Neglecting the Test Body's Effect on the Gravitational Source as a Cause of the Breakdown of Energy-Momentum Conservation

Mueiz Gafer KamalEldeen, Nov. 2026

Abstract

The aim of this brief analysis is to clarify and demonstrate that the effect of a test body on the source of a gravitational field is no less important than the effect of the source on the test body when studying conservation laws. This fact does not depend on—and is entirely unaffected by—the ratio of the test body's mass to the mass of the source. The frequently repeated statement that a gravitational field violates conservation laws, particularly the laws of conservation of momentum, is incorrect. Rather, neglecting the effect of the test body on the gravitational source is what undermines the theoretical foundation on which these laws rest.

Introduction

It is well known that, when studying gravitation, we assume that the mass of the test body is very small compared with that of the source of the field. There is nothing wrong or problematic about this. We consequently conclude that the effect of the test body on the source can be neglected in practice, and also theoretically, as a limit approached as the ratio between the two masses decreases. There is likewise no problem with this as long as we are studying a matter concerning what happens to the gravitational source. However, the situation is different when the subject involves calculating quantities exchanged between the body and the source. In this case, observing the transfer of even the smallest amount of the quantity whose conservation is being investigated from the body to the source is fundamentally different from considering that quantity to have been lost in an unknown manner or to be nonconserved. This is clear, yet it is rarely noticed.

With this clarification in mind, we can readily demonstrate the existence of complete conservation laws, not only for energy but also for momentum, after making modifications to the mathematical structure of their definitions: from their familiar form in the absence of gravity, within the domain of special relativity, to a new form suited to their new setting—the presence of a gravitational field, whose study requires general relativity.

Space is not Symmetric in Every Dimension but it is Symmetric as a Whole

The claim that there is no spatial symmetry permitting a law of conservation of momentum in a gravitational field[1] is incorrect, because this claim effectively equates the absence of simple symmetry with complete chaos and disorder. Space in a gravitational field is regular and simple with respect to time. For this reason, there is a simple conservation law for the time-related component in the matrix that distributes energy and momentum among the dimensions of spacetime. With respect to space, however, it is not equally simple, but it is nevertheless regular. If there is no conservation law in each individual dimension, then there is a conservation law for the dimensions taken together. In general, we can say that, for every form of regularity in spacetime, there is a law of conservation of energy and momentum corresponding to the degree of that regularity and complexity. Even the expansion and contraction of space have conservation laws expressing their regularity and homogeneity.

Definitions of Energy-Momentum Components [2]

Let us define the Energy-Momentum Matrix as:

$$\mathbf{B}_{pq} = \mathbf{m}c^2\left(\frac{dt}{d\mathcal{T}}\right)^2\left(g_{pq}V^pV^q - \Delta\, g_{pq}U^pU^q\right)$$

p=0,1,2,3 and q=0,1,2,3

The B_{00} is the temporal component or Energy E_g:

$$\mathbf{E}_g = \mathbf{m}c^2\left(\frac{dt}{d\mathcal{T}}\right)^2\left(g_{00}V^0V^0 - \Delta\, g_{00}U^0U^0\right)$$

B_{ij} are the spatial components and their sum is the total momentum P_g;

$$\mathbf{P}_g = \sum_{i,j=1,2,3} \mathbf{m}c^2\left(\frac{dt}{d\mathcal{T}}\right)^2\left(g_{ij}V^iV^j - \Delta\, g_{ij}U^iU^j\right)$$

Important Clarification Regarding the Quantities Included in the Definitions of Energy and Momentum

***t* is time as measured by an observer very far and stationary reative to gravitational source**
$\mathcal{T}$ is the proper time of the test body
V^p is the velocity of the test body as measured by an observer very far and stationary rekative to the gravitational source

$\Delta\, g_{pq}U^pU^q$ This quantity represents the gravitational field at the point where the body is located, as determined by calculating the changes that occur in the 16 components of $g_{pq}U^pU^q$ composed of metric and velocityU^p of a body falling from infinity, or from a very distant location, until it reaches the specified point.

To calculate this quantity at a given point, we assume that a body is initially located very far from the gravitational source and begins to fall freely until it reaches the specified point. The initial value of this quantity is zero in all its components, except for the time-time component, which is equal to one, because U^0 eqial to one and metric is also equal to the identity metric at infinity.

As the body approaches the specified point, its other velocity components change (they become the escape velocity components), and the metric also changes.

The delta symbol here therefore represents the difference between the value of this quantity for a body at rest at infinity and its value when the body reaches the specified point.

Subtracting this quantity from the (original0 quantity $g_{pq}V^pV^q$ is a reasonable and physically meaningful procedure, because in this way we isolate the contributions of the gravitational field to this quantity, leaving only the part that has no reason to change. Consequently, the conservation law is satisfied, and this can indeed be demonstrated mathematically.

If we apply these definitions, for example, in Schwarzschild spacetime, we will clearly find that the quantity E_g reduces to the well-known conserved energy E[3] because $\Delta\, g_{00}U^0U^0$ is zero This does not mean that we have merely reproduced an already known result; rather, we have expressed the energy in a form that allows it to be generalized to the other dimensions. This generalization provides us with analogous definitions for momentum, having the same mathematical structure as the energy, but differing from it only in their association with the spatial dimensions rather than the temporal dimension.

The author has previously published material on the idea behind these laws, for example in [reference missing in the original]. However, all previous publications, including this version, although sharing the same general form, contain some errors, either in the mathematical formulation of the law or in the precise interpretation of some of its variables, such as the state and position of the observer measuring the different velocities. The entire formulation has finally been reviewed with great precision in this version. The artificial intelligence tools that have recently emerged have helped me greatly in this regard, particularly through their ability to detect mathematical inconsistencies and perform the calculations necessary to verify the validity of the laws.

Law of Conservation of Energy

The quantity E_g for each body remains constant along any geodesic path in a static gravitational field.

The validity of this law can be demonstrated by applying the geodesic conditions to the definition.

Law of Conservation of the Sum of the Momentum Components

The sum of the momentum components—that is, the nine purely spatial components of the matrix—remains constant when the body moves radially at escape velocity.

At first glance, this appears to be a restriction that diminishes the importance of the law and narrows its scope. However, considering the fact that a body moving in a gravitational field affects the gravitational source and exchanges amounts of momentum with it, what we initially regarded as a restriction is merely a special case of a law applicable to all geodesic motions of the body. This is because any case other than motion at escape velocity involves an interaction between the test body and the gravitational source.

Even if the amount of momentum transferred is small relative to the gravitational source, it is important and consequential for the body. The subject of conservation laws is to account for what each body possesses, what it has lost or gained, and the consistency of this accounting with what remains—not the extent to which one of the bodies responds to the addition or subtraction of that quantity.

Even from this perspective, however, the degree of the effect is the same for the body and the source, as is known from Newton's third law and its generalizations. There is no difference between the large body and the small one in this respect, because any deviation of the source from its constant velocity relative to distant stars is associated with an equal change in the momentum of the test body, whose velocity changes by a large and noticeable amount.

The only case in which no momentum exchange occurs between the orbiting body and the gravitational source is when the body moves radially at a speed equal to the escape velocity. In this case, the body's momentum equals zero in all components, so the body has gravitational energy without any momentum.

There is another case, but it is different: the case of the body rotating in a circular orbit. In this case, we observe that the sum of the components of momentum is constant, while each individual component is not constant. Therefore, this type of variation does not mean that the source does not interact with it at all; rather, it simply means that similar changes occur in the source itself, so that the sum of the components of its momentum is conserved.
An example that illustrates this is the effect of the Moon's rotation on Earth, such as the phenomena of tides and the disturbance of Earth's position in space. This means that

neither the Moon loses momentum nor does Earth gain it; rather, part of the momentum components is transferred in each of them from one component to another.

Conclusion

The laws of conservation of energy and momentum hold without exception in all known physical circumstances, but this becomes apparent only under two conditions: first, that the calculations include all interacting bodies; and second, that the possibility of the transfer of energy–momentum from one component to another is taken into consideration

References

[1] See D Robert M Wald. General Relativity. University of Chicago Press, 1984.

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[3] More about this important quantity in : J B Hartle, Gravity . An Introduction to Einstein's General Relativity, Addison Wesley 2003.