

MATH ERROR

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Abstract

This paper identifies a century old error in math.

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This paper reports an error in the literature for the past century and more, starting with the Riemann-Siegel Formula. The error is the assumption

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s} \quad (1)$$

The series on the right converges absolutely on the half-plane $\sigma > 1$. Hence any permutation $\psi(n)$ of indices produces the same result for $\sigma > 1$. Thus in general

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{\psi(n)^s} \quad (2)$$

This re-arrangement of terms affects all the results that extend the partial sum $\sum_{n < x} n^{-s}$ into the Critical Strip $0 < \sigma < 1$. With the re-arrangement, the definition of $\zeta(s)$ becomes

$$\zeta(s) = \lim_{N \rightarrow \infty} \sum_{n=1}^N \frac{1}{n^s} + \lim_{N \rightarrow \infty} H(s, N) \quad (3)$$

Here $H(s, N)$ is the change the re-arrangement of terms does to the original series $\sum_{n=1}^N n^{-s}$. It is a sum of the selection of terms n^{-s} . Currently, not much is known about $H(s, N)$. Clearly, in the half-plane $\sigma > 1$, one finds $\lim_{N \rightarrow \infty} H(s, N) = 0$ due to the absolute convergence of $\sum_n n^{-s}$. By the results of [2] and [3], it is clear that $H(s, N) \neq 0$ in the Critical Strip in general, nor in the limit.

One of the affected results is [1, eq.3.5.3]. Instead of

$$\zeta(s) - \sum_{n=1}^N \frac{1}{n^s} = s \int_N^{\infty} \frac{\frac{1}{2} - \{x\}}{x^{s+1}} dx + \frac{N^{s-1}}{s-1} - \frac{N^{-s}}{2} \quad (4)$$

it should read

$$\zeta(s) - \sum_{n=1}^N \frac{1}{n^s} - H(s, N) = s \int_N^{\infty} \frac{\frac{1}{2} - \{x\}}{x^{s+1}} dx + \frac{N^{s-1}}{s-1} - \frac{N^{-s}}{2} \quad (5)$$

This changes all the subsequent results in the Critical Strip that involve the partial sum $\sum_{n=1}^N n^{-s}$.

References

- [1] E.C. Titchmarsh: *The Theory of the Riemann Zeta-Function*, Oxford University Press (1986), Second Edition
- [2] I. Hrnčić: *Disproof of the Riemann Hypothesis*, <https://www.vixra.org/abs/2508.0055>
- [3] I. Hrnčić: *Disproof of the Riemann Hypothesis 2*, <https://www.vixra.org/abs/2608.0011>