

Distributivity in Relationship Theory and Other Abstract Relational Structures

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Abstract

The property of distribution allows the mathematical understanding of two elements that have no relation to each other but are both related to a common element. Indeed, it can be generalized to describe any number of elements that have no relation to each other though they are all related to a common element. From accepting the distributive property of relationships meaningful relationships arise that explain reality. Reality says that compound elements can break up into new relationship, this paper shows that with the proper notation, proper symbolism. Relationship theory can explain what is in a box and table as well as what happens in particle colliders. The abstraction of relationships allows the comparison of relationships between diverse phenomena. The reality of human divorce in terms of relationships is no different from many chemical relationships in terms of relationship theory, they are the same in a mathematical abstract concept, as abstractions they have the same mathematical structure. This abstraction is needed for those who desire to understand how the universe operates. Applications of the theory are given with everyday realities like putting things in boxes and on tables. Since existence is about relationships, mathematics needs the abstraction of relationships, and this paper successfully lays that foundation.

Keywords: relationship theory, relational algebra, set-theoretic abstraction, abstract relational structures, mathematical foundations

Subject classification: 03E99 03E75 97E60

1. Introduction

The paper that introduced the concept of relationship of things in an abstract manner, “A formal Mathematical Framework for Relationships”, showed relationships with the property of commutativity. The property of being commutative does not take into account the full range of relationships. Take humans two humans who are on the other side of the Earth, they have no relationship with each other. Both though have a relationship with the Earth.

The aims of this paper are to introduce the property of distribution to the abstractions of relationships. The distributive property allows relationships theory to illustrate how two humans on the other side of the earth to both have a relationship with the Earth but no relationship with each other. Relationships theory has arisen from the need to solve a practical problem, the need to create a mathematical framework for relationships. It is thus vital that it can describe elements that are themselves not related but are all related to a common element.

The practical problem was to understand relationships in an abstract manner given that everything that exists is in a relationship, and that fact nothing can escape it. This maths is needed for those who desire to understand how the universe operates.

What does introducing the property of distribution lead to? As shall be seen, it leads to the reality of more types of relationships that can be explained with the right type of mathematical notation, compound elements can be broken up when they get into a relationship, this is the reality of observation. A mathematical relationships theory must make effort to make an abstraction of all types of relationships in a consistent manner.

The significance of this paper is that it further generalizes the concept of relationships and more types of relationships within a mathematical framework. Relationships theory is a mathematical framework that comes from reality and hence must strive to explain all relationships in a mathematically logical manner given that everything is in a relationship. This paper further adds to mathematically understanding more relationships when using appropriate symbolism.

One will understand that a lion hunting and killing a zebra has a similar mathematical structure as what happens in particle colliders. The mathematical concepts developed go into showing how this abstraction manifests in common relationships like putting things on a table. This paper will show the foundations of reality, relationships, in an abstract manner.

2. What Has Been Established

This section summarizes the key ideas established from the paper that introduced relationship theory “a formal mathematical framework for relationships”.

2.1. An Element

An element E is a thing, anything.

$E_1, E_2, \dots E_{n-1}, E_n \in \mathbb{E} = \sum E_i$

$\mathbb{E}$ is the set of all elements, all things.

$E_i \neq E_j$ no two elements are the same.

r = relationship it is an operator

An element getting into a relationship with itself

$E_1 r E_1 = E_1 \Rightarrow$ an element is its own identity.

A singular element E_i can be considered as a set of one

$\{E_i\} = E_i$

E_i can also be considered as a subset of the universal set

$E_i \subset \mathbb{E}$

2.2. Two Element Relationship

A two-type element relationship is of the type $E_1 r E_2 = \{E_1 E_2\}$ and can be expressed as

$\{E_i E_j\} \subset \mathbb{E}$ or $\{E_i E_j\} \in \mathbb{E}$

$\{E_i E_j\}$ is its own unique element such that

$\{E_i E_j\} \neq E_i \neq E_j$

Letters of the alphabet can be used for ease of visuals. There are more than 26 elements hence the use of E_i

$A, B \in \mathbb{E}$

$A \neq B$

$A r B = \{A B\}$

The process is commutative

$A r B = B r A = \{A B\} = \{B A\} \in \mathbb{E}$

2.2.1 Two Element Inverse Relationship

r^{-1} = inverse relationship it is an operator

$$\{A r^{-1} B\} = \{B r^{-1} A\} = A, B$$

2.3. Three Element Relationship

2.3.1. Basic Relationship

A basic three type element relationship is of the type $E_1 r E_2 r E_3 = \{E_1 E_2 E_3\}$ and can be expressed as

$$\{E_i E_j E_k\} \in \mathbb{E}$$

$\{E_i E_j E_k\}$ is its own unique element such that

$$\{E_i E_j E_k\} \neq E_i \neq E_j \neq E_k$$

Letters of the alphabet can be used

$$A, B, C \in \mathbb{E}$$

$A \neq B \neq C$ all elements are unique.

$$A r B r C = \{A B C\}$$

The process is commutative, it does not matter the order.

2.3.1.2 Basic Three Element Inverse

$$\{A B C\} r^{-1} = A, B, C$$

2.3.2 Relationship of the Type $\{E_i \{E_j E_k\}$ Lack of Apparent need for Associativity

There is no associativity. The reason is because of the way associativity is defined. All maths operations are defined as brackets or parentheses first. Associativity is defined mathematically as

$$(a + b) + c = a + (b + c) \text{ in more abstract terms}$$

$$(a \bullet b) \bullet c = a \bullet (b \bullet c) \text{ where}$$

$\bullet$ is an operation

Relationship theory is about things. $A r B = \{A B\}$ and it is a new element. Say $\{A B\} = Z$ Then

$$C r \{A B\} = C r Z = Z r C = \{Z C\}$$

$$Z = \{A B\}$$

$$\therefore Z r C = \{Z C\}$$

$$\text{Since } Z = \{A B\}$$

$$\Rightarrow Z r C = \{\{A B\} C\}$$

$$\{\{A B\} C\} \neq \{A B C\} = \{C A B\} \text{ because of commutative property}$$

A relationship happens in a moment. Associativity implies one relationship first and then another. Reality of putting three elements say in a container, they all get into a relationship in that moment.

2.3.3 Relationship of type $\{E_i E_k E_j E_k\}$

$$2C r \{A B\} = \{AC BC\}$$

$$\{AC BC\} = \{\{A C\} \{B C\}\} \neq \{C \{A B\}\} = \{\{A B\} C\} \neq \{A B C\} = \{B A C\}$$

A conundrum because $\{A B\}$ is its own element. There is the difficulty of mathematics, a thousand pages or more can be written. It is however clearly an element with parts as most things are made of parts.

$$\{A B\} \in \mathbb{E}$$

$$\{A B\} \subset \mathbb{E}$$

The above logic leads to a generalization

$$(n-1)E_1 r \{E_2, E_3, \dots, E_{n-1}, E_n\} = \{E_1 E_2, E_1 E_3, \dots, E_1 E_{n-1}, E_1 E_n\}$$

3. The Distributive Property and Relationships

3.1 3 Element example

The distributive property of relationships, are relationships of the type $\{E_k [E_i, E_j]\}$ for example

A = a can of tomatoes

B = a cellphone

C = a table

The can of tomatoes, A, is placed on top of the table. Thus, the can of tomatoes has a relationship with the table, C.

$$A r C = \{A C\}$$

The cellphone, B, is also placed on the table and it too has a relationship with the table.

$$B r C = \{B C\}$$

These two relationships can be considered as a set of things with a relationship with the table.

$$\{\{A C\}: \{B C\}\} = \mathfrak{e} \in \mathbb{E}$$

$$\{A C\} \in \mathfrak{e}$$

$$\{B C\} \in \mathfrak{e}$$

An element of something can be considered as a subset of that thing.

$$\{A C\} \subset \mathfrak{e}$$

$$\{B C\} \subset \mathfrak{e}$$

The table is common to both relationships in $\mathfrak{e}$

$$\{C[A: B]\}$$

The use of the colon, is to denote that A and B are not themselves in a relationship, what they have in common is that they are both in a relationship with C.

The cellphone is taken off the table and is therefore no longer in a relationship with the table

$$\{C[A: r^{-1} B]\} = \{C[A: B]\} r_b^{-1} = \{A C\} B$$

Note

$$\{\{A C\}: \{B C\}\} \neq \{\{A C\} \{B C\}\}$$

$\{\{A C\}: \{B C\}\}$ implies there is one C and A and B are both related to this one C. and hence we get $\{C[A: B]\}$

$\{\{A C\} \{B C\}\}$ implies that there are two C's, see 2.3.3 above. It does make sense to simplify where possible. It is still logical to say

$$\{\{A C\} \{B C\}\} = \{C(A B)\}$$

$$\{C(A B)\} \neq \{C[A: B]\}$$

Thus, generalization becomes

$$(n-1)E_1 r \{E_2, E_3 \dots E_{n-1} E_n\} = \{E_1 E_2, E_1 E_3, \dots, E_1 E_{n-1}, E_1 E_n\}$$

$$= \{E_1(E_2 E_3 \dots E_{n-1} E_n)\} \neq \{E_1[E_2: E_3: \dots: E_{n-1}: E_n]\}$$

3.2 4 elements

A = a can of tomatoes

B = a cellphone

C = a remote control

D = A table

The can of tomatoes, A, the cellphone, B, and the remote control are placed on the table, each having a relationship with the table but not with each other.

$$A \text{ r } D = \{A D\}$$

$$B \text{ r } D = \{B D\}$$

$$C \text{ r } D = \{C D\}$$

All three sets above belong to the bigger set $\mathbb{e}$ of all things having a relationship with the table

$$\{\{A D\}: \{B D\}: \{C D\}\} = \mathbb{e} \in \mathbb{E}$$

The table, D, is common to all relationships, thus set $\mathbb{e}$ can be defined as

$$\{D[A: B: C]\}$$

The cellphone is removed from the table and is no longer in a relationship with the table.

$$\{D[A: r^{-1}B: C]\} = \{D[A: B: C]\}r_b^{-1} = \{D[A: C]\} B$$

3.3 Adding some complexity to 4 elements

A = water

B = a cup

C = a cellphone

D = a table

The water, A, is poured into the cup, B. The water and the cup have a relationship

$$A \text{ r } B = \{A B\}$$

The cup with water is then placed on the table and thus there is a relationship with the table

$$\{A B\} \text{ r } D = \{\{A B\} D\}$$

The cellphone, C, is also placed on the table, thus the cellphone has a relationship with the table

$$C \text{ r } D = \{C D\}$$

$\mathbb{e}$ is the set of things having a relationship with that table

$$\{\{\{A B\} D\}: \{C D\}\} = \mathbb{e}$$

The table is common to all relationships, thus $\mathbb{e}$ can be written as

$$\{D[\{A B\}: C]\}$$

3.3.2 removing C from common relationship

The cellphone C, is removed from the table thus it is no longer has a relationship with the table

$$\{D[\{A B\}: r^{-1}C]\} = \{D[\{A B\}: C]\}r_c^{-1} = \{\{A B\} D\} C$$

The cup containing water is removed from the table thus no longer in a relationship with the table

$$\{D[r^{-1}\{A B\}: C]\} = \{D[\{A B\}: C]\}r_{ab}^{-1} = \{C D\} \{A B\}$$

3.4 Further Abstraction example

Though coming from abstraction of reality, the need to understand relationships, any mathematical theory at the end is abstraction, and abstraction can be taken as far as logic allows. Take the reasoning below.

$\{F[\{A\ B\}: C: \{D\ E\}]\}$
 $\{M[\{G\ H\ I\}: \{J\ K\ L\}]\}$
 $\{Q[N: O: P]\}$
 $\{W[R: S: \{T\ U\}: V]\}$
 X, Y

F, M, Q, W, X Y are themselves related to Z. Thus the set of things related to Z can be illustrated as
 $\{Z[\{F[\{A\ B\}: C: \{D\ E\}]\}: \{M[\{G\ H\ I\}: \{J\ K\ L\}]\}: \{Q[N: O: P]\}: \{W[R: S: \{T\ U\}: V]\}: X: Y]\}$

3.4.1 Application

The above is pure abstraction, however, these more complex relationships do exist.

A = tape measure
 B = speed square
 C = claw hammer
 D = chisel
 E = utility knife
 F = Box 1

A, B C, D, E are all put into box 1, F thus there exist the following relationships as all these things will have a relationship with box 1

$A\ r\ F = \{A\ F\}$
 $B\ r\ F = \{B\ F\}$
 $C\ r\ F = \{C\ F\}$
 $D\ r\ F = \{D\ F\}$
 $E\ r\ F = \{E\ F\}$

There exists a set @1 of all things in box 1

$\{\{A\ F\}\ \{B\ F\}\ \{C\ F\}\ \{D\ F\}\ \{E\ F\}\} = @_1$

Box 1 is common to all elements thus

$\{F[A: B: C: D: E]\} = @_1$

There are other elements

G = screwdriver
 H = wood glue
 I = small clamp
 J = 16oz rubber mallet
 K = box 2

G, H, I, J are all put into box 2, K thus there exists the following relationships as all these elements will have a relationship with box 2

$G\ r\ K = \{G\ K\}$
 $H\ r\ K = \{H\ K\}$
 $I\ r\ K = \{I\ K\}$
 $J\ r\ K = \{J\ K\}$

There exists a set @2 of all things in box 2

$$\{\{G\ K\} \{H\ K\} \{I\ K\} \{J\ K\}\} = \mathcal{E}_2$$

Box 2 is common to all elements thus

$$\{K[G: H: I: J]\} = \mathcal{E}_2$$

There are three more elements

L = circular saw

M = reciprocating saw

N = table

Box 1 is placed on the table creating a relationship with the table

$$\{F[A: B: C: D: E]\} \text{ r } N = \{\{F[A: B: C: D: E]\} N\}$$

Box 2 is placed on the table creating a relationship with the table

$$\{K[G: H: I: J]\} \text{ r } N = \{\{K[G: H: I: J]\} N\}$$

The circular saw is placed on the table creating a relationship with the table

$$L \text{ r } N = \{L N\} = \mathcal{E}_3$$

The reciprocating saw is placed on the table creating a relationship with the table

$$M \text{ r } N = \{M N\} = \mathcal{E}_4$$

There is a set $\mathcal{E}_5$ of elements having a relationship with the table, N.

$$\{\{F[A: B: C: D: E]\} N\} \{K[G: H: I: J]\} N\} \{L N\} \{M N\} = \mathcal{E}_5$$

As the table N is common relationship

$$\{N[\{F[A: B: C: D: E]\}: \{K[G: H: I: J]\}: L: M]\}$$

3.4.2 Inverse relationship

Take $\{N[\{F[A: B: C: D: E]\}: \{K[G: H: I: J]\}: L: M]\}$

The wood glue, H, is taken out of box 2 and put on the table separately, such that

$$\{N[\{F[A: B: C: D: E]\}: \{K[G: r^{-1} H: I: J]\}: L: M]\} = \{N[\{F[A: B: C: D: E]\}: \{K[G: I: J]\}: H: L: M]\}$$

The relationship could be such that the wood glue, H, is taken out of box 2 and placed somewhere else outside the table such that H has no relationship with that particular table

$$\{N[\{F[A: B: C: D: E]\}: \{K[G: r^{-1} H: I: J]\}: L: M]\} = \{N[\{F[A: B: C: D: E]\}: \{K[G: I: J]\}: L: M]\} \{H\} \text{ or } \{N[\{F[A: B: C: D: E]\}: \{K[G: r^{-1} H: I: J]\}: L: M]\} = \{N[\{F[A: B: C: D: E]\}, \{K[G: I: J]\}: L: M]\} H$$

4. Capture type relationships

Capture type relationships can be described as

$$\{E_i E_j\} \text{ r}_{E_j} E_k = E_i \{E_j E_k\}$$

There are two humans A and B and they get into a relationship.

$$A \text{ r } B = \{A B\}$$

C is another human and they seduce B causing a split between A and B

$$\{A B\} \text{ r}_b C = A \{B C\}$$

4.1 Extension of capture type relationships

There are three humans, A, B, and C. They get into a relationship

$$A \text{ r } B \text{ r } C = \{A B C\}$$

D is another human and she seduces C causing C to break from this relationship
 $\{A B C\} r_c D = \{A B\} \{C D\}$

Mathematics is about abstraction. There can be two human beings in a relationship, A and B

$A r B = \{A B\}$

Another two human beings C and D get into a relationship

$C r D = \{C D\}$

A seduces B, and D seduces B

$\{A B\} \{r_a C: r_b D\} = \{A C\} \{B D\}$

This need not be humans, it could be nations, military commanders in history have been known to change sides.

Chemical reactions are known to capture certain elements. The principle is abstract, but it applies to all such types of relationships.

5. Catalyst type relationships

The catalyst type of relationship is the type

$\{E_i E_k\} r E_j = E_i E_k E_j$

An example can be

Take four elements, A, B, C, and D. They are in a relationship say

$\{A B C D\}$

A catalyst E is introduced that induces one of the elements to break free from the relationship

$\{A, B, C, D\} r E = \{\{A B, C\} D E\}$

The catalyst itself is not affected in the relationship.

Another type of catalyst relationship is that things can not react together unless a catalyst is introduced such that

$\{A B C\} r D = \{A B C\} D$

A catalyst E is introduced

$\{A B C\} r D r E = \{\{A B C D\} E\}$

If catalyst is there $\{A B C D\}$ can exist but the catalyst does not react and remains E.

1 000's of pages can be written, aims of this paper is to introduce key principles. Do not forget mathematics can be drawn out into incredible abstraction. These are examples to understand the basic principles.

Take for example

$\{D[A: B: C]\}$

A catalyst E is introduced

$\{D[A: B: C]\} r E = \{D[A: B] C E\}$

Note the catalyst though in the relationship remains E but in the set.

6. Annihilation type relationships

If a lion feeds on Zebras and it catches a zebra, the zebra will cease to exist

A = lion

B = zebra

$$A \text{ r } B = A \{b_1, b_2, \dots, b_n\}$$

$$b_i \in B$$

b_i are the parts that make up B. The lions do not eat the entire zebra. Some of the zebra's blood goes into the soil, the bones are not eaten by the lions. Because the lion eats some of the zebra not all, it would be more technically correct to say

$$A \text{ r } B = \{A \ b_i\} \ \{b_n - b_i\} \text{ where}$$

$\{b_n - b_i\}$ are the parts the lions do not eat.

Zebra might survive

$$A \text{ r } B = A \ B$$

6.1 Further abstraction of lion zebra relationship

Returning to the lion killing the zebra

$$A \text{ r } B = A \{b_1, b_2, \dots, b_n\}$$

This reasoning can be extended in an abstract manner to

$$A \text{ r } B = \{a_1, a_2, \dots, a_n\} \ \{b_1, b_2, \dots, b_n\}$$

$$a_i \in A, b_i \in B$$

The above can be extended to relationship of type

$$E_{i1} \text{ r } E_{i2} = \{e_1, e_2, \dots, e_n\}$$

Why $\{e_1, e_2, \dots, e_n\}$

This is because E_i has the same structure and is built up from the same structure as opposed to say E_j .

To clarify

$$A_1 \text{ r } A_2 = \{a_1, a_2, \dots, a_n\}$$

In practical terms A can be considered as neutrons.

$$A_i \in A = \text{set of neutrons } \{A_1, A_2, \dots, A_n\}$$

A_1 is a neutron as well as A_2 . They are smashed together in a particle collider and break into their constituent parts hence

$$A_1 \text{ r } A_2 = \{a_1, a_2, \dots, a_n\}$$

6.1.1 relationship of constituent parts

Given

$$A_1 \text{ r } A_2 = \{a_1, a_2, \dots, a_n\}$$

Take section 3.4.1 above. The tape measure from box 1 could be taken out and placed in box 2 thus creating a new relationship. Or the tape measure could be put on top of the utility knife inside box 1, thus creating a relationship within box 1.

Given

$$A_1 \text{ r } A_2 = \{a_1, a_2, \dots, a_n\}$$

The a_i 's can have relationships with each other.

Thus

$$a_i \text{ r } a_j = B$$

$$a_i \neq a_j$$

$$B_i \neq A_i$$

To be technically/ logically clear

$a_i r a_j = B \{a_n - \{a_i r a_j\}\} = B \{a_n - B\}$ where
 $\{a_n - B\}$ are the elements that do not go into creating B

The reality of this within particle colliders is neutrons can be smashed together, and a myriad of other particles come to be from the smashed constituents.

Conclusion

This paper set out to build on the basic principles set out in the paper “A Formal Mathematical Framework for Relationships”. It has done that though still leaving relationship theory at its infancy. However, the development has been substantial in the conceptualization of relationship theory. It is substantial by abstraction of relationships, showing that the principles of a chainsaw on the table are not far of from what happens in particle colliders. In abstract language of relationship theory. In showing that relationships have similar mathematical structure, this paper has done its job, though abstraction itself can be taken far from what exists. Relationships have similar structure even with things that seem far removed from each other.

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