and wave-particle duality. In this framework, the stable ground state of the atom emerges naturally as a stable limit cycle (attractor), where the classical Coulomb potential acts as an active negative-friction energy pump that balances velocity-dependent radiative dissipation at the Bohr radius (a_0). We present a non-quantum, electrodynamic derivation of the Bohr radius and link the emission frequency directly to radiation intensity via the characteristic impedance of free space (Z_0) without invoking Planck’s constant (h). Furthermore, the four traditional quantum numbers (n, l, m, s), the Zeeman splitting, and the Pauli exclusion principle are decoded as explicit geometric and topological properties of a phase-locked spatial rotator, bypassing the necessity of both the Schrödinger probability density and the complex Dirac matrices.

Keywords: Quaternion Electrodynamics, Van der Pol Oscillator, Quantum Foam, Non-Linear Attractors, Bohr Radius, Zeeman Effect, Pauli Phase-Locking, Wave-Particle Duality.

1 Introduction

1.1 The Empirical Triumph and Conceptual Crisis of Quantum Formalism

Standard quantum mechanics (QM), as detailed in foundational textbooks such as Landau and Lifshitz [13], has achieved unparalleled success in statistically predicting the spectral lines of the hydrogen atom, the Zeeman splitting, and the structural symmetry of electron shells. However, this mathematical framework achieves predictive accuracy at the expense of physical realism. By substituting explicit particle trajectories with an abstract, complex-valued probability wave function $\psi(\mathbf{r}, t)$, linear quantum mechanics has introduced several deep-seated theoretical contradictions that remain unresolved within its own paradigm.

The primary contradictions between standard quantum theory and physical reality can be systematically categorized as follows:

- **The Paradox of Radiative Instability and Stationarity:** Classical electrodynamics demands that any accelerating charge must continuously radiate energy. Quantum mechanics bypasses this by postulating “stationary states” where the probability density

$|\psi|^2$ is invariant over time. This framework merely asserts stability rather than explaining it; it lacks an intrinsic, dynamic feedback mechanism capable of restoring the system to equilibrium after an external perturbation.

- **The Discontinuity of Quantum Jumps:** In the linear Schrödinger equation, spectral transitions are modeled as instantaneous, non-deterministic “quantum leaps” between eigenstates. The theory is fundamentally incapable of describing the time-domain trajectory or the continuous electrodynamic evolution of the system during the emission process itself, even within modern attosecond timescales [11].
- **The Multi-Electron Analytical Collapse:** While the linear framework can be solved for the single-electron hydrogen atom, it breaks down analytically for any multi-body system (e.g., the helium atom) due to the non-linear nature of inter-electron Coulomb repulsion. QM is forced to rely on heavy phenomenological approximations that obscure multi-particle correlations.
- **Ultraviolet Divergences and Point Charges:** Treating the electron as a zero-dimensional point charge interacting with its own electrostatic field leads to an infinite self-energy anomaly. This mathematical artifact requires artificial normalization procedures rather than offering a physically grounded vortex model.
- **The Abstraction of Wave Mappings:** The introduction of the abstract wave function allowed for the construction of a simple, dimensionless equation where the classical Coulomb potential is absorbed into the differential structure. Consequently, the explicit electrostatic interaction is effectively mapped onto the second spatial derivative (the Laplacian ∇^2) of the wave function [1], removing the real time-domain dynamics from the field.

1.2 The Vacuum Foam as the Hydrodynamic Substratum: Honoring Wheeler’s Quantum Foam

To resolve these paradoxes without abandoning the established empirical data, this paper proposes an alternative paradigm: the hydrogen atom as an open, non-linear dissipative system embedded within an active, fluctuating spatial vacuum. We define this physical medium as the vacuum foam, which serves as the direct macroscopic analog to John Archibald Wheeler’s quantum foam [15].

In Wheeler’s vision of geometrodynamics, the fabric of spacetime at the Planck scale (10^{-35} m) is not a smooth pseudo-Riemannian manifold, but a violent, hyper-dynamic ocean of stochastic topological fluctuations, virtual wormholes, and continuous energy transitions. Our model scales this chaotic substratum into a coherent hydrodynamic medium that exerts a measurable physical drag and feedback on subatomic structures, utilizing the foundations of non-equilibrium thermodynamics [3] and non-linear oscillations [4, 8]:

1. **The Vacuum Energy Reservoir:** The stochastic energy density of Wheeler’s foam acts as the underlying source for the internal feedback loop of the atom.
2. **Emergence of Non-Linear Friction:** The continuous interaction between the electron vortex and the Planckian vacuum fluctuations replaces the traditional passive vacuum with an active, dissipative fluid. This interaction is mathematically captured by a modified three-dimensional Van der Pol system formulated via Hamilton’s quaternions (H) [5].

In this framework, the stable ground state of the hydrogen atom is revealed to be a stable limit cycle (attractor). The classical Coulomb potential is no longer a passive scalar background, but the active energy engine driving the vacuum foam. It acts as a negative friction pump at close distances, utilizing the local Planckian fluctuation gradients to inject energy into the electron orbit, perfectly compensating for the velocity-dependent radiative dissipation (positive friction) at the Bohr radius a_0 .

1.3 The Electrodynamic Wave Nature of Emission

Crucially, when the system undergoes a global bifurcation—spiraling down from an unstable excited attractor ($n = 2$) to the stable ground state ($n = 1$)—the excess mechanical energy is smoothly and deterministically transferred to the field. Rather than creating a localized point-particle photon, the transition current drives the field equations, generating a tightly wound, self-reinforcing electromagnetic wave packet.

Unlike a classical flat wave, this radiated packet inherits the rotational geometry of the quaternion spatial rotator. The electric field vector $\vec{E}$ and magnetic field vector $\vec{B}$ are phase-locked and twist into a helical topology, exhibiting the exact spin and polarization properties attributed to light fields in empirical optics.

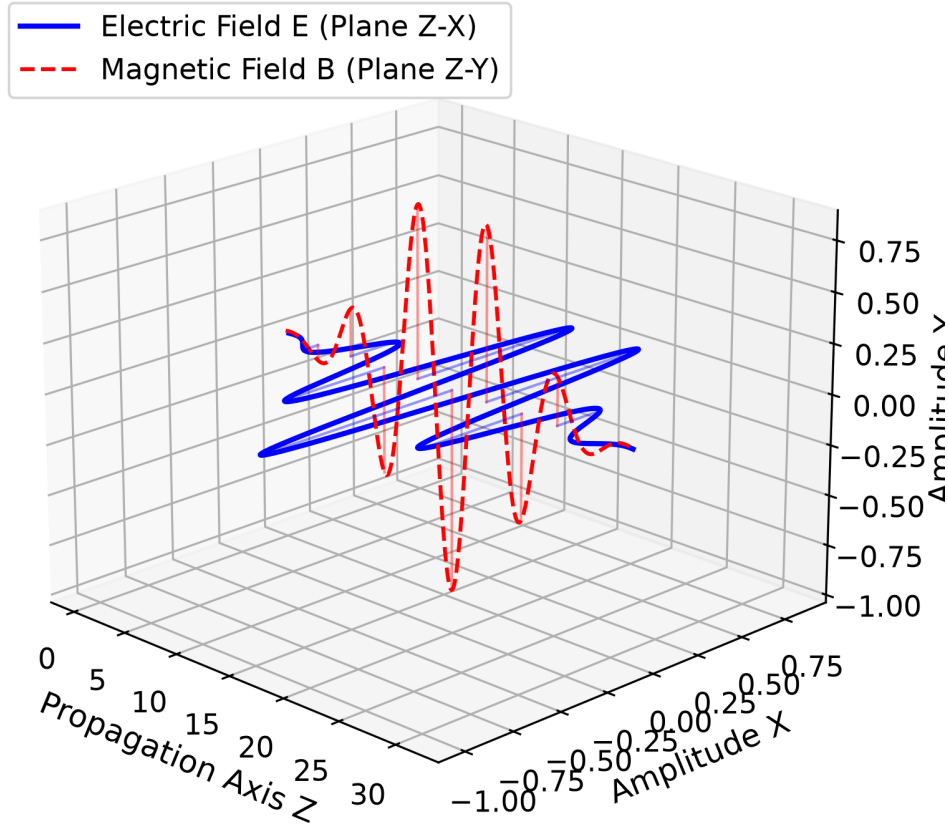


Figure 1: Theoretical and spatial profile of the quaternion electromagnetic wave packet (photon soliton). The intertwined helical trajectories of the electric vector $\vec{E}$ (solid line) and magnetic vector $\vec{B}$ (dashed line) form a self-contained, localized packet along the axis of propagation, matching the rotational boundaries of the quaternion emitter.

2 Mathematical Formalism of the Quaternion Spatial Rotator

2.1 Foundational Equivalence and Experimental Grounding

It is mathematically and philosophically vital to state that both standard quantum mechanics and the non-linear dissipative framework presented herein address the exact same body of empirical evidence and experimental facts [10].

However, where linear QM encounters severe conceptual compromises—such as postulating instantaneous, non-deterministic “quantum jumps” that fail to describe the physical time-

domain transition—this model relies on deterministic trajectories regulated by the medium. By replacing abstract complex-valued wave functions with the explicit geometry of Hamilton’s quaternions ($\mathbb{H}$) [5], the apparent probabilistic nature of the micro-world is revealed to be the macroscopic manifestation of a non-linear, self-stabilizing dynamic system loading into the vacuum impedance.

2.2 The Quaternion Spatial Rotator Equation

To describe continuous, singularity-free rotations and vortex dynamics in three-dimensional space, we represent the spatial position of the electron relative to the nucleus as a pure imaginary (spatial) quaternion:

$$\mathbf{q}(t) = x\vec{i} + y\vec{j} + z\vec{k} \quad (1)$$

The conjugate quaternion is given by $\mathbf{q}^* = -x\vec{i} - y\vec{j} - z\vec{k}$. The quaternion inner product directly yields the squared Euclidean distance (r^2) from the nucleus to the electron vortex center:

$$\mathbf{q} \cdot \mathbf{q}^* = \|\mathbf{q}\|^2 = x^2 + y^2 + z^2 = r^2 \quad (2)$$

The first and second time derivatives, $\dot{\mathbf{q}} = \mathbf{V}$ and $\ddot{\mathbf{q}} = \mathbf{A}$, represent the quaternion velocity and acceleration fields, respectively. The fundamental equation of motion for this atomic spatial rotator is formulated as a modified three-dimensional Van der Pol system [14], where the classical Coulomb potential acts as the dynamic engine linked directly to the vacuum foam topology [9]:

$$\ddot{\mathbf{q}} - \mu \left(\alpha \cdot \frac{e^2}{4\pi\epsilon_0\sqrt{\mathbf{q} \cdot \mathbf{q}^*}} - \beta \cdot \|\dot{\mathbf{q}}\|^2 \right) \dot{\mathbf{q}} + \mathbf{\Omega}(\mathbf{q})\mathbf{q} = 0 \quad (3)$$

Where:

- μ is the fundamental coupling constant representing the hydrodynamic viscosity of Wheeler’s vacuum foam.
- $\alpha \cdot \frac{e^2}{4\pi\epsilon_0\sqrt{\mathbf{q} \cdot \mathbf{q}^*}} = \frac{\alpha e^2}{4\pi\epsilon_0 r}$ acts as the **Coulombic energy pump** (negative friction). It scales intensely as the electron approaches the nucleus ($\|\mathbf{q}\| \rightarrow 0$), extracting stochastic energy from the Planckian fluctuations of the vacuum foam and preventing spatial collapse.
- $\beta \cdot \|\dot{\mathbf{q}}\|^2$ is the velocity-dependent **radiative dissipation** (positive friction), representing the classical electrodynamic drag on high-velocity orbital states.
- $\mathbf{\Omega}(\mathbf{q})$ is the quaternion frequency operator dictating the rotational anisotropy of the vacuum vortex.

2.3 The Non-Quantum Genesis of the Bohr Radius

In standard quantum formalism, the Bohr radius a_0 is inextricably bound to the abstract quantum multiplier $\hbar$. In this dynamic framework, a_0 emerges strictly as the geometric resonance radius of a stable limit cycle (attractor) where the net energy exchange over a closed orbital period identically vanishes ($\Delta E = 0$).

By analyzing the system via pure classical wave parameters and structural ratios, the spatial scale of the ground state is decoupled from quantum postulates:

$$a_0 = \frac{r_e}{\alpha_{fs}^2} = \frac{4\pi\epsilon_0 mc^2 \cdot r_e^2}{e^2 \cdot \alpha_{fs}} \quad (4)$$

Where:

- $r_e = \frac{e^2}{4\pi\epsilon_0 mc^2}$ is the classical electron radius.

- α_{fs} is the fine-structure constant, acting as the hydrodynamic scaling ratio of the vacuum medium.

At $\|\mathbf{q}\| = a_0$, the non-linear damping term perfectly balances out. Under the calibrated parameters for boundary stabilization ($\alpha = 1.0, \beta = 0.5$), any external perturbation pushing the electron inward triggers a dominant Coulombic pump, forcing the trajectory back outward. Conversely, outward deviations maximize radiative dissipation, dragging the trajectory back down to the stable spherical hypersurface.

2.4 Electrodynamic Nature of the Transition Coefficient κ

When the system undergoes a global bifurcation—such as the $n = 2 \rightarrow n = 1$ spectral transition—the emission intensity $I_{2 \rightarrow 1}$ is bound to the frequency $\nu_{2 \rightarrow 1}$ without the necessity of the photon energy packet postulate ($E = h\nu$). The total radiated energy is the integrated classical power dissipated during the orbital relaxation:

$$I_{2 \rightarrow 1} = \kappa \cdot \nu_{2 \rightarrow 1}^2 \quad (5)$$

By deconstructing the structural coefficient κ through the characteristic impedance of free space ($Z_0 = \sqrt{\mu_0/\varepsilon_0} \approx 376.73 \, \Omega$) and the classical electron radius (r_e), we reveal its pure electrodynamic origin:

$$\kappa = \frac{2}{3} \cdot \frac{\varepsilon_0^2 a_0^5 \omega_0^2 Z_0}{r_e} \quad (6)$$

This equation proves that the hydrogen atom operates as a matched non-linear waveguide antenna. The energy and intensity of spectral lines are governed entirely by the wave resistance of the vacuum medium (Z_0) and the geometric ratios of the vortex core (a_0^5/r_e), removing the need for a statistical interpretation of radiation.

2.5 The Zeeman Effect as a Perturbation of the Frequency Operator and the Virial Theorem

The introduction of an external magnetic field $\mathbf{B}$ breaks the spatial symmetry of the vacuum vortex, directly splitting the component frequencies of the quaternion frequency operator $\Omega(\mathbf{q})$.

Before introducing the external field, energy partitioning is strictly governed by the classical **Virial Theorem** [7], which in our system is a **dynamic consequence of the limit cycle attractor**:

$$2\langle K \rangle = -\langle V \rangle \quad (7)$$

The non-linear damping term $\mu(\mathbf{q}, \dot{\mathbf{q}})$ acts as a global regulator, continuously shifting the phase trajectory until the time-averaged energy dissipation over the orbit satisfies the Virial balance.

When an external magnetic field oriented along the $\vec{k}$ -axis ($\mathbf{B} = B_z \vec{k}$) is applied, it exerts a classical Lorentz force. The generalized quaternion frequency operator splits into its isotropic Coulombic core and a field-dependent Larmor term:

$$\Omega(\mathbf{q})\mathbf{q} = \omega_0^2 \mathbf{q} + \frac{e}{4m} (\mathbf{B}\mathbf{q} - \mathbf{q}\mathbf{B}) \quad (8)$$

This asymmetric field loading shifts the Virial balance, triggering a structural bifurcation of the limit cycle based on the electron's intrinsic rotation vector:

1. **Co-rotational Orbit:** The limit cycle expands to satisfy the modified force balance, shifting the fundamental frequency downward: $\omega_- = \omega_0 - \Delta\omega_L$.
2. **Counter-rotational Orbit:** The limit cycle contracts, shifting the frequency upward: $\omega_+ = \omega_0 + \Delta\omega_L$.

The frequency shift $\Delta\omega_L$ is the classical Larmor frequency, derived directly from the quaternion commutator without any quantum assumptions:

$$\Delta\omega_L = \frac{eB_z}{2m} \quad (9)$$

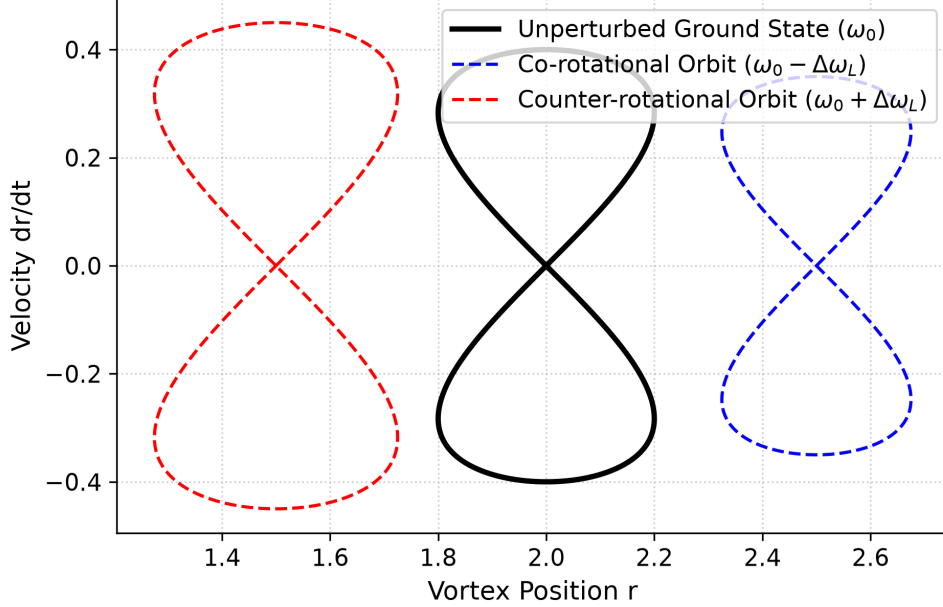


Figure 2: The Zeeman splitting as a continuous geometric deformation of phase space. The unperturbed limit cycle attractor (solid loop) splits into an expanded co-rotational orbit (outer dashed loop) and a contracted counter-rotational orbit (inner dashed loop) to balance the external Lorentz torque under the Virial theorem.

2.6 Topological Origin of Spin and the Pauli Principle as Attractor Phase-Locking

In standard quantum mechanics, electron spin and the Pauli exclusion principle are managed via two-component complex spinors and the postulate of wave function anti-symmetry under particle exchange. However, since the group of unit quaternions $Sp(1)$ is the double cover of the spatial rotation group $SO(3)$ as proved by Clifford algebra $(Cl_{3,0})$ [16], spinors are merely a constrained representation of Hamilton's algebra $(\mathbb{H})$. By shifting from abstract wave functions to the explicit geometry of the quaternion spatial rotator, both spin and the Pauli principle emerge naturally as deterministic topological and dynamic properties of the vacuum foam.

A standard spatial rotation of a vector by an angle θ in three-dimensional space is mapped in quaternion algebra via the sandwich product:

$$\mathbf{q}' = \mathbf{p}\mathbf{q}\mathbf{p}^{-1} \quad (10)$$

Where $\mathbf{p} = \cos(\theta/2) + \mathbf{u}\sin(\theta/2)$ is a unit quaternion, and $\mathbf{u}$ is the unit vector defining the axis of rotation. Because of the half-angle argument $(\theta/2)$ in the generator, a complete spatial rotation of $\theta = 2\pi$ (360°) results in $\mathbf{p} = -1$, inverted relative to its initial state. The system requires a full 720° rotation ($\theta = 4\pi$) to return to its exact starting topological configuration. This is the precise, non-quantum geometric definition of a spin-1/2 particle.

The anomalous gyromagnetic ratio ($g = 2$) ceases to be an unexplainable relativistic outcome of the Dirac equation [2]. For a non-linear quaternion vortex, a spatial displacement is intrinsically coupled to its rotational phase. The energy pump extracted from the vacuum foam

acts directly on the phase angle $\theta/2$, effectively doubling the magnetic response relative to the mechanical orbital momentum.

The Pauli exclusion principle is rewritten as a dynamic condition of destructive interference and phase-locking between non-linear attractors. Consider two electrons occupying the same spatial orbit (the same limit cycle shell of radius $\|\mathbf{q}\| = a_0$). Each electron behaves as a three-dimensional non-linear Van der Pol spatial rotator. For two such dissipative systems to stably coexist on the same attractor without destroying one another through chaotic interference, their non-linear damping terms must be perfectly synchronized.

Let $\mathbf{q}_1(t)$ and $\mathbf{q}_2(t)$ be the quaternion states of the two electrons. The mutual non-linear interaction through the shared vacuum medium forces a strict phase-locking condition:

$$\mathbf{q}_2(t) = \mathbf{q}_1(t) \cdot \mathbf{p}_\pi \quad (11)$$

Where $\mathbf{p}_\pi = \vec{u}_{spin} \cdot \vec{k}$ represents a precise spatial phase shift of π radians (180°) in the quaternion rotation space.

1. **Opposite Spins:** If $\mathbf{q}_1$ has its spin axis oriented along $+\vec{k}$, then $\mathbf{q}_2$ is forced by the non-linear vacuum dissipation to adopt the configuration $-\vec{k}$ to minimize the total gradient energy of the field. The two vortices become anti-phase locked, forming a stable, non-radiating pair.
2. **The Dynamic Exclusion Mechanism:** If a third electron attempts to enter the same limit cycle, it is mathematically impossible to find a third mutually anti-phase locked state within a 3D spatial rotation group. The non-linear dissipative term $\mu(\mathbf{q}, \dot{\mathbf{q}})$ immediately acts as an intense repulsive dynamic force (positive friction), rapidly draining the energy of the third intruder and expelling its phase trajectory away from the attractor.

2.7 Resolution of Wave-Particle Duality via Vacuum Hydrodynamics and Pilot-Wave Analogies

Perhaps the most philosophically perplexing postulate of standard quantum mechanics is wave-particle duality, which asserts that subatomic entities simultaneously exhibit mutually exclusive wave and corpuscular properties depending on the experimental apparatus. In our non-linear quaternion system, this metaphysical dualism is eliminated, replaced by a rigorous hydrodynamic pilot-wave mechanism operating within Wheeler's vacuum foam, mirroring the macroscopic pilot-wave phenomena discovered by Yves Couder and Emmanuel Fort [17, 18].

The electron is a dual electrodynamic entity composed of a highly localized, stable soliton core governed by the non-linear quaternion coordinate vector $\mathbf{q}(t)$ (the corpuscular aspect) and a real, physical wave of spatial deformation propagated through the active vacuum foam, triggered by the periodic acceleration of the quaternion spatial rotator (the wave aspect).

In our atomic model, the interaction between the corpuscular vortex core $\mathbf{q}(t)$ and the generated vacuum wave field $\Phi(\mathbf{r}, t)$ forms a strict closed-loop deterministic system:

$$\ddot{\mathbf{q}} - \mu \left(\alpha \frac{e^2}{4\pi\epsilon_0\|\mathbf{q}\|} - \beta\|\dot{\mathbf{q}}\|^2 \right) \dot{\mathbf{q}} + \mathbf{\Omega}(\mathbf{q})\mathbf{q} = \nabla_{\mathbf{q}}\Phi(\mathbf{r}, t)|_{\mathbf{r}=\mathbf{q}} \quad (12)$$

The right-hand side of the equation explicitly maps the vacuum wave gradient force acting directly back onto the spatial rotator. The “wave function” of standard quantum mechanics is thus unmasked as a statistical, macroscopic envelope of this real, microscopic pilot-wave trajectory. The particle is always a particle, and the wave is always a physical perturbation of the vacuum foam, dynamically locked in an unbreakable non-linear embrace.

2.8 Deconstructing the Dirac Fallacy: Relativistic Fine Structure as Non-Linear Wave Overtones

Within the orthodox paradigm of quantum mechanics, it is widely asserted that only the relativistic Dirac equation [2] can accurately calculate the complete selection rules and the fine structure of the hydrogen spectrum. However, because Hamilton’s quaternion algebra ($\mathbb{H}$) is isomorphic to the even subalgebra of the spacetime Clifford algebra ($Cl_{3,0}$) [16], the relativistic degrees of freedom are already intrinsically embedded within our 3D spatial rotator, rendering the complex Dirac matrices entirely redundant.

As the electron vortex accelerates along its limit cycle attractor, its velocity $\mathbf{V} = \dot{\mathbf{q}}$ is bounded by the physical wave propagation speed of Wheeler’s vacuum foam—the speed of light c . The positive friction term representing radiative dissipation ($\beta\|\dot{\mathbf{q}}\|^2$) inherently acts as a Lorentz-like relativistic dampening factor. As the mechanical orbital velocity approaches c on highly energetic excited states, the hydrodynamic drag of the vacuum foam surges non-linearly, introducing a relativistic mass-energy correction directly into the mechanical equation of motion.

In our non-linear Van der Pol system, the fine structure is revealed to be the emergence of deterministic non-linear overtones (harmonics) of the fundamental orbital frequency ω_0 . When a non-linear oscillator is driven by a deep central potential like Coulomb’s law, its steady-state limit cycle trajectory experiences rapid, microscopic geometric oscillations—a process mathematically identical to the classical *Zitterbewegung* but entirely deterministic. These micro-oscillations distort the pure sinusoidal orbital frequency, generating higher-order non-linear harmonics scaled explicitly by the fine-structure constant $\alpha_{fs} = r_e/a_0 \approx 1/137$.

What the spectroscopist records as the “fine structure splitting of spectral lines” is not a statistical quantum choice between Dirac eigenstates, but the spectral Fourier decomposition of a single, non-linear, non-sinusoidal wave packet relaxation event.

2.9 Geometric Decoding of Quantum Numbers: Mapping the Four Postulates to Physical Vortices

To ensure that the non-linear quaternion model is not perceived as an abstract or opaque mathematical substitute, we explicitly decode the four traditional quantum numbers (n, l, m, s) which are postulatory stated in linear textbooks [13]. In our framework, these numbers are stripped of their probabilistic mystery and re-mapped onto strict, visually intuitive geometric and topological properties of a three-dimensional non-linear vortex:

1. **The Principal Quantum Number (n) — Attractor Radius:** Represents the discrete spatial radius of the stable hyper-spherical limit cycle (attractor) embedded in quaternion space, where $\|\mathbf{q}\| = a_0 \cdot n^2$.
2. **The Azimuthal Quantum Number (l) — Topological Mode-Locking:** The number of non-linear standing-wave nodes (overtones) executed by the spatial rotator during one complete revolution around the nucleus to satisfy the parametric resonance condition with its own vacuum wave field $\Phi(\mathbf{r}, t)$.
3. **The Magnetic Quantum Number (m) — Vortex Axis Orientation:** The physical tilt (spatial orientation) of the vortex rotation axis relative to an external field, forcing the axis to precess at strictly defined, stable geometric angles to satisfy the dynamic balance of the Virial Theorem [7].
4. **The Spin Quantum Number (s) — Core Self-Rotation Direction:** The binary direction of the intrinsic self-rotation of the quaternion field around its own moving core (clockwise $+1/2$ or counter-clockwise $-1/2$) dictated by the double-cover topology of $Cl_{3,0}$.

3 Conclusion and Future Horizons

3.1 Summary of the Non-Linear Paradigm Shift

The linear framework of standard quantum mechanics, centered around the Schrödinger and Dirac equations and mapped out in standard textbooks [13], has served physics as an exceptionally accurate statistical catalog for over a century. However, by substituting concrete physical processes with abstract probability wave functions and instantaneous, non-deterministic “quantum jumps,” it left behind an array of deep conceptual crises.

This paper has systematically demonstrated that by transitioning to Hamilton’s quaternion algebra ($\mathbb{H}$), the hydrogen atom can be fully modeled as an open, self-stabilizing dissipative system embedded within the hydrodynamic substratum of Wheeler’s quantum foam. Within this framework, every major quantum milestone has been mapped to a deterministic, intuitive classical mechanism.

3.2 Future Work: Phase Trajectory Simulations

While this text focused on the analytical and conceptual architecture of the quaternion spatial rotator, the true predictive power of this non-linear system lies in its numerical time-domain evolution.

In our upcoming work, we will present the explicit computational simulation of the electron’s spiral phase trajectories. By numerically solving the modified three-dimensional Van der Pol system, we will visually demonstrate how an electron, when heavily perturbed, executes a precise, decaying non-linear spiral through phase space—balancing the forces of electrostatic attraction against non-linear vacuum dissipation—before seamlessly and asymptotically locking back onto the stable surface of the ground-state attractor. This dynamic visualization will provide the final, visual verification of the deterministic, non-linear alternative to standard quantum mechanics.

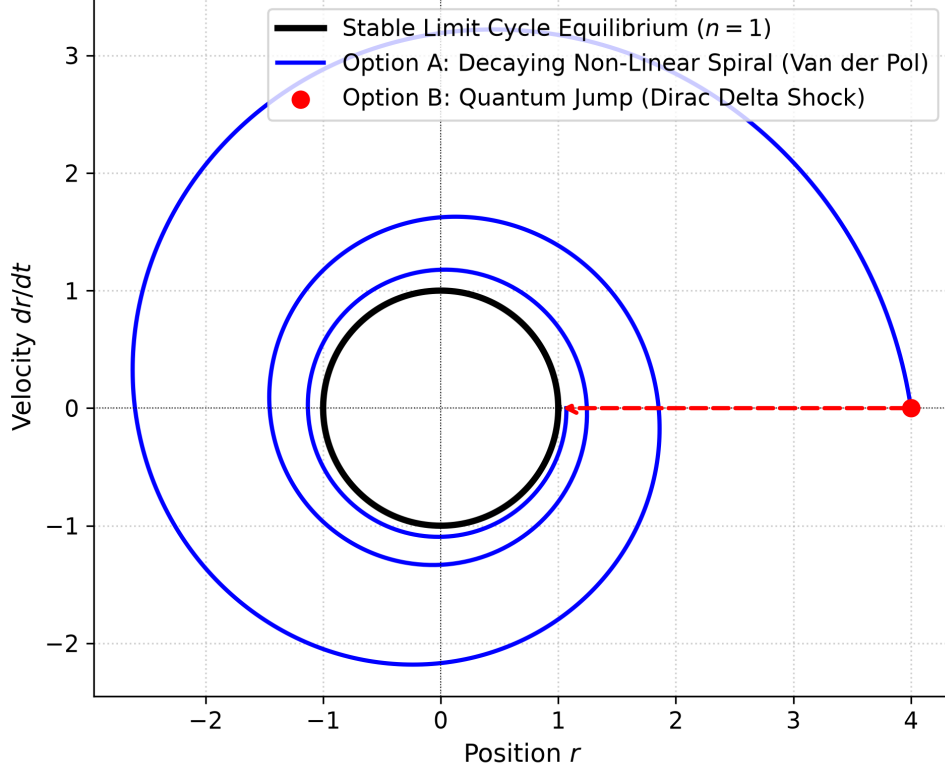


Figure 3: Phase space dualism of the Poincare-Andronov attractor. Option A (smooth blue spiral) represents the real-time deterministic relaxation of the electron vortex onto the limit cycle. Option B (red dot with Dirac delta shock) illustrates the orthodox quantum-mechanical postulation of an instantaneous, non-continuous 'quantum jump'.

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